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THE DIRECTOR OF PUBLIC INSTRUCTION, EASTERN BENGAL AND ASSAM,
as an Alternative Text-book for Aided Schools
IN THE THREE DIVISIONS OF EAST BENGAL.
Vide—E. B. & A. Gazette, Sept. 2, 1908

THE

Elements of Arithmetic

BY

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Ninth Edition.

PUBLISHED BY
N. C. DUTTA & Co.,
32, COLLEGE STREET, CALCUTTA.

1908.

Calcutta:
PRINTED BY M. C. CHAKRAVARTI,
COLLEGE PRESS:
117/1, BOW BAZAR STREET.

PREFACE.

:—o—:

The 'Elements of Arithmetic' will be found to differ in certain respects from the many excellent treatises which are already in the field. In respect of the arrangement of Chapters, care has been taken to make it really progressive, so that the student may have no difficulty in passing from the elementary to the more advanced portions of the work. With this end in view, the same subject has in many cases been treated of in more places than one, the several stages of progress being thereby made both easy and convenient.

The proper way to grasp firmly the principles of arithmetic and to acquire the necessary amount of accuracy, clearness and ingenuity, is to study and master a large number of different types. It is hoped that the number and variety of the types worked out as well as of the examples set for exercise will be found amply sufficient for the purpose.

Some importance has been attached to the subject of Mental Arithmetic, the study of which appears to be unduly neglected in our public schools.

Chapters 37 and 39 contain subjects which are not usually found in works of a similar kind. They will be found to be of considerable use to the student, both in theory and practice, and will, it is hoped, form a valuable feature of the book.

The Unitary Method, which is so highly appreciated by experienced teachers as being calculated to bring out the intellectual powers of the pupil and strengthen his reasoning faculties, has been sufficiently developed and elucidated. At the same time the old Method of Proportion has also been retained, as it is very useful in certain classes of problems, especially in the way of simplifying labour and abridging work.

In a book dealing with so many figures, it has been found impossible to keep clear of errors and inaccuracies. It is hoped that in the next edition, such errors and inaccuracies will be altogether avoided or, at any rate, minimised. Meanwhile any correction or suggestion communicated to me will be gratefully received.

In conclusion I have to thank Babu Narendra Lal De, M.A., Professor, General Assembly's Institution, for having kindly revised the proofsheets; Babu Kisori Mohan Sen Gupta, M. A., Lecturer, Hughly College, for having tested many of the answers; and Babu Nivaran Chundra Ghosh, Mathematical Teacher, Calcutta Institution for the invaluable help which I have received from him both in preparing the work and in getting it through the press.

Calcutta : February, 1893. B. V. GUPTA.

PREFACE TO THE SECOND EDITION.

The few alterations that have been made here and there will, it is hoped, enhance to some extent the usefulness of the work. An earnest endeavour has also been made to remove all the errors and inaccuracies that had unavoidably crept into the first edition.

The author gladly takes the present opportunity to express his warm acknowledgments to those heads of institutions who have kindly introduced the book into their schools. His thanks are also due to those teachers and students who have taken the trouble to point out its errors and offer suggestions for its improvement.

Calcutta : March, 1894.

PREFACE TO THE FOURTH EDITION.

In this edition also, the author has made a few additions and alterations Such of the University Examination Papers as have now grown almost out of date have been omitted, all the interesting and important questions being, however, retained On the other hand, a few specimen papers from the Previous Examinations of the Cambridge University have been added and these will, it is believed, more than make up for the papers taken out.

Calcutta : January, 1897.

PREFACE TO THE SIXTH EDITION.

Some new examples of an elementary character have been added in this edition to afford more exercise to the student in the mechanical portion of the subject The arrangement however has not been in any way interfered with the new examples standing by themselves in separate groups at their proper places in the book.

Problems marked with asterisks may be omitted at the first reading.

Purulia : August, 1901.

PREFACE TO THE NINTH EDITION.

The Metric System has been fully treated in this edition and added at the end of the book with numerous examples Some new examples have also been added in different parts of the book without interfering with the previous arrangements.

Cuttack : November, 1908.

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THE COM-

(for Addition Subtrac-

1	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
2	1	3	5	7	9	11	13	15	17	19
	2	*4*	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*
3	1	4	7	10	13	16	19	22	25	28
	2	5	8	11	14	17	20	23	26	29
	3	*6*	*9*	*12*	*15*	*18*	*21*	*24*	*27*	*30*
4	1	5	9	13	17	21	25	29	33	37
	2	6	10	14	18	22	26	30	34	38
	3	7	11	15	19	23	27	31	35	39
	4	*8*	*12*	*16*	*20*	*24*	*28*	*32*	*36*	*40*
5	1	6	11	16	21	26	31	36	41	46
	2	7	12	17	22	27	32	37	42	47
	3	8	13	18	23	28	33	38	43	48
	4	9	14	19	24	29	34	39	44	49
	5	*10*	*15*	*20*	*25*	*30*	*35*	*40*	*45*	*50*
6	1	7	13	19	25	31	37	43	49	55
	2	8	14	20	26	32	38	44	50	56
	3	9	15	21	27	33	39	45	51	57
	4	10	16	22	28	34	40	46	52	58
	5	11	17	23	29	35	41	47	53	59
	6	*12*	*18*	*24*	*30*	*36*	*42*	*48*	*54*	*60*

Note 1. Any number in the first column (the one in thick type) added

Note 2. Any number in the first column subtracted from any number in

Note 3. The lines in italics give the MULTIPLICATION TABLE.

PLETE TABLE.

tion and Multiplication.)

7	1	8	15	22	29	36	43	50	57	64
	2	9	16	23	30	37	44	51	58	65
	3	10	17	24	31	38	45	52	59	66
	4	11	18	25	32	39	46	53	60	67
	5	12	19	26	33	40	47	54	61	68
	6	13	20	27	34	41	48	55	62	69
	7	*14*	*21*	*28*	*35*	*42*	*49*	*56*	*63*	*70*
8	1	9	17	25	33	41	49	57	65	73
	2	10	18	26	34	42	50	58	66	74
	3	11	19	27	35	43	51	59	67	75
	4	12	20	28	36	44	52	60	68	76
	5	13	21	29	37	45	53	61	69	77
	6	14	22	30	38	46	54	62	70	78
	7	15	23	31	39	47	55	63	71	79
	8	*16*	*24*	*32*	*40*	*48*	*56*	*64*	*72*	*80*
9	1	10	19	28	37	46	55	64	73	82
	2	11	20	29	38	47	56	65	74	83
	3	12	21	30	39	48	57	66	75	84
	4	13	22	31	40	49	58	67	76	85
	5	14	23	32	41	50	59	68	77	86
	6	15	24	33	42	51	60	69	78	87
	7	16	25	34	43	52	61	70	79	88
	8	17	26	35	44	53	62	71	80	89
	9	*18*	*27*	*36*	*45*	*54*	*63*	*72*	*81*	*90*

to any n umber in the TABLE gives the next number in the same line.
the TABLE gives the *preceding* number in the same line.

PART I. INTRODUCTION.

CHAPTER I. DEFINITIONS & CONVENTIONS.

1. Anything which may be regarded as made up of parts like the whole is called a **quantity.**

Thus, a sum of money, a certain length or distance, a number of men, a period of time, the weight of a bag of wheat, are *quantities.*

2. Quantities of the same kind are called **like** quantities.

Three mangoes and six mangoes are *like* quantities : five days and eighteen days are also *like* quantities. But six mangoes and eighteen days are *unlike* quantities.

3. A **unit** of quantity or, more simply, a **unit** is a quantity which is used to compare the magnitudes of other quantities of the same kind.

Thus, when a sum of money is spoken of as two *rupees,* a rupee is the unit of money used.

4. A **number** is that which is applied to the unit to denote the magnitude of any quantity. The *numerical measure* or **measure** of any quantity is the number of units which it contains.

Thus, when a certain time is expressed as fifteen hours, the number *fifteen* is applied to the unit *an hour*, to indicate the magnitude of the time. Also *fifteen* is the numerical measure of the time.

The number and the unit together indicate the *absolute* magnitude of a quantity ; the number alone its *relative* magnitude.

5. Numbers are of two kinds, **abstract** and **concrete.**

An *abstract* number is one which is not applied to any particular unit ; a *concrete* number is one which is applied to a unit.

Thus, *one*, *two, four* are abstract numbers ; *one* man, *two* feet, *four* maunds are concrete numbers.

6. **Arithmetic** is the science of numbers and the art of combining them.

7. To the unit is applied the number **one** ; to one unit and one unit, **two** ; to two units and one unit, **three** ; and so on. In this way the numbers **one, two, three, four, five, six, seven, eight, nine,** are obtained in order, any one of these being *one* more than the number immediately preceding it. The absence of any quantity is called **zero, cypher** or *nought*.

8. The **figures** or *digits* **1, 2, 3, 4, 5, 6, 7, 8, 9** denote the numbers *one, two, three, four, five, six, seven, eight,* and *nine* respectively. The *zero* is denoted by the figure **0.**

The figures from 1 to 9 are called **significant** digits.

All numbers are formed by the above figures and their combinations, as will be explained below.

9. *Nine* and *one* is **ten,** and *ten* and *one* is **eleven.** But these numbers, instead of being represented by single digits are denoted by 10 and 11. When *two* figures are placed together side by side, as here, it is *assumed* that the figure to the right should indicate the number of *units,* and that to the left the number of *tens.*

Thus, **10** indicates *one* ten and *no* unit, *i. e., ten* ; **11** indicates 1 ten and 1 unit, *i e., eleven.* So **12** would denote 1 ten and 2 units, or one unit more than *eleven, i. e.,* **twelve** Similarly 24 would denote 2 tens and 4 units, and 99 would denote 9 tens and 9 units.

Ten *tens* are called a **hundred** ; it is 1 unit more than 99, and is denoted by 100. When *three* figures are placed side by side, the third figure from the right is assumed to indicate the number of *hundreds.*

Thus, 432 denotes 4 hundreds, 3 tens, and 2 units.

Ten *hundreds* are called a **thousand** (1000). In any arrangement of figures, the *fourth* figure from the right indicates the number of *thousands.*

In the above way we get *ten thousand, hundred thousand,* **million,** *ten million,* &c., each of which is *ten* times the one immediately preceding it. The *fifth, sixth, seventh, eighth* figures from the right, show how often these are respectively contained in any particular number.

A million million is called a **billion.***

* In America a thousand million is called a billion, a thousand billion is called a trillion, &c. A billion in England is, therefore, the same as a trillion in America.

10. The following Table, called the **Table of Numeration** will, we hope, be now easily understood.

hundreds of thousands of billions	tens of thousands of billions	*thousands* of billions	hundreds of billions	tens of billions	**Billions**	hundreds of thousands of millions	tens of thousands of millions	*thousands* of millions	hundreds of millions	tens of millions	**Millions**	hundreds of thousands	tens of thousands	*thousands*	hundreds	tens	**Units**
6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1

The **periods** which follow the *billion* are called **trillion, quadrillion, quintillion, sextillion, septillion, &c.**

11. A figure has, therefore, two values,—**simple** or *intrinsic* and **local** or *accidental*. The value which a figure has when it stands alone is called its *simple* value; the value which it happens to possess in consequence of its position with relation to other figures, is called its *local* value. Thus in 847, the 8 means 8 *hundreds* which is quite different from its intrinsic value 8. The intrinsic value is, however, obviously unaltered when the figure occupies the place to the extreme right, *i. e.*, the unit's place.

12. Ten, hundred, thousand, ten thousand, &c., are called the first, second, third, fourth **powers** of *ten*, respectively. It will be easily seen that a *million* is the 6th power of ten, a *billion* the 12th power, and so on.

13. As the above system of Numeration is based upon the number **ten** (Latin *decem*), it is often called the **Decimal System** of Numeration.

[The Hindus first invented the above mode of representing numbers by means of only ten figures and their combinations. The method was learned from them by the Arabs, who later on introduced it into Europe. Hence the Europeans erroneously call it the *Arabic Notation*.]

CHAP. II. NUMERATION.

14. **Numeration** is the art of expressing in words a number which is already given in figures.

15. ART. 8 has taught the beginner how to express the numbers represented by **one** figure; the following Table will teach him how to express numbers given in **two** figures.

10 ten	40 forty	70 seventy
11 eleven	41 forty-one	71 seventy-one
12 twelve	42 forty-two	72 seventy-two
13 thirteen	43 forty-three	73 seventy-three
14 fourteen	44 forty-four	74 seventy-four
15 fifteen	45 forty-five	75 seventy-five
16 sixteen	46 forty-six	76 seventy-six
17 seventeen	47 forty-seven	77 seventy-seven
18 eighteen	48 forty-eight	78 seventy-eight
19 nineteen	49 forty-nine	79 seventy-nine
20 twenty	50 fifty	80 eighty
21 twenty-one	51 fifty-one	81 eighty-one
22 twenty-two	52 fifty-two	82 eighty-two
23 twenty-three	53 fifty-three	83 eighty-three
24 twenty-four	54 fifty-four	84 eighty-four
25 twenty-five	55 fifty-five	85 eighty-five
26 twenty-six	56 fifty-six	86 eighty-six
27 twenty-seven	57 fifty-seven	87 eighty-seven
28 twenty-eight	58 fifty-eight	88 eighty-eight
29 twenty-nine	59 fifty-nine	89 eighty-nine
30 thirty	60 sixty	90 ninety
31 thirty-one	61 sixty-one	91 ninety-one
32 thirty-two	62 sixty-two	92 ninety-two
33 thirty-three	63 sixty-three	93 ninety-three
34 thirty-four	64 sixty-four	94 ninety-four
35 thirty-five	65 sixty-five	95 ninety-five
36 thirty-six	66 sixty-six	96 ninety-six
37 thirty-seven	67 sixty-seven	97 ninety-seven
38 thirty-eight	68 sixty-eight	98 ninety-eight
39 thirty-nine	69 sixty-nine	99 ninety-nine

16. In a number consisting of **three** figures, the *third* figure from the right is read as so many *hundred*, and the *two* following figures are read *together* as in the above Table.

Thus, 230 is read two hundred and thirty;
576 is read five hundred and seventy-six;
709 is read seven hundred and nine.

Examples 1.

Write down in words the following numbers:

1. 15; 29; 51; 76; 85; 92; 49; 97; 33; 25; 67; 45; 30.
2. 318; 707; 594; 420; 576; 510; 832; 245; 208; 407; 860; 701.

17. It will appear from Art. 10 that the unit, the million, the billion, &c., divide the figures into periods of *six* each; and that a *thousand* subdivides each such period into two groups of 3 figures each. This will be made more clear by the following diagram :—

Thousand	Billion	Thousand	Million	Thousand	Units
3	0 4 2	4 0 9	3 0 0	7 0 2	0 0 1

The following Rule is thus obtained :—Where a number is denoted by more than *three* figures, count off the places in *threes* by commas, beginning from the *right*. Then read on, from left to right, in the order of the *above diagram*, remembering that the *thousand* comes in once between the million and the unit, once between the billion and the million, once between the trillion and the billion, &c.

Thus,	1,000	is read	one thousand.
	2,500	,,	two thousand, five hundred.
	5,670	,,	five thousand, six hundred and seventy.
	7,090	,,	seven thousand and ninety.
	20,572	,,	twenty thousand, five hundred and seventy-two.
	31,059	,,	thirty-one thousand, and fifty-nine.
	349,025	,,	three hundred forty-nine thousand, twenty-five.
	5,230,647	,,	five million, two hundred thirty thousand, six hundred and forty-seven.
	960,703,201	,,	nine hundred and sixty million, seven hundred three thousand, two hundred and one.
	6,302,810,500	,,	six thousand three hundred two million, eight hundred ten thousand, and five hundred.
	17,305,860,012,503	,,	seventeen billion, three hundred five thousand, eight hundred sixty million, twelve thousand, five hundred and three.

Examples 2.

Call out and write down in words each of the following numbers :

1. 7903; 8025; 2107; 3003; 9067; 5988; 6250.
2. 29061; 70635; 29143; 39672; 69002; 80103.
3. 123456; 706709; 880087; 567890; 800724.
4. 5189879; 9095056; 4094704; 5206650; 4870320.
5. 79038025; 80753045; 19800769; 36850067.
6. 805566931; 860370635; 900300300.
7. 87296391 67; 2906450216; 3727529061.
8. 302169177720; 21900186692; 70080090300.
9. 960530975643; 7006398007346.

***10.** What is the local value of each of the significant digits in the following numbers? 92; 435; 3705; 30725; 92375; 103595; 100075; 30[illegible]6; 6730205890.

***11.** Write down in words the greatest number of seven figures and the least number of eight figures.

***12.** Two boys were told to read 20,080,700; the first read it as two million eight thousand and seventy, the other as twenty million eight hundred thousand and seven hundred. Point out the mistakes that were made.

CHAP. III. NOTATION.

18. **Notation** is the art of expressing a number already given in words by means of figures.

ARTS. 8, 15 and 16 will shew the student how to write down in figures any number less than one thousand. It should be remembered that an absence of *tens* is always indicated by putting a zero, in the *second* place from the right.

Thus,	forty-five is expressed in figures as		45,
	three hundred eighty-two	,, ,,	382,
	five hundred seven	,, ,,	507.

Examples 3.

Set down the following numbers in figures:

1. Fifty-three; seventy-two; thirty-seven; eighty-nine; seventeen; ninety-five; forty-four; twenty-six; sixty.

2. Three hundred; four hundred and five; nine hundred and sixty; five hundred and three; six hundred and twenty-four; four hundred and three; eight hundred and seven.

19. In expressing a large number by means of figures, write down (from left to right) the figures in successive groups (see *diagram*, ART. 17); and put down the required number of **0**'s at the left when less than 3 figures are found in any group.

Thus, in expressing by figures the number, twenty-seven thousand nine hundred and four billion, eight hundred forty thousand twenty-one million, two hundred thousand and five, write in order 27, 904, 840, 21, 200 and 5. Since there are only *two* figures in 21 and *one* figure in 5, put *one* zero before 21 and *two* before 5, so that groups of three may be obtained.

thousand	billion	thousand	million	thousand	units.
27,	904,	840,	021,	200,	005

When an entire group or more are missing, their places should be filled up with *three* cyphers for each group.

Thus, 'twenty billion, one hundred thousand and three', is written as

billion	thousand	million	thousand	units.
20,	000,	000,	100,	003

Examples 4.

Put down the following numbers in figures :

1. Eight thousand ; seven thousand four hundred ; six thousand two ; two thousand five hundred and thirty; four thousand seven hundred and forty-four ; five thousand and one.

2. Fifty-seven thousand, nine hundred and seventy-four ; thirty-nine thousand, one hundred and eighty ; forty-eight thousand, two hundred and seven ; sixty-seven thousand and seventy-three ; forty-thousand seven hundred ; twenty-three thousand and nine.

3. Nine hundred thirty-five thousand, four hundred and seventy-three ; seven hundred twenty-four thousand. and eight ; six hundred two thousand and forty-five ; four hundred thousand, four hundred and four ; nine hundred nine thousand and ninety.

4. Four million, seven hundred thirty-eight thousand, six hundred and eighty-five ; seven million, twenty thousand, and ninety-nine.

5. Nine million, two hundred three thousand, one hundred and sixty-one ; five million, nineteen thousand three hundred and eighty ; seven million, six thousand and nine

6. Seventy-two million, three hundred sixteen thousand, one hundred and fifteen ; ninety-nine million, nine hundred nine thousand, and ninety ; seventy million seventy-seven thousand and seven ; thirty million, forty thousand, and eight.

7. Three hundred seventy-two million, seventy-two thousand, seven hundred and twenty ; two hundred nine million, one hundred and ten ; two hundred ninety-six million, eight thousand and sixty.

8. One thousand, two hundred sixty million, seven hundred five thousand, two hundred and one ; seven thousand, five million, fifty-two thousand and seventy ; nine thousand million, nine thousand nine.

9 Thirty thousand, three hundred seventy million, three hundred sixty thousand, three hundred and six ; fifty-two thousand, twelve million, one hundred nine thousand and ninety-eight.

10. Eight hundred ninety thousand, one hundred and twenty-three million, five hundred ninety-six thousand and ninety-six ; seven hundred thousand, seventy million, seven thousand and seven.

11. Eight billion, eighty million, eighty thousand and eighty ; seven billion, seven hundred and seven ; five billion, five million, five thousands and five.

12. Seventy-five billion, five hundred sixty-seven thousand, nine hundred seventy-two million, four hundred ninety-nine thousand, seven hundred and twenty-five.

13. Five hundred ten billion, seventy-nine million, seven thousand and seven ; seven hundred and three billion, fifty-six thousand, seventy million, eight hundred and twenty thousand.

***14.** Write in figures the greatest number of ten digits, and the least number of twelve digits.

***15.** A teacher told his boys to write sixty million, seven hundred three thousand, nine hundred and two ; but one of them wrote 67030902, and another wrote 607392. What were the mistakes made ?

CHAP. IV. HINDU NOTATION.

20. The following Table gives the local values of figures according to the Hindu method of Numeration and Notation :—

Figure	Place
3	Brinda (hundred crore) বৃন্দ
2	Arbuda অর্বুদ
1	**Koti** or **crore** (কোটি)
7	Nijuta (million) নিযুত
6	**Laksha (lac)** লক্ষ
5	Ajuta অযুত
4	**Sahasra** (thousand) সহস্র
3	Shata (hundred) শত
2	Dasha (ten) দশ
1	**Ekak** (unit) একক

The above number would be read :—Three hundred and twenty-one crores, seventy-six lacs, fifty-four thousand, three hundred and twenty-one.

The powers of ten which follow in the above Table are *kharva, nikharva, sankha, padma, jaladhi, antya, madhya, parárdha.*

Examples 5.

Call out the following numbers according to Hindu numeration :

1. 24392 ; 75381 ; 90613 ; 90611 ; 53545 ; 72131.

2. 2906096 ; 3201706 ; 334056 ; 13070006 ; 900194.

3. 30216917 ; 304024340 ; 29600836 ; 99999999.

4. 77196176 ; 58026260 ; 883070000.

Put down in figures :

5. Five lacs; nineteen thousand six hundred and four ; thirty-one lacs, eight thousand and fifty-four ; nine hundred fifty-seven lacs, eighty thousand, seven hundred and five ; seven crores, five lacs, fifty-seven crores, thirty lacs, nine thousand and five.

6. One hundred and seventy crores, fifty lacs and three ; three thousand and ninety-nine crores, five lacs, one thousand and twenty.

7. How many lacs are there in ten millions ? How many thousands are there in thirty lacs ? How many millions are there in fifty crores ?

***8.** Express a *parárdha* in English, and a *billion* in Hindu, Notation.

***9.** A boy when told to write seven crores, five lacs, two thousand and fifty-four, wrote 700520054. What mistakes did he commit ?

***10.** Ram was told to write nine crores two lacs, three thousand five hundred and two, and he put down 902300502. Find out his mistakes.

CHAP. V. ROMAN NOTATION.

21. Before the introduction of the Hindu system of Notation by the Arabs, the Roman system was generally used in Europe.

In this system the symbols chiefly used are I*, V, X, L, C, D and M, which correspond respectively to 1, 5, 10, 50, 100, 500, and 1000 in the Arabic system. Again, when a bar (—) is placed over any one of the above letters, it increases the value of that letter a thousandfold. Thus $\overline{D}$ represents 500,000, $\overline{M}$ represents 1,000,000.

The following Table will enable the boys to represent numbers by means of the above symbols.

I.	1	XV.	15	CC.	200
II.	2	XVI.	16	CCC.	300
III.	3	XVII.	17	CD.	400
IV.	4	XVIII.	18	D.	500
V.	5	XIX.	19	DC.	600
VI.	6	XX.	20	DCC.	700
VII.	7	XXX.	30	DCCC.	800
VIII.	8	XL.	40	CM.	900
IX.	9	L.	50	M.	1000
X.	10	LX.	60	MCD.	1400
XI.	11	LXX.	70	MDCCCLXXXIX.	1889
XII.	12	LXXX.	80	MCM.	1900
XIII.	13	XC.	90	MM.	2000
XIV.	14	C.	100	$\overline{DLX}$DCCXLII.	560742

Examples 6.

Express each of the following numbers in the Arabic Notation :

1. VI; XIV; XXIV; XCIX; XCVIII.
2. CLIX; CCLXXVI; CCCXXXIX; DXL; DXC.
3. DCXC; DCXXVIII; DCIX; CDXCIX; DCCXXI; CDXXIX.

Express each of the following numbers in the Roman Notation.

4. 8, 15, 27, 35, 46, 51, 62, 77, 94, 108, 127.
5. 19, 29, 39, 49, 59, 69, 79, 89, 99, 109, 139.
6. 217, 494, 567, 2139, 2768, 8953, 67241, 40069.

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* In very old times, a finger was used to denote 1, two fingers to denote 2, a hand with its five fingers to denote 5, and the two hands together to denote 10. In fact, in the Roman system, **I** represents a finger, **II** two fingers, **V** a hand (with the four fingers joined together on one side and the thumb on the other), and **X**, 2**V**'s joined together. Hence a figure is called a **digit**, which word, in Latin, means a finger.

PART II. THE FOUR RULES.

CHAP. VI. ADDITION

22. **Addition** is the process of finding a single number, which is equal to *two* or *more* numbers taken together.

When two or more numbers are added together, the result is called their **sum** or **amount**; and the numbers to be added are called *summands*.

The numbers to be added must be either all *abstract* numbers or all *concrete* numbers of the *same kind.*

23. There are two kinds of addition, **simple** and **compound.**

If the numbers to be added are all *abstract* numbers or all *concrete* numbers *of the same denomination* (*i.e*, all *rupees,* or all *maunds,* or all *yards*) the operation is called *Simple Addition.*

If the numbers to be added are all *concrete* numbers of the same kind, but of *different* denominations of that kind, the operation is called *Compound Addition.*

24. The sign +, placed between two or more numbers, denotes that these numbers are to be added together. It is read **plus.**

Thus, $5+7+9$ denotes that 5, 7, and 9 are to be added together and is read five *plus* seven *plus* nine.

The sign =, is called the **sign of equality.** It is read **equals** or **is equal to.**

Thus, $5+4=9$ expresses that the sum of 5 and 4 is 9, and is read five *plus* four *equals* nine. So, $3+4=5+2$ indicates that the sum of 3 and 4 is the same as the sum of 5 and 2.

25. The student should have the following **Addition Table** committed to memory. It will materially help him to add small numbers mentally.

SIMPLE ADDITION.

1 and	2 and	3 and	4 and	5 and	6 and	7 and	8 and	9 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 = 10
2... 3	2... 4	2... 5	2... 6	2... 7	2... 8	2... 9	2... 10	2...11
3... 4	3... 5	3... 6	3... 7	3... 8	3... 9	3... 10	3... 11	3...12
4... 5	4... 6	4... 7	4... 8	4... 9	4... 10	4... 11	4... 12	4...13
5... 6	5... 7	5... 8	5... 9	5... 10	5... 11	5... 12	5... 13	5...14
6... 7	6... 8	6... 9	6... 10	6... 11	6... 12	6... 13	6... 14	6...15
7... 8	7... 9	7... 10	7... 11	7... 12	7... 13	7... 14	7... 15	7...16
8... 9	8... 10	8... 11	8... 12	8... 13	8... 14	8... 15	8... 16	8...17
9... 10	9... 11	9... 12	9... 13	9... 14	9... 15	9... 16	9... 17	9...18

Note that $1+10=11$, $2+9=11$, $3+8=11$, $4+7=11$, $5+6=11$; $2+10=12$, $3+9=12$, $4+8=12$, $5+7=12$, $6+6=12$; $3+10=13$, $4+9=13$, $5+8=13$, $6+7=13$; $4+10=14$, $5+9=14$, $8+6=14$, $7+7=14$; $10+5=15$, $9+6=15$, $8+7=15$; $10+6=16$, $9+7=16$, $8+8=16$; $7+10=17$, $9+8=17$; $8+10=18$, $9+9=18$; $9+10=19$.

26. **Small numbers are mentally added thus.**

Ex. Add together 23, 36 and 29.

$23=2$ tens and 3 units and $36=3$ tens and 6 units;

$\therefore$ $23+36=5$ tens and 9 units.

Again $29=2$ tens and 9 units;

$\therefore$ $23+36+29=5$ tens and 9 units $+$ 2 tens and 9 units
$=7$ tens and 18 units $=7$ tens and 1 tens and 8 units
$=8$ tens and 8 units $=$ **88**.

Examples 7. Mental Addition.

1. State the sums of:

(1) 2 and 3; 3 and 4; 4 and 5; 7 and 8; 2 and 9; 6 and 5.
(2) 4 6; 4 „ 7; 4 „ 9; 5 „ 7; 5 „ 8; 5 „ 9.
(3) 6 3; 6 „ 7; 6 „ 9; 6 „ 10; 6 „ 13; 6 „ 15.
(4) 7 7; 7 „ 9; 7 „ 10; 7 „ 11; 7 „ 12; 7 „ 13.
(5) 8 ; 8 „ 10; 8 „ 12; 8 „ 13; 8 „ 15; 8 „ 16.
(6) 9 9; 9 „ 10; 9 „ 11; 9 „ 13; 9 „ 18.
(7) 10 10; 10 „ 12; 10 „ 14; 10 „ 16; 10 „ 18.
(8) 11 11; 11 „ 12; 11 „ 13; 11 „ 15; 11 „ 17.
(9) 12 12; 12 „ 14; 12 „ 16; 12 „ 18; 12 „ 20.
(10) 13 13; 13 „ 14; 13 „ 15; 13 „ 16; 13 „ 17.
14 14; 14 „ 15; 14 „ 16; 14 „ 17; 14 „ 18.

2 (1) Add 4 to 9, to 19, to 29, to 39, to 49, to 59, to 69, &c.
(2) Add 7 to 7, to 17, to 27, to 37, to 47, to 57, to 67, &c.
(3) Add 3 to 15, to 25, to 35, to 45, to 55, to 65, to 75, &c.

3. (1) Count aloud by increments of 5, starting at 7, at 9, at 15.

(2) Count aloud by increments of 8, starting at 5, at 9, at 11, at 13, at 15, at 17, at 19, at 21, and at 25.

4. I gave 17 rupees to Ram and 19 rupees to Shyam. What money did I spend?

5. I have two pockets : into one I have put 13 marbles and into the other 22. How many marbles have I altogether?

6. I took away 9 rupees from my purse and had 19 rupees left. What sum had I at first?

7. I bought on a certain day 27 oranges and 17 apples. How many fruits did I buy altogether?

8. The age of a son is 14 years, and his father is 32 years older than he. Find the age of the father.

9. In one rupee there are 16 annas. How many annas are there in 2 rupees?

10. I first took out 16 books from a shelf, and then 7. How many books did I take out?

11. There are two rows of trees in a road ; there are 12 in the first row and 9 in the second. How many trees are there in the road?

12. Ram and Shyam are respectively 23 and 27 years old. Find the sum of their ages.

13. A boy is now 14 years old ; how old will he be after 16 years?

14. Shyam lost 15 marbles and had then 27 left. How many marbles had he at first?

15. There were 17 mangoes in a basket; first 9, and then 12 are put in. Find the total number of mangoes in the basket.

16. A school has three classes ; the first class has 7 boys, the second has 16, and the third 10. Find the number of boys in the school.

17. A box has two divisions ; in one there are 13 rupees, and in the other 5 rupees more. Find the number of rupees in the box.

18. In a game, I begin to play with 27 rupees and win 15 rupees. How many rupees have I at the end?

19. One page of a book contains 35 lines and another 3 more. How many lines are there in the two pages?

20. Jadu bought a dozen chairs for 45 rupees and gained 7 rupees by selling them. For how much were they sold?

21. What is that number from which if I take first 11 and then 17, there will remain 29?

22. From a piece of cloth I cut off 14 yards and had 25 yards left. Find the length of the piece.

23. I paid 2 rupees for cooly hire, 6 rupees for carriage hire, and 7 rupees for Railway fare. How much did I pay altogether?

24. From a piece of cloth I prepared a shirt with 3 yards, a coat with 4 yards and there were 33 yards remaining. Find its length.

***25.** I gave to Ram 7 rupees, to Shyam 3 rupees more than to Ram, and to Hem 5 rupees more than to Shyam. What did Hem get?

27. When the numbers are large, the process of addition is as follows:

Write down the given numbers one under another, so that units come under units, tens under tens, hundreds under hundreds, and so on; and then draw a line under the lowest line of figures. Add all the numbers in the column of *units*; set down under *the same column* the figure in the unit's place of the sum. **Carry** to the next column, *i.e.* include in the *tens*, the other figure or figures of the sum, and add to it all the numbers in that column. Put down the units figure of the sum, carry the other figure or figures to the column of hundreds and add to it the figures in that column. Continue this process until there are no more figures left.

Ex. Add together 327, 1085, 1837, and 2956.

Arrange the numbers according to the above rule, and proceed to add the columns, commencing from the column of units.

$$\begin{array}{r} 327\\ 1085\\ 1837\\ \underline{2956}\\ 6205 \end{array}$$

The sum of 7, 5, 7, and 6 is 25. Put down the **5** under the column of units below the line, and carry the 2 on to the column of tens.

The sum of 2, 2, 8, 3 and 5 is 20. Put down the **0** in the column of tens, and carry the 2 on to the column of hundreds.

The sum of 2, 3, 0, 8 and 9 is 22. Put down the **2** in the column of hundreds, and carry the 2 on to the column of thousands.

The sum of 2, 1, 1 and 2 is 6. Put down this **6** in the column of thousands. Thus the entire sum is **6205**.

28. The following method can be advantageously employed in **long** additions.

During the process of adding the figures in any column, whenever the sum exceeds nine, reject the ten and add the unit figure to the succeeding figures in the unit place, putting a mark at each step where a ten is thus left; finally put down the unit figure under the line. Then find the number of tens by the marks, and add to it the figures in the column of tens, putting a mark wherever an occasion arises, as before.

$$\begin{array}{r} 296\\ \dot{3}1\dot{8}\\ 24\dot{6}\\ 3\dot{7}2\\ \dot{9}64\\ 32\dot{7}\\ \dot{9}\dot{4}6\\ \dot{5}1\dot{9}\\ 23\dot{9}\\ 51\dot{8}\\ \dot{9}2\dot{9}\\ \dot{3}\dot{7}3\\ 40\dot{9}\\ \underline{50\dot{8}}\\ 6964 \end{array}$$

In the above example $6+8=14$; a ten is left out at 8 which is therefore marked, and the remaining figure 4 is added to 6. The sum is 10 and therefore 6 is also marked. As we have nothing in hand, we add 2, 4 and 7 and their sum being 13, we mark 7 and add 3 to the following figure. The process is continued in this way.

Examples 8.

1. Add together:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
96	29	52	75	79	30	93	87	16	99	35	92
18	18	72	38	63	42	23	46	18	44	83	7
46	26	77	92	59	71	40	92	29	77	42	81

(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
764	239	707	696	456	46	316	926	419	7908	8456
327	518	70	804	89	312	205	218	527	5988	3092
946	929	777	593	87	407	57	377	619	7966	5361
512	373	707	123	546	692	8	946	328	8644	8585

(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)
8025	2107	1096	7206	926	7215	9215	29061
967	3206	3003	9025	7209	8603	683	70635
5067	960	7967	8396	76	9206	7095	92186
15	158	8296	7215	4800	5946	87	59147
9295	9	3915	9604	713	7003	6201	92081

(32)	(33)	(34)	(35)	(36)	(37)	(38)
72136	29137	39061	69002	96044	70123	302196
9063	7029	72096	7358	3726	9686	75194
70032	571	59185	521	96925	500	99098
9063	25	72096	92015	72156	72156	157969
72336	78205	3095	81007	9213	36363	888808

(39)	(40)	(41)	(42)	(43)	(44)
510051	908076	906017	900196	370037	30216917
325686	504321	29603	37295	58051	219003856
72980	123607	5654	9209	99684	38216306
59066	790102	943	3960	672011	49036305
971017	803209	25	7070	238258	9203160

(45)	(46)	(47)	(48)
700070007	7206317	72038370	29609721
70077070	92061315	59623630	31203716
7700700	7290376	67216162	29065531
888888888	57101371	46384217	3044702
999999999	2005006	72003176	90605

2. Find the value of:

(1) 41725 + 8376 + 38961 + 90075 + 7219 + 27072 + 5967 + 66968 + 59666.

(2) 818 + 3039067 + 59176 + 382952 + 8007 + 900867 + 309150.

(3) 5991776 + 96031 + 970603 + 317 + 9260072 + 7137 + 9263.

(4) 79909 + 36921 + 12304 + 77788 + 315796 + 87159 + 38372 + 9603124 + 79638 + 99999.

(5) 92107 + 31763 + 9020202 + 77003322 + 67891 + 3031 + 36 + 869698.

(6) 353612 + 50021 + 721 + 56 + 4 + 900267 + 56420152 + 79267345001

3. Find the sum of :

(1) 370613, 901234, 666777, 38638, 979201, 2517005 and 156.

(2) 9208030, 9709790, 3456789, 456, 370037, 13825 and 9.

(3) 75319019, 35682003, 376989, 79386946, 37020012067, 9508176, 2200220022 and 87078780.

(4) 71107, 386967, 596831, 8070, 781567, 26043, 215, 72120, 75623 and 1459602.

(5) 37, 128, 1596, 25692, 579215, 2467325, 12345678, and 993567829.

(6) 9, 19, 129, 1239, 12349, 123459, 1234569, 12345679, 123456789, and 999999999999.

4 Add together twelve thousand, four hundred and eighty-three; seven hundred and ninety-six; fifty-nine: seventy thousand; nine hundred and ninety-four; ten thousand three hundred and twenty-eight; and eighty-two thousand and two.

5. Add together seventeen thousand and four; fifty-eight hundred and nine; eleven thousand, seven hundred and thirty; four hundred thousand, nine hundred and six; forty-two thousand and ninety-seven; eight thousand and seventy.

6. Find the sum of eighty-five thousand one hundred forty-six; seventy thousand two hundred and ninety-two: four hundred twelve thousand, nine hundred and fifty; and three hundred nine thousand, seven hundred and four.

7. Add together seven millions, ninety-nine thousand and forty; eighty-three millions, nine thousand and seven; twelve hundred and four; forty-five millions, twenty-nine thousand, six hundred and two; thirteen millions, thirteen thousand and thirteen; and seventy thousand seven hundred and seventy.

8. Add together twenty-seven millions, nine hundred thousand four hundred and three: nine hundred and seventy-three millions, forty-four thousand and fifty-five; three hundred and seventeen millions four hundred and ninety thousand and ten; three thousand fifty-six millions, eight thousand and nine; one thousand two hundred and ten millions, one thousand and thirty-six; fifty-five billions, three thousand three hundred and four.

9 Find the sum of two hundred and fifty-four crores, seven lacs; one crore, four lacs, six hundred and seven; three hundred and ninety-four crores, twenty-five lacs and nineteen thousand; three crores, five lacs, six thousand, two hundred and seven; and fifty-five crores, seventy-seven lacs, eighty-eight thousand and eighty-eight.

10. A school has nine classes; in the first there are 65 boys, in the second 72, in the third 61, in the fourth 57, in the fifth 92, in the sixth 85, in the seventh 60, in the eighth 54, and in the ninth 76. Find the total number of boys in the school.

11. In a four-storied house there are 70 windows in the fourth story, 64 in the third, 60 in the second, and 92 in the first. How many windows are there in the house ?

12. The population of Calcutta is four lacs, eighty-seven thousand, four hundred and five ; that of Hooghly one lac, twenty thousand and sixty-four ; that of Burdwan two lacs, ninety-five thousand, nine hundred and eighty-seven ; that of Midnapur two lacs, three hundred and four ; and that of the Twenty-four Parganas 12 lacs, fifty thousand, four hundred and two. Find the total population.

13. In a garden there are 756 mango trees, 307 jack trees, 452 roseberry trees, 740 cocoanut trees, 315 date trees, 207 tamarind trees, 57 orange trees, 104 lichi trees and 215 peach trees. How many trees are there in the garden ?

14. January has 31 days, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, and December 31. Find the total number of days in a year.

15. A gentleman before his death ordered that his property should be distributed in the following manner: 10550 Rs to his eldest son, 8956 Rs. to his second son, 5290 Rs. to each of his three youngest sons, 4900 Rs. to each of his four daughters, 3617 Rs. to charitable purposes, and 2700 Rs. to his servants. Find the value of his property.

16. On Monday 2562 passengers went to Tarakeswar by the East Indian Railway, on Tuesday 1502, on Wednesday 715, on Thursday 512, on Friday 1015, on Saturday 15710, and on Sunday 2745. How many people went to Tarakeswar in the week ?

17. A man was born in 1809 ; in what year was he 69 years old ?

18. A man has five sons ; the youngest son is 7 years old, the fourth 5 years older than the youngest, the third 7 years older than the fourth, the second 3 years older than the third, the first 4 years older than the second, and the father himself 21 years older than his eldest son. Find the father's age and that of the third son.

19. There are 17640 words in each of the 4 pages of the Daily News. How many words are there in the issues of six days ?

20. I first took out from my box 576098 rupees then 40325 rupees, and yet had 38921 rupees left. What did the box contain ?

21. In a school of six classes there are 32, 45, 57, 60, 75, 92 pupils respectively ; if 11 new pupils be admitted to each of the classes, how many pupils will there be in the school ?

22. 75690275 rupees are left, when 99063704 rupees are taken away from a certain sum. Find that sum.

23. Find the sum of 3 times 1198+4 times 4736+5 times 7989.

24. A train consists of 5 carriages, which contain 34, 63, 42, 51 and 44 passengers respectively. If 25 men now enter into each of the first two carriages and 17 into the third, how many passengers will there be in the train altogether ?

25. In one regiment there are 2560 soldiers, in another 3204, and in a third 1392. If the regiments be respectively joined by 725, 1810 and 1204 soldiers more, how many soldiers will there be altogether ?

CHAP. VII. SUBTRACTION.

29. **Subtraction** is the method of finding the number by which the greater of two given numbers exceeds the less. It is, therefore, the number which should be added to the smaller of the two numbers that the sum may be equal to the larger number.

30. There are two kinds of subtraction, **simple** and **compound**.

If the numbers are both *abstract* numbers or both *concrete* numbers of the *same* denomination, the operation is called *Simple Subtraction*.

If the numbers are both *concrete* numbers of the same kind but of *different* denominations, the operation is called *Compound Subtraction.*

31. The sign —, placed between two numbers, signifies that the latter number is to be subtracted from the former. It is read **minus.**

The greater of the two given numbers is called the **minuend,** and the less is called the **subtrahend.** The number by which the *minuend* exceeds the *subtrahend*, is called the **remainder** or **difference.**

Thus, $7-4=3$. This is read, seven *minus* four *equals* three, and states that when 4 is subtracted from 7, the remainder is 3. 7 is called the *minuend,* 4 the *subtrahend,* and 3 the *remainder.*

32. Since the *remainder* is the number which should be **added** to the smaller number in order that the **sum** may be equal to the larger number, **subtraction** is sometimes called **Complementary Addition.**

SIMPLE SUBTRACTION.

Examples 9. Mental Subtraction.

1. (1) Subtract 2 from 3, 3 from 4, 4 from 5, 5 from 6, 6 from 7, &c.

(2) „ 3 „ 5, 5 „ 7, 7 „ 9, 9 „ 11, &c.

(3) „ 4 „ 7, 7 „ 10, 10 „ 13, 13 „ 16, &c.

(4) „ 17 „ 21, 23 „ 27, 29 „ 33, 35 „ 39, &c.

(5) „ 3 „ 8, 4 „ 6, 2 „ 9, 5 „ 15, 7 from 10.

2. Find the difference between

(1) 11 and 9, 15 and 8, 17 and 6, 21 and 12, 33 and 25, 30 and 27.

(2) 32 and 10, 16 and 3, 27 and 12, 45 and 40, 49 and 9, 69 and 13.

3. By how much does 9 exceed 5, 12 exceed 7, 20 exceed 17, 25 exceed 19, 39 exceed 29, 38 exceed 25, 45 exceed 21, 67 exceed 47, 59 exceed 50, and 82 exceed 67?

4. Count by decrements of 2, 4, and 6, commencing at 80.

5. Subtract 9 five times from 100, 7 three times from 37, and 8 seven times from 70.

6. Take 6 from $5+8$; 8 from $5+6$; 12 from $20+13$; 15 from $10+30$; 20 from $60+28$; and 27 from $18+43$.

7. Out of 20 marbles Jadu gives Hari 9; how many has he left?

8. I am now 35 years old; how old was I 18 years back?

9. Ram is 35 years old, and Shyam is 15 years younger than he; how old is Shyam?

10. I have 40 rupees in my purse from which I give 9 rupees to a poor man; how many rupees are left?

11. There were 60 leaves in my exercise book from which one of my class-fellows tore out 20; how many leaves were left?

12. A shop-keeper bought 4 maunds of sugar for 48 rupees, and sold them for 61 rupees; what did he gain thereby?

***13.** I am 14 years younger than my elder brother, who will be 32 years old 8 years hence. What is my age?

14. Out of 30 rupees, I gave 7 rupees to Shyam and 9 rupees to Ram. How much can I spend in other ways?

***15.** Ram has 24 marbles; if I had 15 more than what I have, I would have 5 more than he; how many marbles have I?

***16.** Your little sister was born when you were only 3 years old, and you were born when your father was 27 years old. If your father is now 60 years old, how old is your sister now?

33. When the numbers are large, the operation of subtraction is as follows: Place the less number under the greater, so that units come under units, tens under tens, hundreds under hundreds, and so on; then draw a line under the lower line of figures.

(*a*) When the figures in the lower line are none of them greater than the corresponding figures in the upper line, subtract (commencing from the right) each figure of the lower line from the figure which stands directly over it, and set down the difference under the line drawn, units under units, tens under tens, hundreds under hundreds, and so on.

Ex. 1. Subtract 57 from 199

$$\begin{array}{r} 199 \\ 57 \\ \hline 142 \end{array}$$

Place the smaller number 57 under 199, and draw a line under it. First take 7 from 9, and set down the difference **2** in the place of units below the line; next take 5 from 9, and put down the difference **4** in the place of tens under the line. Lastly, bring down **1**, since there is *nothing* under it. Thus the remainder is **142**.

Ex. 2. Subtract 343 from 547.

$$\begin{array}{r} 547 \\ 343 \\ \hline 204 \end{array}$$

As before, put 343 under 547, and draw a line. Taking 3 from 7, the remainder is **4**; 4 subtracted from 4 leaves *nothing* or **0** as remainder; lastly, 3 is subtracted from 5, the remainder being **2**. Thus the remainder is **204**.

(*b*) Where any figure in the lower line is greater than the corresponding figure in the upper line, add **ten** to the upper figure and subtract the lower number from the *sum thus obtained.* Put the remainder down as before, and then **carry 1,** *i.e*, *add* 1 to the *next lower* figure, and subtract, if possible, the number thus increased from the number above it. If not, add *ten* to the upper number, and continue as before.

Ex. 1. Subtract 152 from 348.

Subtract 2 from 8, and set down the remainder **6.** 5 is greater than 4; so 10 is added to 4, and from the sum 14, 5 is subtracted. The remainder is **9.** 1 is now added to the next lower figure 1; the sum is 2, which is subtracted from 3, the remainder being **1.** Thus the result is **196.**

$$\begin{array}{r} 348 \\ 152 \\ \hline 196 \end{array}$$

Ex. 2. Subtract 7533 from 28423.

$3-3=$**0.** Since 3 is greater than 2 2 is made 12 by adding 10. $12-3=$**9.** $5+1=6$, which is greater than 4; 6 is therefore subtracted from $4+10$ or 14, so that the remainder is **8.** $7+1=8$, and $8-8=$**0.** Lastly, **2** is brought down. Thus the remainder is **20890.**

$$\begin{array}{r} 28423 \\ 7533 \\ \hline 20890 \end{array}$$

N. B. It will be easily seen that adding 10 to an upper figure is equivalent to *increasing the next upper figure* by 1. When the figures under this latter is increased by 1, the difference therefore remains unaltered.

Examples 10.

1. Find the difference between the following pairs of numbers:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
89	62	315	997	789	643	514	523	3447	2020
52	32	105	565	661	221	307	132	3239	1850

(11)	(12)	(13)	(14)	(15)	(16)
96175	79061	97316	69736	70313	15902
43023	43023	63201	29367	15579	10023

(17)	(18)	(19)	(20)	(21)
9321321	15600713	3533021	17006157	7201307
7094556	596007	729682	929159	2109109

(22)	(23)	(24)	(25)	(26)
7103115	50050015	19920030	8012960	7152136
379680	3123716	7206031	1390972	372195

(27)	(28)	(29)	(30)	(31)
6036031	5960317	2906137	3031327	9010400
603932	3337129	728659	791998	2737

(32)	(33)	(34)	(35)	(36)
8603176	8603176	5001297	5090502	3219063
90937	1276952	30988	509055	1985197

(37)	(38)	(39)	(40)
93213916	796030205	807126026	455173226
70904569	259383765	545928397	299407329

2. Find the value of :

(1) 1271083316 − 94034397 ; 601106016 − 365365065.
(2) 19190019 − 7191197 ; 72013018 − 35316077.
(3) 987654321 − 123456789 ; 80310036 − 26494729.
(4) 706320301 − 387495696 ; 208304040 − 72405043.
(5) 363036066 − 50868338 ; 100200200 − 90070350.

3. Find the difference between :

(1) 298889566 and 356247338 ; 30208735 and 29487697.
(2) 599386585 and 970302060 ; 80000102 and 8765320.
(3) 20010025 and 9050042 ; 38209504 and 73506701.
(4) 143043076 and 39648079 ; 8729039 and 33603306.

4. By how much is 6036034 greater than 602932 ?

5. By how much does a million exceed 905602 ? By how much is a crore greater than 57 lacs, and 248426 less than 13 lacs ?

6. What number must be added to each of the following numbers to make the sum equal to a crore ? 229 ; 504 ; 7632 ; 77905 ; and 230004.

7. What number must be subtracted from 7000000 so as to leave 3506087 as remainder ?

8. Calcutta has 872541 inhabitants and Bombay 812305 ; how many inhabitants has the one more than the other ?

***9.** The first battle of Paniput was fought in the year 1526, and the second battle of Paniput in 1556 ; how many years after the first, did the second battle take place ?

10. I took out at various times 344, 719, 1204, 1907 and 1209 rupees from a box containing 17954 rupees ; how much was left in the box ?

11. A water-pipe supplies 6452 gallons of water, of which only 4052 gallons are used ; how much water runs waste ?

12. A merchant sold goods for 25679 rupees, thereby gaining 5264 rupees ; what did he pay for them ?

13. A house and its furniture are together worth 501207 rupees ; if the house alone be worth 496250 rupees, find the price of the furniture.

14. A man, who is 84 years of age, was 32 years old when his eldest son was born ; what is the son's present age ?

***15.** Akbar was born in 1542, and died in 1605 ; how old was he at the time of his death ?

16. A merchant bought goods for 9205 rupees, and sold them for 10719 rupees ; find his profit.

17. A boy wrote 25690075 words when he was told to write 19742673 words ; how many more words did he write?

18 Shyam was born when Ram was 37 years old ; what will be Shyam's age when Ram is 85 years old ?

19. Had I 9465 rupees more, I would have been able to buy a house which was sold for 25796 rupees ; what money had I ?

20. A man has 7256021 rupees ; how much must he borrow that he may buy an estate worth 8295624 rupees ?

***21.** A man has 2560261 rupees, and gives 562100 rupees to each of his three sons, 59620 rupees to each of his four daughters, and the remainder to his wife ; how much is given to the wife ?

22. A rope is 259 yards long, 65 yards are coloured red, 37 black and the rest yellow. How many yards are coloured yellow ?

23. In a basket there are 729 mangoes, lichies, and oranges in all. The number of mangoes and oranges taken together is 332 ; the number of oranges and lichies is 678. How many more lichies are there than oranges ?

24. A basket contained 1223 oranges and lichies, 979 lichies and nuts, and 788 oranges and nuts. How many fruits were there in all ? How many more lichies were there than oranges ?

25. There are peaches, lichies, and oranges in a basket ; the number of peaches and lichies is 189 and the number of peaches and oranges is 211. How many more oranges are there than lichies ?

26. The Diamond Jubilee of the late Queen Victoria was celebrated in the 60th year of her reign in the year 1897. When was the queen crowned ? The Golden Jubilee of the queen was celebrated in the 50th year of her reign. When was it celebrated ?

27. A boy was told to write five hundred and thirty-one million, four thousand and forty-seven, but he wrote 503140047 ; how much more or less did he write ?

28. The greater of the two numbers is 972134 and their difference is 84769, find the other number.

34. When a number is preceded by + (the *plus* sign), it is called a **positive** number. When it is preceded by − (the *minus* sign), it is called a **negative** number. A number which has *no* sign affixed to it, is considered as *positive.*

35. When two or more numbers are connected by the sign + or −, the whole is called an **expression** ; and the numbers themselves are called **terms.**

Thus, $7-3+4+2-5$ is an *expression* ; 7, 4, and 2 are *positive* ; 3 and 5 are *negative* ; and 7, 3, 4, 2, and 5 are *terms.*

36. $7-5+3=5$; $7+3-5=5$; $\therefore 7-5+3=7+3-5$.

It therefore appears that an alteration in the order in which the terms are added or subtracted makes no difference in the

result. In finding the value of an expression, therefore, add together *all* the positive terms and *all* the negative terms, separately; subtract the latter sum from the former; then the remainder will be the value required.

Ex. Find the value of $20-15-7+8+9$.

$20+8+9=37$, and $15+7=22$; also $37-22=15$.

Therefore the required value $=$ **15**.

36 A. The complement of a number is the defect of the number from 10 times its highest order. Thus, 3 is the complement of 7; 192 is the complement of 808; for $10-7=3$. and $1000-808=192$.

Examples 11.

Find the value of:

1. $4-7-2+13-4$.

2. $15-24+17-14+30$.

3. $342-422+362-110+740-247$.

4. $2464-749+219-824+3624-1214$.

5. $12567+324-9424-3240+7420+24900-824-665$.

6. What number must be added to the difference of 8472 and 9565 to make the result equal to the sum of 702, 47 and 819?

7. What are the complements of 19, 324, 4032, and 56789?

CHAPTER VIII. MULTIPLICATION.

37. **Multiplication** is a short method of finding the sum of a given number repeated as many times as there are units in another given number.

Thus, when 26 is multiplied by 3, the result is $26+26+26$ or 78.

38. The number to be repeated or *multiplied*, is called the **multiplicand**; the number which tells us how often the other number is to be repeated, *i. e.*, which *multiplies*, is called the **multiplier**; and the sum or resulting number is called the **product.** Also the *multiplicand* and the *multiplier* are both called **factors** of the product.

Thus, in the above example, 26 is the *multiplicand*, 3 the *multiplier*, and 78 the *product*; also 26 and 3 are *factors* of 78.

39. There are two kinds of multiplication, **simple** and **compound.**

In *Simple Multiplication*, the multiplicand is either an *abstract* number or a *concrete* number of *one* denomination.

In *Compound Multiplication*, the multiplicand is a *concrete* number of *more than one* denomination, but of the *same* kind.

The multiplier merely tells us how often the multiplicand is to be repeated and is therefore always an *abstract* number.

40. 6 repeated 5 times$=6+6+6+6+6=30$; 5 repeated 6 times$=5+5+5+5+5+5=30$. It, therefore, appears that 5 times 6 is the same as 6 times 5.

Thus, *the multiplicand and the multiplier may be interchanged without altering the product.*

N. B. When any number and 0 are multiplied together, the product is always 0.

For, $0\times5=0+0+0+0+0=0$.

41. The sign of multiplication $\times$, signifies that the numbers between which it stands are to be multiplied together. It is read **into** or **times** or *multiplied by*.

Thus, 5×8 means that 5 and 8 are to be multiplied together, and is read 5 *into* 8, or 5 *times* 8, or 5 *multiplied by* 8.

The symbols $\because$ and $\therefore$ stands for *because* and *therefore*, respectively.

42. When in an expression $\times$ occurs with $+$ or $-$, the operation of multiplication must be performed before those of addition or subtraction.

Thus, to find the value of $26\times3+5$, we must proceed thus;

$$26\times3=26+26+26=78;$$
$$\therefore\ 26\times3+5=78+5=83.$$

SIMPLE MULTIPLICATION.

43. The student must first commit to memory the following

Multiplication Tables.

Table 1.

	1	2	3	4	5	6	7	8	9	10
Once	**1**	2	3	4	5	6	7	8	9	10
Twice	2	**4**	6	8	10	12	14	16	18	20
Thrice	3	6	**9**	12	15	18	21	24	27	30
4 times	4	8	12	**16**	20	24	28	32	36	40
5 times	5	10	15	20	**25**	30	35	40	45	50
6 times	6	12	18	24	30	**36**	42	48	54	60
7 times	7	14	21	28	35	42	**49**	56	63	70
8 times	8	16	24	32	40	48	56	**64**	72	80
9 times	9	18	27	36	45	54	63	72	**81**	90
10 times	10	20	30	40	50	60	70	80	90	**100**

Table 2.

	1	2	3	4	5	6	7	8	9	10
11 times	11	22	33	44	55	66	77	88	99	110
12 times	12	24	36	48	60	72	84	96	108	120
13 times	13	26	39	52	65	78	91	104	117	130
14 times	14	28	42	56	70	84	98	112	126	140
15 times	15	30	45	60	75	90	105	120	135	150
16 times	16	32	48	64	80	96	112	128	144	160
17 times	17	34	51	68	85	102	119	136	153	170
18 times	18	36	54	72	90	108	126	144	162	180
19 times	19	38	57	76	95	114	133	152	171	190
20 times	20	40	60	80	100	120	140	160	180	200

Table 3.

	11	12	13	14	15	16	17	18	19	20
11 times	**121**	132	143	154	165	176	187	198	209	220
12 times		**144**	156	168	180	192	204	216	228	240
13 times			**169**	182	195	208	221	234	247	260
14 times				**196**	210	224	238	252	266	280
15 times					**225**	240	255	270	285	300
16 times						**256**	272	288	304	320
17 times							**289**	306	323	340
18 times								**324**	342	360
19 times									**361**	380
20 times										**400**

Examples 12. Mental Multiplication.

1. How many are?

(1) 6 times 3; 7 times 8; 4 times 9; 5 times 8; 9 times 6; 7 times 2.

(2) 7 „ 13; 6 „ 14; 5 „ 15; 8 „ 12; 9 „ 16.

(3) 5 „ 16; 6 „ 17; 7 „ 18; 9 „ 15; 3 „ 19.

(4) 11×12; 10×19; 12×13; 13×14; 14×15; 15×16; 12×16; 13×17; 14×18; 15×19; 16×20.

2. On the road I met with 13 beggars, and gave 5 pice to each; how many pice did I give altogether?

3. A yard of silk costs 5 rupees; find the cost of 17 yards.

4. In a class there are 16 boys; to each of whom I give 9 oranges; how many are given altogether?

5. I can walk 4 miles an hour; how far can I walk in 18 hours?

6. The daily wages of a labourer is 6 annas; what would be the daily wages of 17 labourers?

7. A box has 15 divisions, in each of which there are 13 rupees. Find the number of rupees in the box.

8. The cost of a watch is 15 rupees; what must I pay for 16 watches?

9. Mangoes are 17 for a rupee; how many can I buy for 9 rupees?

10. There are 18 horses in a stable; how many legs have they?

11. A horse runs 15 miles an hour; how far can it run in 15 hours?

12. By how much is 9 times 12 under 120, and 11 times 14 above 120? What number exceeds 12 times 12 by 10?

13. There are 7 days in a week; how many days are there in 18 weeks?

14. Find the price of 16 maunds of sugar at 13 rupees per maund.

15. 18 pages of a book contain 20 lines each, and 19 pages 13 lines each; how many lines are there in the book?

***16.** The boys of a school can be arranged in 17 lines of 16 each, with 12 over; how many boys are there in the school?

44. To multiply a large number by a number of **one** figure the following method should be adopted.

Put down the multiplier under the unit's figure of the multiplicand, and draw a straight line under it. Multiply each figure of the multiplicand separately, beginning with the unit's, by the multiplier; set down the unit's figure under the line; and *carry* as in addition.

Ex. Multiply 5762 by 7.

$$\begin{array}{r} 5762 \\ \underline{7} \\ 40334 \end{array}$$

7 times 2 is 14. Put down **4** in the unit's place and carry 1. 7 times 6 is 42, and $42+1=43$; set down **3** in the ten's place and carry 4. 7×7 is 49, 49 and 4 are 53. Put down **3** in the hundred's place and carry 5. Lastly, multiply 5 by 7 and add 5 to the product 35, so that the sum is 40. Set down **40**. The product is therefore **40334**.

The above will be understood by considering what follows :

$5762 = 5$ thousands $+ 7$ hundreds $+ 6$ tens $+ 2$ units ;

$\therefore 5762 \times 7 = 7 \times 5$ thousands $+ 7 \times 7$ hundreds $+ 7 \times 6$ tens $+ 7 \times 2$
$= 35$ thousands $+ 49$ hundreds $+ 42$ tens $+ 14$
$= 35$ thousands $+ 49$ hundreds $+ 43$ tens $+ 4$
$= 35$ thousands $+ 53$ hundreds $+ 3$ tens $+ 4$
$= 40$ thousands $+ 3$ hundreds $+ 3$ tens $+ 4$
$= \mathbf{40334}$.

45. $24256 \times 1000 = 24256 \times 1$ thousand $= 24256$ thousands $= 24256000$. We therefore easily get the following.

RULE. In multiplying by 10, 100, 1000, &c., add as many cyphers to the multiplicand as there are cyphers in the multiplier.

46. The methods of ART. 44 and ART. 45 can also be used in multiplying a number by 20, 30, &c. ; by 200, 300, &c. ; by 2000, 3000, &c.

Thus, (1) $62 \times 30 = 62 \times 3$ tens
$= 186$ tens
$= \mathbf{1860}$.

$$\begin{array}{r} 62 \\ \underline{30} \\ 186,0 \end{array}$$

(2) $574 \times 4000 = 574 \times 4$ thousands
$= 2296$ thousands
$= \mathbf{2296000}$

$$\begin{array}{r} 574 \\ \underline{4,000} \\ 2296,000 \end{array}$$

Thus the result will be same as if we multiply the number by only the *significant* figure in the multiplier, and affix to the product the number of cyphers at the end of the multiplier.[1]

47. When the multiplier is greater than 9 but less than 20, the method of ART. 44 will also apply.

Ex. Multiply 375 by 12.

$$\begin{array}{r} 375 \\ \underline{12} \\ 4500 \end{array}$$

$5 \times 12 = 60$
$7 \times 12 + 6 = 84 + 6 = 90$
$3 \times 12 + 9 = 36 + 9 = 45$

Thus the product is 4500.

Examples 13.

1. Multiply

(1) 32156 separately by 2, 3, 4, 5, 6, 7, 8 and 9.

(2) 976543 „ „ 7, 8, 9, 11, 12, 13, 14, 15.

(3) 5769092 „ „ 3, 5, 7, 9, 11, 13, 15, 17, 19.

(4) 9292193 „ „ 2, 4, 6, 8, 10, 12, 14, 16, 18, 19, 20.

2. Find the value of :

(1)	864302 × 7 ;	932370 × 8 ;	1015145 × 9.
(2)	8903251 × 6 ;	735215 × 11 ;	932548 × 13.
(3)	254967 × 12 ;	478142 × 14 ;	7214615 × 15.
(4)	54201 × 16 ;	42146 × 17 ;	100459210 × 19.

3. Multiply

(1)	672978 separately	by	10, 100, 1000, 10000.
(2)	734965 ,,	,,	20, 200, 2000, 20000.
(3)	694857 ,,	,,	30, 40, 50, 60, 70, 80.
(4)	694857 ,,	,,	300, 400, 500, 600, 900.
(5)	835492 ,,	,,	110, 1200, 13000, 160000.

4. Multiply 3732 times 13 by 1600, and 12 times 2760 by 18000.

5. Add 5 times 374592 to 8 times 897261, and subtract 6 times 82973 from 16 times 46932.

6. Find the value of 1875213 + 893725 repeated 5 times.

7. Find the price of 17539 watches at 7 rupees each.

8. A steamer plies at the rate of 240 miles per day; what distance will it pass over in 9 days?

***9** A gentleman had three sons; to the second he gave 1572 rupees, to the first 5 times 438 rupees, and to the youngest 9 times the share of the second. How much did his sons get respectively?

10. A merchant bought 1765 yards of cloth at 5 rupees a yard and retailed them at 8 rupees a yard. Find his gain.

48. To multiply one large number by another, proceed as in the Example below.

Ex. Multiply 5234 by 325.

```
   5234
    325
  26170
 10468
15702
1701050
```

First multiply 5234 by 5 and put down the product 26170. Then multiply 5234 by 2, and set down the product 10468 so that 8 may come under 7, 6 under 1, 4 under 6, and so on. Lastly multiply 5234 by 3, and set down the product 15702 so that 2 may fall under 6, 0 under 4, 7 under 0, and so on. Then add.

49. The following consideration will shew the correctness of the method indicated above.

By ARTS. 44, 45 and 46, the product of 5234 and 325 may be found thus:

```
   5234          325 = 5 + 20 + 300 :
    325
  26170          5234 ×   5 =   26170
 104680          5234 ×  20 =  104680
1570200          5234 × 300 = 1570200
1701050       ∴  5234 × 325 = 1701050
```

Ex. Multiply 597 by 905.

597	597
905	905
2985	2985
0000	5373
5373	540285
540285	

In this Example after multiplying by 5 and putting down the result, we proceed to multiply by 0: but anything multiplied by zero is 0, (ART. 40). We next multiply by 9, and set down the product according to the rule given in ART. 48.

The result is therefore the same as if we leave out the 0 altogether, and put down the first figure of the product by 9 under the *third* figure of the line above it. *W[illegible] are cyphers in the multiplier, the product by the next significant digit must therefore be moved by an equal number of places to the left.*

50. If the multiplier, or multiplicand, or both, end with cyphers, these may be omitted in the working; care must be taken, however, to affix to the product as many cyphers as have been omitted in the multiplier, or multiplicand, or both.

Thus, if 507 be multiplied by 2500, 2600 by 79, and 520 by 6200 we have,

507	2600	520
2500	79	6200
2535	234	104
1014	182	312
1267500	205400	3224000

51. In multiplication, it is generally convenient to make the smaller number the multiplier and the larger number the multiplicand. By such interchange of positions the product is not altered; see ART. 40.

52. We may test the correctness of a multiplication by either of the following ways:

I. Multiply the multiplier by the multiplicand; the product *must be the same.* (ART. 40).

II. By **casting out the nines.**

(*a*) Add successively the figures of the *multiplicand*, and whenever a sum exceeds 9, *cast out* or omit the 9, so that a remainder is finally obtained less than 9.

(*b*) *Cast out the nines* in the multiplier in the same way, and obtain a remainder less than 9.

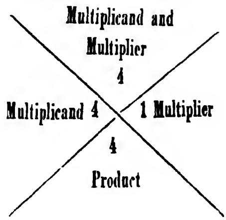
(*c*) Multiply these two remainders, and *cast out the nines* in their product.

(*d*) *Cast out the nines* in the number obtained as the product of the two given numbers.

Then if the remainder in (c) is *not* the same as the remainder in (d), the answer *must be incorrect.*

Thus, multiply 84271 by 2953.

```
     84271
      2953
    252813
   421355
  758439
 168542
 248852263
```



(a) $8+4=9+3$; $3+2+7=9+3$; $3+1=\mathbf{4}$.
(b) $2+9=9+2$; $2+5+3=9+1$; $\mathbf{1}$. (c) $4\times 1=\mathbf{4}$.
(d) $2+4+8=9+5$; $5+8=9+4$; $4+5=9$; $2+2+6=9+1$; $1+3=\mathbf{4}$.

The remainders, therefore, agree.

Examples 14.

1. Multiply :

(1) 921939 separately by 45, 67, 89 ; 19579 separately by 678, 907.
(2) 926985 „ „ 709, 876 ; 6963519 „ „ 6570, 8090.
(3) 869109 „ „ 6809, 9080 ; 695870 „ „ 8300, 9007.

2. Find the product of

(1)	(2)	(3)	(4)	(5)
7040957	3010925	8219543	7051136	6644585
925719	289576	702761	903106	437980
(6)	(7)	(8)	(9)	(10)
3721967	7152136	1001297	50050016	8012960
90700	370195	30968	3132716	1390972
(11)	(12)	(13)	(14)	(15)
30014973	10000101	10901709	57570057	55103301
903182	7356180	9796484	5086831	47013907
(16)	(17)	(18)	(19)	(20)
6908754	3100031	60000100	19190019	43296851
390283	300043	572130	7191197	390283

3. Find the values of :

(1) 5678×5679; 6009×3729.

(2) 52064×15672; 200917×29565.

(3) 9927869×796080; 8097658×580039.

(4) $972654321 \times 123456789$; $790025001 \times 42621500$.

(5) $100700900500 \times 7000800100$; $4512400415 \times 800900700600$.

4. A mile contains 1760 yards: how many yards are there in 952 miles ?

5. Ghee is sold at 36 rupees a maund; what is the value of 9763 mds. ?

6. A book contains 1192 pages and each page contains 256 words; find the number of words in the book.

7. If 2512010 letters are posted daily in the General Post Office, how many are posted there in a month of 31 days ?

8. A loaded truck weighs 273 maunds ; what is the weight of a train consisting of 59 such trucks and an engine weighing 2927 maunds ?

9. One rupee is equivalent to 192 pies ; how many pies are there in 9250 rupees ?

10. A contractor sold to the Municipality 7625 horses, at 575 rupees each ; what money did he receive ?

11. The engines on the East India Railway consume 41190 maunds of coke daily ; how many maunds lasted them the year 1896, which contained 366 days ?

12. If 41320 persons cross the Hooghly Bridge daily, how many will cross it in a year of 365 days ?

13. Each basket contains 925 mangoes ; how many mangoes are there in 627 such baskets ?

14. Sixty minutes make one hour; how many minutes are there in 893291 hours ?

***15.** A cistern has three spouts ; by the first 240 seers of water, by the second 180 seers, and by the third 125 seers, flow out every hour. It, when the cistern is full, it is emptied by them all in 912 hours. how much water can the cistern hold ?

53. When three or more numbers are multiplied together, *i.e.*, when the product of the first and the second is multiplied by the third, the result by the fourth, and so on, the final result is called the **Continued product** of the numbers, and each of these numbers is called a **factor** of the product.

Thus, the *continued product* of 2, 3, 4, $5 = 2 \times 3 \times 4 \times 5 = 6 \times 4 \times 5 = 24 \times 5 = 120$. Also 2, 3, 4, 5 are called *factors* of 120.

54. Since, $2 \times 3 \times 7 \times 4 = 6 \times 7 \times 4 = 42 \times 4 = 168$,

and $3 \times 4 \times 2 \times 7 = 12 \times 2 \times 7 = 24 \times 7 = 168$;

$\therefore 2 \times 3 \times 7 \times 4 = 3 \times 4 \times 2 \times 7$.

Thus the continued product of any numbers will be the same in whatever order we multiply them.

Examples 15.

1. Find the continued products of :

(1) 12, 15, 10, 13. (2) 6, 7, 8, 5, 6. (3) 16, 18, 19, 20.
(4) 11, 7, 3, 10, 29. (5) 13, 14, 15, 16, 17. (6) 25, 20, 24, 8, 3.
(7) 193, 180, 125. (8) 250, 115, 120. (9) 349, 102, 312, 124.
(10) 9, 111, 25, 100. (11) 875, 340, 500, 63. (12) 785, 212, 305, 401.

2. In a school there are 6 classes ; in every class there are 24 boys ; and in every boy's pocket there are 5 marbles. How many marbles are there altogether ?

3. A Railway train has 47 carriages; in each carriage there are 5 compartments ; in each compartment there are 2 benches ; and on each bench there are seated 5 persons. Find the total number of passengers.

4. In a building there are 40 rooms; in each room there are 18 windows ; in each window there are 28 panes of glass ; how many panes are there in the whole building ?

5. A book contains 875 pages, each page 35 lines, each line 63 letters ; how many letters are there in the book ?

6. A maund contains 40 seers, each seer 16 chhatáks, each chhaták 4 kánchás ; how many kánchás are there in 843 maunds ?

7. In a garden there are 225 mango trees ; each tree has 65 branches, each branch 25 mangoes ; how many mangoes are there altogether ?

8. A piece of stuff contains 112 yards, and each yard costs 25 rupees. Find the price of 56 such pieces of stuff.

55. If a number be multiplied by itself, the product is called the **second power** or **square** of the number.

Thus, 4×4 or $4^2 = 16$; $\therefore$ 16 is the *square* or *second power* of 4.

If a number be twice multiplied by itself, the product is called the **third power** or **cube** of that number.

Thus $4 \times 4 \times 4$ or $4^3 = 64$; $\therefore$ 64 is the *third power* or *cube* of 4.

And so on.

Examples 16.

1. Find the square of :

(1) 1, 2, 3, 4...20 ; 30, 50, 60, 80, 110, 130. (2) 125, 325, 525, 725.

2. Find the cube of :

(1) 1, 2, 3, 4...20 ; 30, 50, 90. (2) 73, 95, 117, 139, 257.

3. Find the values of :

(1) $2^3+3^2-4^2+5^2-6^2+7^2$. (3) $15^2 \div 7^3+14^2-2^3+16^2$.
(2) $2^3+3^3-4^3-5^2+8^2$. (4) $9^4+10^3-16^2+8^3-2^3$.

4. Simplify the following expressions :

(1) $47 \times 2 - 18 \times 7 + 109 \times 11 - 17 \times 4$.

(2) $365 - 75 \times 4 + 44 \times 9 - 39 \times 9$.

(3) $5 \times 6 \times 3 + 4 \times 3 \times 0 - 2 \times 1 \times 4 + 3 \times 6 \times 4$.

(4) $6 \times 3 \times 12 - 2 \times 3 \times 4 - 2 \times 16 \times 2 - 3 \times 2 \times 8 \times 0$.

(5) $-9 \times 6 \times 3 \times 4 + 4 \times 6 \times 3 \times 4 + 3 \times 3 \times 4 \times 2 + 4 \times 3 \times 24$.

(6) $8^2 - 3 \times 4^2 + 9 \times 7^2 - 10 \times 5^2 - 2 \times 14^2 + 5 \times 55^2$.

Examples A.

1. Find the product of the sum and difference of 2479 and 3804.

2. Find the difference between 239 times 4379 and 484 times 2407.

3. How much must be subtracted from the product of 2736 and 895 to produce 7309 ?

4. How much must be added to the product of 7530 and 894 to produce 70000000 ?

5. *A* supplied 638 horses, *B* 794 mules, and *C* 1136 oxen to the Municipality. The average price per head was 72 rupees; find the amount paid.

6. In a meeting hall there are 220 rows of chairs ; each row contains 70 chairs. If 1250 persons attend, how many seats will remain unoccupied ?

7. A gentleman in distributing a sum of money among 625 beggars at 96 rupees each, found that 75 rupees were wanting. Find the sum he had.

8. In distributing a sum of money among 739 poor people at 117 rupees each, I find that I have 112 rupees left. What sum have I ?

9. A farmer sold 216 horses at 567 rupees each and 517 oxen at 45 rupees each, and he bought 412 horses at 495 rupees each. Did he save or borrow money, and how much ?

10. A boy worked 57 exercises, each containing 8 sums, and got 51 sums wrong ; how many did he get right ?

11. A farmer sold 257 horses at 465 rupees each and 439 oxen at 64 rupees each ; if with the proceeds he bought 675 cows at 56 rupees each, how much had he left ?

12. A ship has to sail 15000 miles. She sails 12 miles a day for 12 days ; how much farther has she to sail ?

13. Two steamers ply at the rate of 236 miles and 340 miles respectively per day. By what distance does the one fall behind the other after 136 days ?

CHAP. IX. DIVISION.

56. **Division** is the method of finding how often one given number may be subtracted from another given number, until there is left either nothing or a number less than itself.

The number which *divides* or is subtracted is called the **divisor** ; the number which is *divided* or from which we

subtract, is called the **dividend**; the number which is left is called the **remainder**; and the number of times that the subtraction is performed is called the **quotient.**

Thus, $22-6=16$; $16-6=10$; $10-6=4$; hence 6 can be subtracted 3 times from 22, 4 being left. Here 22 is the *dividend*, 6 is the *divisor*, 3 is the *quotient*, and 4 is the *remainder.*

57. Division is of two kinds, **Simple and Compound.**

The division is called *simple* when the dividend and divisor are either both *abstract* numbers, or both *concrete* numbers of one and the same denomination; or when the divisor is an *abstract* number and the dividend a *concrete* number of one denomination.

The division is called *compound* when the dividend is a *concrete* number of the same kind but of different denominations of that kind, and the divisor is an *abstract* number; or when the dividend and divisor are both *concrete* numbers of the same kind, and one of them at least is of different denominations of that kind.

58. The symbol $\div$, placed between two numbers, denotes that the former is to be divided by the latter. It is read **divided by,** or simply, **by.**

Thus, $24 \div 8$ denotes that 24 is to be divided by 8, and is read 24 *divided by* 8 or 24 *by* 8.

59. It follows immediately from the definition in ART. 56 that Divisor $\times$ Quotient $+$ Remainder $=$ Dividend.

60. When no remainder is left, the division is said to be **exact**; otherwise it is called **inexact.**

Thus, if 15 be divided by 5, the subtraction can be performed 3 times without leaving any remainder, hence the division is *exact*. But when 14 is divided by 3, after 3 is subtracted 4 times from 14, 2 will remain over; hence the division is in this case *inexact.*

When no remainder is left or the division is *exact*, the dividend is the product of the divisor and the quotient; and if the dividend and divisor are both abstract numbers or both concrete numbers of the same kind, the divisor is said to be *contained* in the dividend a number of times equal to the quotient. In such cases, the dividend is called a **multiple** of the divisor, and the divisor a **part** or **sub-multiple** of the dividend.

Thus in the first of the above Examples, $15=5\times3$; therefore 5 is *contained* in 15, 3 times. Also 15 is a *multiple* of 5, and 5 a *part* of 15. Similarly, $24=6\times4$; therefore 6 rupees is *contained* 4 times in 24 rupees, 24 rupees is a *multiple* of 6 rupees, and 6 rupees a *part* of 24 rupees.

SIMPLE DIVISION.

61. When the divisor does not exceed 20 nor the dividend 400, the division may be mentally effected by means of the

Multiplication Tables in pp. 23-24. The following Examples will shew how the operation is performed in such cases.

Ex. 1. Divide 56 by 8.

$8\times7=56$; $\therefore$ $56\div8$ gives **7** as quotient, and **0** as remainder.

Ex. 2. Divide 62 by 8.

$8\times7=56$ which is less than 62; but $8\times8=64$ which is greater than 62. Also $62-56=6$. Therefore $62\div8$ gives **7** as quotient and **6** as remainder.

Ex. 3. Divide 217 by 13.

$13\times16=208$, but $13\times17=221$. Also $217-208=9$. Therefore $217\div13$ gives **16** as quotient and **9** as remainder.

Examples 17.

1. Divide mentally:

(1) 9 by 3; 12 by 4; 24 by 6; 36 by 9; 60 by 10; 44 by 11; 64 by 16.

(2) 48 separately by 2, 3, 4, 5, 6, 8, 12, 14 and 16.

(3) 64 " " 2, 4, 6, 8, 10, 12, 14, 16 and 18.

(4) 108 " " 3, 5, 7, 9, 11, 13, 15, 17 and 19.

(5) 180 " " 4, 6, 7, 9, 10, 12, 13, 14, 15, 16, 17 and 19.

(6) 365 " " 10, 11, 12, 13, 15, 16, 17, 18, 19 and 20.

(7) 226 by 11; 248 by 12; 269 by 13; 277 by 14; 300 by 15.

(8) 225 by 16; 232 by 17; 250 by 18; 361 by 19; 390 by 20.

2. How many times is 16 contained in 148 and how many over? How many times 12 in 190, and how nany over? How many times 15 in 221, and how many over?

3. What is the 7th part of each of the following numbers?

28; 49; 77; 84; 98; 112; 126; 133 and 140.

4. If 20 marbles be divided equally among 5 boys, how many marbles will each boy get?

5. If 120 rupees be distributed equally among 8 men, how much will each man get?

6. 95 oranges are equally divided among a certain number of boys and each boy gets 5 oranges. Find the number of boys.

7. A man can travel 6 miles in one hour; in what time will he travel 96 miles?

8. The sum of 180 rupees was distributed among some beggars, each of them getting 15 rupees. How many beggars were there?

9. How many maunds of rice can be bought for 65 rupees when a maund of rice costs 5 rupees?

10. In a class of 18, 198 mangoes were distributed equally among the boys. How many did each boy get?

11. A seer contains 16 chhatáks; how many seers are there in 192 chhatáks?

12. The monthly consumption of rice in a family is 13 maunds: find in how many months 195 maunds will be consumed.

13. How many men have 140 fingers? how many horses 92 legs?

14. There are 228 boys in a school; in how many lines of 12 each can they be arranged?

***15** A fruiterer bought 240 mangoes at 12 for the rupee and 280 more at 14 for the rupee, and sold the whole lot at 13 for the rupee: what did he gain or lose?

***16.** A man sold for 126 rupees some chairs which had cost him 98 rupees, and gained 2 rupees on every chair; what was the cost price of each chair?

62. **Short Division.** When the dividend is a large number, but the divisor less than 20, proceed as below.

Ex. Divide 3376 by 7.

Starting from the left of the dividend cut off a number greater than 7 but less than 70 (here 33, which represents 33 hundreds): 7 is contained 4 times in 33, and 5 over; therefore it is contained 4 hundred times in 33 hundreds, with 5 hundreds or 50 tens over. Place **400** in the quotient, and carry the 50 tens to the next figure which is 7 and represents 7 tens. Now 50 tens+7 tens=57 tens; and 7 is contained in 57, 8 times with 1 over, therefore in 57 tens 80 times, with 1 ten or 10 over. Place **80** in the quotient, and carry 10. Now 10+6 is 16, in which 7 is contained 2 times with 2 over. Putting **2** in the quotient, we find we have subtracted 7, first 400 times, then 80 times, and then 2 times with 2 over. Thus the quotient is 400+80+2 or 482, and the remainder 2.

The operation may be indicated in the following ways.

$$\begin{array}{r} 7)\underline{3376} \\ 482-2 \end{array} \qquad\qquad \begin{array}{r} 7)3376(482 \\ \underline{28} \\ 57 \\ \underline{56} \\ 16 \\ \underline{14} \\ 2 \end{array}$$

Mental process:

4 times 7, 28, remainder 5
8 " 7, 56, " 1
2 " 7, 14, " **2**

The quotient is **482**, and the remainder **2**.

Examples 18.

1. Divide:

(1) 738 by 2, by 6, by 9; 8432 by 4, by 11.

(2) 97354 by 3, by 5, by 7, by 9, by 11, by 15.

(3) 230625 by 8, by 12, by 13, by 15, by 16, by 17.

(4) 1821077 by 8, by 9, by 12, by 14, by 16, by 18, by 19.

2. Shew that I can divide 2310 apples equally amongst 5 boys, as well as amongst 7 boys, 10 boys, 11 boys, 14 boys, and 15 boys.

3. A man travels 18 miles a day ; in how many days will he travel 32436 miles.

4. A boy gets a certain number of pice from his father every day, and at the end of 16 days finds he has got 496 pice. How much does he get per day ?

***5.** The number of trees in 13 orchards of the same size is 75075 ; if there are 15 rows of trees in each, find the number of trees in each row.

***6.** 4865 rupees are divided between Ram and 13 other persons so that what remains after division goes to Ram. What does Ram get, and what do the others each get ?

63. **Long Division.** When the dividend and divisor are both large numbers, proceed as in the following Examples :

Ex. Divide 433418162 by 615.

```
615)433418162(704744
    4305
    ----
      2918
      2460
      ----
       4581
       4305
       ----
        2766
        2460
        ----
         3062
         2460
         ----
          602
```

Here 433 is less than 615, but 4334 is greater. Now 6 (the first figure of the divisor) goes into 43 no more than 7 times ; hence we put down **7** in the quotient. 615 × 7 is 4305, which being subtracted from 4334 leaves the remainder 29 ; next we bring down 1 from the dividend, put it down to the right of 29 and thus get 291. 291 is smaller than 615 ; therefore we place a **0** to the right of 7 in the quotient, bring down 8 from the dividend, and place it to the right of 291. We thus get 2918. Now 29 does not contain 6 more than 4 times, hence **4** is placed to the right of 0 in the quotient. 615 × 4 = 2460 which is subtracted from 2918, leaving a remainder 458. Proceeding thus until there are no more figures to bring down, we get **704744** as quotient and **602** as remainder.

N. B. In operations, like the foregoing if the lower number happens in any case to be greater than the upper number, the *figure taken in the quotient must be diminished.*

Ex. Divide 28247 by 96.

Here 28 being less than 96, we take 282. 9 goes into 28, 3 times ; but 96 × 3 is 288 which is greater than 282 ; hence 2 should be put in the quotient, and *not* 3.

Note. When the second figure (from the left) of the divisor is 8 or 9, *one* should be added to the first figure of the divisor for a *trial* divisor ; in this case, however, the student must watch that the remainder is not greater than the divisor.

Ex. Divide 21346 by 38.

Here 213 being cut off, we have to see how often 38 goes into it. Now 3 goes into 21, 7 times, while the proper quotient is only 5. If, however we change the 3 in the divisor (because the *second* figure is 8) to 4, we immediately get the quotient wanted, namely, 5.

64. We can prove the correctness of a division in either of the following two ways :

I. Multiply the quotient by the divisor, and add the remainder to the product. The sum should be equal to the dividend.

II. By **casting out the nines.**

(*a*) Divide the sums of the figures in the divisor and the quotient separately by 9, and multiply the remainders.

(*b*) Divide the sum of the figures in the remainder by 9.

(*c*) Add the remainder in (*b*) to the product in (*a*), and divide the sum by 9.

(*d*) Add the figures in the dividend, and divide the sum by 9. If the remainders in (*c*) and (*d*) do not agree, the *division is incorrect.*

In Example, ART. 63,

6+1+5=12, *remainder* 3 ; 7+4+7+4+4=26, *remainder* 8 ; 3×8=24 ;

6+0+2=8, *remainder* 8 ;

24+8=32, *remainder* 5 ;

4+3+3+4+1+8+1+6+2=32, *remainder* 5.

N. B. It is obvious that this test will fail when the mistake made is such that the sum of the figures either remains unaltered, or is increased or diminished by 9 or a multiple of 9.

Thus, in the same Example, if the quotient obtained be 704474 or 7047044 or 7807445, we shall in every case get the same remainder after division by 9 and the test will consequently fail.

Examples 19.

1. Divide :

(1) 1463049 by 27. (2) 2156875 by 35. (3) 9316468 by 49.
(4) 5713848 by 54. (5) 84149802 by 66. (6) 57138543 by 99.
(7) 7086498 by 87. (8) 82151568 by 72. (9) 6848734656 by 96.
(10) 131047560 by 15, by 35, by 57, by 69, by 72, by 85, and by 98.

2. Find the value of :

(1) 28894545+123. (2) 8419448+144. (3) 433418175+615.
(4) 4461387+132. (5) 7153961+512. (6) 6901213+840.
(7) 61201703+795. (8) 60213169+961. (9) 713061311+773.
(10) 8861594+1026. (11) 47603136+2149. (12) 86043759+3916.
(13) 46021307+4601. (14) 721906+1960. (15) 596802+5760.
(16) 62934820+52312. (17) 7050429+61073. (18) 51980631+19890.

(19) 11021031 ÷ 69380.
(20) 11021031 ÷ 21570.
(21) 8230124037 ÷ 463025.
(22) 12345678901 ÷ 456789.
(23) 6343945̂67 ÷ 463025.
(24) 1220225292 ÷ 20563.
(25) 742892741 5293 ÷ 8496417.
(26) 604350745368845 ÷ 79094451.
(27) 932580093425 ÷ 42473211.
(28) 75328345621 ÷ 3331456.
(29) 8976543210987 ÷ 42573211.
(30) 6634567890012345 ÷ 21545678.

3. How often are 23, 34 and 47 separately contained in 36754 ?

4. If 30321 rupees be equally divided amongst 27 men, how much will each receive ?

5. Sound travels at the rate of 1132 feet per second ; how many seconds will elapse before it travels 101880 feet ?

6. The yearly consumption of flour in a garrison containing 98765 soldiers was 1185180 sacks ; how much provision was allowed to each soldier yearly ?

7. 18144 eggs are to be packed in boxes, each box holding 144 eggs; how many boxes are required ?

***8**. What number besides 123, will exactly divide 28894545 ?

***9**. The yearly death-rate of a town containing 295600 people is 1 in 50 ; find the total number of deaths in a year.

10. In a class of 67, 5760 oranges were distributed equally among the boys ; how many were left after the distribution ?

11. The annual expenditure of a family is 14600 rupees ; if a year consist of 365 days, find the daily expenditure.

12. A year consists of 12 months ; how many years are equal to 29472 months ?

13. From a garden of 345 trees, I gathered 1715340 fruits ; how many fruits on the average did I gather from each tree ?

14. A ship sails at the rate of 168 miles a day ; how long will it take to sail from Bombay to New York, a distance of 20328 miles ?

15. If horses are worth 715 rupees each, how many horses can be bought for 228800 rupees ?

***16**. Prove that 1701050 rupees can be equally divided between 25 persons, between 26 persons and also between 2617 persons.

65. The operation of division may be cut short, when the divisor ends with cyphers.

Ex. Divide 796224 by 3300, and 865400 by 21000.

```
3300)796224(241        33)7962,24(241        21)865,400(41
     6600                 66                    84
     -----                ----                  ----
     13622                136
     13200                132
     -----                ---
      4224                 42                      4,400
      3300                 33
      ----                ----
       924                 9,24
```

We may therefore cut off the cyphers at the end of the divisor, mark off an equal number of figures at the end of the dividend, and divide the remaining figures of the dividend by the remaining figures of the divisor. The quotient thus obtained will be the required quotient, and the number formed by the figures of the remainder followed by the figures omitted, will be the required remainder.

The truth of the above process will be clear from the following considerations. Taking the first of the above two Examples,

796224 = 7962 hundreds + 24. 7962 hundreds contains 33 hundreds 241 times with 9 hundreds over; and as 9 hundreds together with 24 cannot contain 33 hundreds, the quotient remains 241, with the remainder 9 hundreds + 24 or 924.

66. Division by factors. Proceed thus:

Ex. Divide 9385437 by 462.

$$462 = 6 \times 7 \times 11.$$

$$\begin{array}{r|l} 6 & 9385437 \\ \hline 7 & 1564239 - 3 \\ \hline 11 & 223462 - 5 \\ \hline & 20314 \;\; - 8 \end{array}$$

Thus the quotient is **20314**.
The remainder is found thus:
$7 \times 8 + 5 = 61$; $6 \times 61 + 3 =$ **369.**

The RULE for finding the remainder is this: *Counting upwards from below,* multiply the second factor by the first remainder and add the second remainder to the product: multiply the sum by the third factor and add the third remainder to the product. Proceed thus, until the last factor and the last remainder are disposed of.

Examples 20.

1. Divide:

(1) 45678 separately by 10, 20, 30, 50, 70, 90.
(2) 89012 „ „ 100, 200, 400, 500, 600.
(3) 4967123 „ „ 1100, 1200, 1300, 1600.
(4) 8795412 „ „ 1700, 1900, 21000, 427000.
(5) 62938206 „ „ 52300, 58000, 72900.
(6) 413780 „ „ 4600, 1760, 2810, 9710.
(7) 47891900 „ „ 67000, 376000, 729000.

2. Divide by factors:

(1) 429561 by 65, by 72, by 84, and by 96.
(2) 5699251 separately by 108, 162, 228, 405.
(3) 789462151 „ „ 105, 462, 504, 760.
(4) 942816415 „ „ 840, 960, 1080, 1320.

66(*a*). When the signs × and ÷ follow each other in an expression, perform the operations in the order in which they occur.

Ex. 1. $5 \times 24 \div 12 = 120 \div 12 = \mathbf{10}$.

Ex. 2. $128 \div 8 \times 7 = 16 \times 7 = \mathbf{112}$.

66(*b*). When in an expression $\times$ and $\div$ occur with $+$ and $-$, the operations of multiplication and division must be performed before those of addition and subtraction.

Ex. $120 - 17 \times 4 + 890 \div 5 = 120 - 68 + 178 = 230$.

Examples B.

1. $22 \times 12 \div 33$.

2. $34 \times 15 \div 51$.

3. $125 \times 16 \div 20 \div 5$.

4. $220 \times 24 \times 60 \times 18 \div 99$.

5. $325 \div 13 \times 32$.

6. $980 \div 70 \times 29$.

7. $660 \div 44 \times 58 \div 87 \times 5$.

8. $625 \div 25 \times 64 \div 80 \times 22$.

9. $8 \times 12 + 9 \times 5 + 11 \times 20 + 17 \times 14$.

10. $12 \times 15 - 20 \times 17 + 18 \times 13 - 14 \times 4 + 19 \times 16$.

11. $225 \div 15 + 114 \div 16 + 153 \div 17 + 290 \div 58$.

12. $18 \times 15 \div 45 - 180 \div 20 \times 3 + 280 \div 7 \times 18 - 220 \div 55 \times 8$.

13. $22 \times 12 \div 33 - 34 \times 15 \div 51 + 325 \div 13 \times 32 - 980 \div 70 \times 29$.

14. $525 \times 7 - 10320 \div 40 + 360 \div 6 - 1280 \div 80 + 15 \times 8$.

15. $6480 - 1020 \times 7 + 12760 \div 11 - 2700 \times 8 \div 30 + 580 \div 29 \times 72$.

CHAP. X. BRACKETS.

67. When two or more numbers, connected by $+$, $-$, $\times$ or $\div$, are to be taken collectively, they are either enclosed in the signs ( ), { }, [ ], or put under a line ——. The signs are called **brackets**, and the line a **vinculum**.

NOTE.—All the rules given below in respect of *brackets* apply equally to the vinculum.

68. When an expression is enclosed in a pair of brackets, the operations indicated inside the brackets should be performed before the brackets are removed.

Ex. (1) $8 - \overline{3+4} = 8 - 7 = \mathbf{1}$.

(2) $18 - (3 + 4 - 2 + 1) = 18 - (8 - 2) = 18 - 6 = \mathbf{12}$.

(3) $(7 \times 4 - 30 \div 3) + 6 = (28 - 10) + 6 = 18 + 6 = \mathbf{24}$.

69. When a number is put *immediately* before an expression within a pair of brackets, this number multiplies the number obtained after the brackets are removed.

Thus, $3 + 2(7 - 5) = 3 + 2 \times 2 = \mathbf{7}$; $5(9 - 3) = 5 \times 6 = \mathbf{30}$.

70. If more pairs of brackets than one are used, the operations within an *inner* pair should be performed and the brackets removed, before anything is done to the outer brackets.

Thus, (1) $2+\{23-2(5+7-6)+3\}$
$=2+\{23-2(12-6)+3\}$
$=2+\{23-12+3\}$
$=2+14=\mathbf{16}.$

(2) $5-[28+7-\{5\times 7-(7-\overline{5-3})\}]$
$=5-[35-\{35-(7-2)\}]$
$=5-[35-\{35-5\}]$
$=5-[35-30]=5-5=\mathbf{0}.$

Note. As a rule the vinculum is used inside the brackets; ( ) inside the other brackets; and { } inside [ ].

71. When a pair of brackets is preceded by + (*plus*), the brackets may be removed without affecting the result.

$\because\ 5+(8-3)=5+5=10$, and $5+8-3=13-3=10$.
$\therefore\ 5+(8-3)=5+8-3.$

71. When a pair of brackets is preceded by —(*minus*), it may be removed after changing, inside the brackets, all the positive signs into negative and all the negative into positive signs.

$24-(8+7-3)=24-12=12$;
also $24-8-7+3=27-15=12$;
$\therefore\ 24-(8+7-3)=24-8-7+3.$

Examples 21.

1. Find the values of :

(1) $(5+2)-(4+3)+(10-3)-(14+6)+(18+7).$

(2) $6+4(14-6)-(6-4).$

(3) $4+3+2+(4-3+2)-(4+\overline{3-2})-(4-\overline{3-2}).$

(4) $16-[(16-6)-\{12-(12-3)\}+3].$

(5) $18+26-[18+6+\{26-(16-\overline{8-6})\}].$

(6) $87642+798762-\{368764+4201-(15706-3207)\}.$

2. Find the difference between $(2917-932)$ and $(954-467)$.

3. What number subtracted from $(2471+56)$ will leave $(2754-2683)$ as remainder ?

4. What is left when 1874 is subtracted from $(56739-2864)$?

5. By how much does the sum of $(5879-989)$ and 7003 exceed their difference ?

6. Find the difference between
$(8671-432)-187$ and $(539-64)-(8+430)$.

7. Find the values of :

(1) $10 \times 250 - 15(2071 - 1993) + (2183 - 2090) - 120(80 - 79)$.

(2) $5629 - (3 \times 450 - 5 \times 204) \div 11 + 264(980 - 644)$.

(3) $(4 \times 5 + 3 \times 5) + (3 \times 6 + 2 \times 6) + (4 \times 5 - 3 \times 5) - (3 \times 6 - 2 \times 6)$.

(4) $2 \times 560 + \{405 \times 410 + 2 \times 410 + 500) - 4 \times 500\} + 256 \div 16$
$- 3\{2 \times 560 - (3 \times 410 - 500)\}$.

(5) $4 \times 2 \times 15 + \{14 \times 5 - (4^2 + 3^2)\} - \{3^2(4^2 - 4 \times 3) + 43 \times 3\}$.

(6) $4 \times 10 - [(4 \times 10 - 2 \times 9) - \{3 \times 10 - (2 \times 9 - 2 \times 3)\}]$.

(7) $16 \times 15 - [16 \times 15 - \{2 \times 16 \times 15 - 2 \times 16 \times 15 - 15^2)\}]$.

(8) $2 \times 21 + [2 \times 31 - \{5 \times 21 - 6 \times 31 + (2 \times 31 + 3 \times 21)\}]$.

(9) $\{2 \times 5 \times 7^2 - (9^2 - 7 \times 9)\} - \{9^2 \times 2 - (4 \times 7 \times 9 - 4 \times 7^2)\}$
$+ 4\{2 \times 9^2 - (3 \times 7 \times 9 - 7^2)\}$

CHAP. XI. MISCELLANEOUS PROPOSITIONS.

The following rules and methods will be found very useful in working out examples.

73. Sum, difference, &c.

(1) When the sum and difference of two numbers are given, to find the numbers.

RULE. *Add the given sum and the given difference, and divide the result by 2.* The quotient is the larger of the two numbers required.

Ex. 1. The sum and difference of two numbers are 84932 and 326, respectively ; find the numbers.

The larger number $= (84932 + 326) \div 2 = 85258 \div 2 =$ **42629**.

The smaller number $= (84932 - 42629) =$ **42303**.

Ex. 2. Divide 1280 rupees between two men so that one of them may get 320 rupees more than the other.

The sum of the two shares is 1280 rupees and the difference 320 rupees.

$\left.\begin{array}{l}\therefore \text{ the share of the first man} = (1280 + 320) \div 2 \text{ or } \mathbf{800} \text{ rupees ;} \\ \text{and the share of the other ,, } = (800 - 320) \quad \text{ or } \mathbf{480} \quad \text{ ,,}\end{array}\right\} \textit{Ans.}$

(2) The sums of every two or three given numbers being given, to find the numbers.

RULE. *Add together the three given sums, divide the result by 2, and from the quotient subtract the three sums separately.*

Ex. 1. The sum of the first and the second of 3 numbers is 50 ; that of the first and the third is 40 ; and that of the second and the third is 42. Find the numbers.

$(50 + 40 + 42) \div 2 = 132 \div 2 = 66.$

$\left.\begin{array}{ll}\therefore \text{ the first number} & = 66 - 42 = 24, \\ \text{,, second ,,} & = 66 - 40 = 26, \\ \text{and ,, third ,,} & = 66 - 50 = 16.\end{array}\right\} \textit{Ans.}$

Ex. 2. Ram and Hem had together 125 rupees, Ram and Shyam together 360 rupees and Hem and Shyam together 375 rupees. Find the sum of money which each of them had.

*Rs.*125 + *Rs.*360 + *Rs.*375 = *Rs.*860.

∴ they together had *Rs.*860 ÷ 2 = *Rs.*430.

∴ Ram had *Rs.*430 − *Rs.*375 = *Rs.*55,
Hem „ „ 430 − „ 360 = „ 70,
and Shyam „ „ 430 − „ 125 = „ 305. } *Ans.*

74. Product, quotient, remainder, &c.

(1) When the divisor, the quotient, and the remainder are given, to find the dividend.

RULE. *Multiply together the divisor and the quotient, and add the remainder to the product.*

Ex. 1. A certain number being divided by 423, gives 534 as quotient and 57 as remainder; find the number.

The reqd. number (dividend) = 423 × 534 + 57 = **225930**.

Ex. 2. A certain number of oranges is divided amongst 24 boys; each boy gets 31 oranges and 7 are left. Find the number of oranges.

The reqd. number of oranges = 24 × 31 + 7 = **751**. *Ans.*

(2) When the dividend, the quotient, and the remainder are given, to find the divisor.

RULE. *Subtract the remainder from the dividend, and divide the difference by the quotient.*

Ex. 1. The dividend is 195063, the quotient 495, and the remainder 33; what is the divisor?

The divisor = (195063 − 33) ÷ 495
= 195030 ÷ 495 = **394**.

Ex. 2. 4857 bricks are divided into heaps of 64 each, with 57 over; find the number of heaps.

The required number = (4857 − 57) ÷ 64 = 4800 ÷ 64 = **75**.

(3) What *least number* must be added to a given number to make it exactly divisible by a second given number?

RULE. *Divide the first number by the second, and subtract the remainder from the second given number.*

Ex. 1. What least number must be added to 84325 to make it exactly divisible by 326?

84325 ÷ 326 gives 258 as quotient and 217 as remainder:
∴ the number to be added = 326 − 217 = **109**.

Ex. 2. What least number of 4 digits has 39 for a factor?

The least number of 4 digits is 1000, which when divided by 39, gives the remainder 25.

∴ the reqd. no. = (39 − 25) + 1000 = **1014**.

(4) What *least number* must be subtracted from a given number to make it divisible by a second given number ?

RULE. *Divide the first given number by the second, and find the remainder.*

Ex. What least number must be subtracted from 720 to make it exactly divisible by 25 ?

720÷25 gives 28 as quotient and 20 as remainder.
∴ the number to be subtracted=**20**.

72. **Progressive series.**

To find the sum of any given numbers increasing by 1.

RULE. *Add the two extreme numbers, multiply the sum by the number of terms, and divide the product by 2.*

Ex. 1. Add together $1+2+3+4+5+...+20$.
The number of terms is obviously 20.
∴ the sum$=20\times(20+1)\div 2=$**210**.

Ex. 2. Find the sum of $5+6+7+8+...+35$.
Here the number of terms will be found to be 31.
∴ the sum$=31\times(5+35)\div 2=$**620**.

Note. The above method also holds good in adding any given numbers increasing by some fixed number.

Examples 22.

1. 5624 and 2490 are respectively the sum and difference of two numbers ; what are those numbers ?

2. The sum and difference of two numbers are 825 and 729 respectively ; find the numbers.

3. Divide 1000 apples between two persons *A* and *B*, so that *A* may receive 236 apples more than *B*.

4. I bought a house and a garden for 25629 rupees, the garden costing 6229 rupees less than the house ; find the price of each.

5. The ages of Ram and Gopal are together 74 years, those of Ram and Hari together 58 years, and those of Gopal and Hari together 52 years. Find their respective ages.

6. A horse and an ox are together worth 750 rupees, a horse and a cow together 650 rupees, and an ox and a cow together 196 rupees. Find the price of a horse, of an ox, and of a cow.

7. A certain number being divided by 274, gives 599 as quotient and 133 as remainder ; find the number.

8. The quotient arising from the division of a certain number by 5264 is 429 and the remainder is 2420 : find the number.

9. The quotient arising from the division of 150906 by a certain number is 465 and the remainder is 246. Find the divisor.

10. By what number must 243915 be divided so that the quotient may be 562, and 321 over?

11. What least number must be added to 3764259 to make it exactly divisible by 724?

12. Find the least number of rupees that should be added to 74625 rupees to make the sum equally divisible among 2372 persons.

13. What least number must be subtracted from 728914 to make it exactly divisible by 275?

14. What least number must be taken from 92146 so that the remainder may be exactly divisible by 625?

15. Add together $1+2+3+4+\ldots+40$.

***16.** Add together $7+8+9+\ldots\ldots+65$.

***17.** Add together $8+12+16+\ldots\ldots+80$.

***18.** How many times will a clock strike during 24 hours?

***19** Add together all the *odd* numbers, between 1 and 101, and all the *even* numbers between 2 and 100.

***20.** A boy saves 1 pice on the first day, 3 pice on the second day, 5 pice on the third, and so on; how much will he save in 10 days?

***21.** Find the least and greatest numbers of 5 digits each, which are exactly divisible by 347.

***22.** Find the greatest and least numbers of 8 digits each, which are exactly divisible by 2946.

76. Subtraction.

To subtract a given number from another number consisting of 1 followed by cyphers only.

RULE. *Set down as many nines as there are cyphers in excess of the number of figures in the subtrahend; then* (beginning from the left) *put down successively the differences of the several figures from nine and of the* unit *figure from ten.*

Ex. Subtract 8019 from 10000000.

There are 7 cyphers in the minuend, and 4 figures in the subtrahend: hence we set down 999. Again, 8 from 9 is **1**, 0 from 9 is **0**, 1 from 9 is **8**, 9 from 10 is **1**. Therefore the required difference is **999198**.

N. B. The foregoing Rule will be of great use to the students when working out examples in Decimals.

77. Abbreviated methods of Multiplication.

(1) To multiply a number by 5.

RULE. *Affix a cypher to the right of the multiplicand, and divide the result by 2.*

Ex. Multiply 12546 by 5.

The required product $=125460\div2=$ **62730**.

$[12546\times5=12546\times10\div2=125460\div2]$.

(2) To multiply a number by 25 or 5^2.

RULE. *Affix 2 cyphers to the right of the multiplicand, and divide the result by 4.*

Ex. Multiply 654211 by 25.

$$4\underline{|65421100}$$
$$16355275 = \text{the required product.} \quad \textit{Ans.}$$

[$654211 \times 25 = 654211 \times 100 \div 4 = 65421100 \div 4$.]

(3) To multiply a number by 125 or 5^3.

RULE. *Affix 3 cyphers to the right of the multiplicand, and divide the result by 8.*

Ex. Multiply 37483 by 125.

$$8\underline{|37483000}$$
$$4685375 = \text{the required product.} \quad \textit{Ans.}$$

[$37483 \times 125 = 37483 \times 1000 \div 8 = 37483000 \div 8$.]

(4) To find the square of a number which ends in 5.

RULE. *Leave out the 5. Multiply the number formed by the remaining figures by the next higher number, and affix 25 to the product.*

Ex. Multiply 165 by itself.

$$16 \times 17 = 272.$$

$\therefore$ the required product is **27225**.

[The square of a number like 124 may be found thus:

The unit figure $= 4$. $124 + 4 = 128$; $124 - 4 = 120$.

$124^2 = 128 \times 120 + 4^2 = 15360 + 16 = \mathbf{15376}$.]

(5) To find the product of two numbers one of which differs by a small number from 100, 1000, 10000, &c., or from 50, 500, &c.

Proceed as in the following Examples.

Ex. 1. Multiply 46828 by 96, and by 994.

$96 = 100 - 4$.	$994 = 1000 - 6$.
$46828 \times 100 = 4682800$	$46828 \times 1000 = 46828000$.
$46828 \times 4 = 187312$	$46828 \times 6 = 280968$.
$\therefore$ the product $=$ **4495488**	$\therefore$ the product $=$ **46547032**.

Ex. 2. Multiply 3867 by 498.

$498 = 500 - 2$.

$3867 \times 5{,}00 = 3867000 \div 2 = 1933500$

$3867 \times 2 \qquad = \quad 7734$

$\therefore$ the required product $= \mathbf{1925766}$.

(6) To multiply one number by another which can be resolved into factors each less than 20.

Proceed thus :

Ex. Multiply 82547 by 462.

$462 = 11 \times 7 \times 6.$

$$\begin{array}{r} 82547 \\ 11 \\ \hline 908017 \end{array} \qquad \begin{array}{r} 908017 \\ 7 \\ \hline 6356119 \end{array} \qquad \begin{array}{r} 6356119 \\ 6 \\ \hline \mathbf{38136714} \end{array}$$

(7) When the multiplier is such that any group of figures in it is a multiple of any figure or group of figures by which the multiplicand has been previously multiplied, the process of multiplication may be considerably shortened as will be seen from the Examples below.

Ex. 1. Multiply 85683 by 426.

$$\begin{array}{r} 85683 \\ 426 \\ \hline 514098 \\ 3598686 \\ \hline \mathbf{36500958} \end{array} \qquad [42 = 7 \times 6 \; ; \; \therefore 85683 \times 42 = 514098 \times 7.]$$

Ex. 2. Multiply 546185 by 168217.

$$\begin{array}{r} 546185 \\ 168217 \\ \hline 3823295 \\ 11469885 \\ 91759080 \\ \hline \mathbf{91877602145} \end{array} \qquad \begin{array}{l} [21 = 7 \times 3 \; ; \; \therefore 546185 \times 21 = 3823295 \times 3.] \\ [168 = 21 \times 8 \; ; \; \therefore 546185 \times 168 = 11469885 \times 8.] \end{array}$$

Note. Since 21 consists of 2 figures, therefore the product by 168 has been removed by 2 places to the left, instead of by only 1 place as ordinarily. See ART. 49.

Examples 23.

1. Multiply :

(1) 83932 separately by 5, 25, 75, 125.

(2) 1932765 „ „ 25, 125, 625.

(3) 845215 64 „ „ 99, 999, 9999.

(4) 7486 „ „ 93, 992, 9991, 9994.

2. Find the square of :

(1) 35, 45, 55, 65, 75, 85, 95, 115, 125, 145, 265, 425.

(2) 42, 58, 83, 97, 106, 132, 156, 117, 172, 183, 195, 197.

3. Multiply by factors :

786 by 504 ; 1256 by 585 ; 1625 by 1224.

4. Subtract 428 from 1000000; 72946 from 1000000000.

5. Multiply 38964 by 328; 52847 by 369; and 12345 by 252426.

78. Abbreviated methods of Division.

(1) To divide a number by 5, by 50, by 500, &c.

RULE. *Multiply the dividend by 2, and divide the product by 10, by 100, by 1000, &c.*, as in ART. 65. The true remainder, however, is the remainder so obtained, divided by 2.

Ex. Divide 34892 by 5, and 486937 by 500.

34892 × 2	486937 × 2
69784 ÷ 10	973874 ÷ 1000
6978,4	973,874
∴ the quotient is **6978**, and the remainder **2**.	∴ the quotient is **973**, and the remainder **437**.

(2) To divide a number by 25, by 75, &c.

RULE. *Multiply the dividend by 4, and divide the product by 100, by 300, &c.*, as in ART. 65. The true remainder is the remainder so obtained, divided by 4.

Ex. Divide 312478 by 25, and 43781 by 75.

312478 × 4	43781 × 4
1249912 ÷ 100	175124 ÷ 300
12499,12	583,224
∴ the quotient is **12499**, and the remainder **3**.	∴ the quotient is **583**, and the remainder **56**.

(3) To divide a number by 125, 375, &c.

RULE. *Multiply the dividend by 8, and divide the product by 1000, by 3000, &c.*, as in ART. 65. For the true remainder, divide the remainder so obtained by 8.

Ex. Divide 92478 by 125, and 7821354 by 375.

92478 × 8	7821354 × 8
739824 ÷ 1000	62570832 ÷ 3000
739,824	20856,2832
∴ the quotient is **739**, and the remainder **103**.	∴ the quotient is **20856**, and the remainder **354**.

(4) The process of Division may be performed much more quickly by proceeding as follows:

Ex. Divide 253848 by 346.

```
346)253848(733
     1164
      1268
       230
```

Mentally thus: 7 × 6 = 42 and **6**, 48;
carry 4, add 7 × 4, 32 „ **1**, 33;
carry 3, add 7 × 3, 24 „ **1**, 25.
Similarly for the other figures in the quotient.

N. B. This process, if mastered, will be of very great use in quickly working out division sums.

Examples 24.

Divide :

1. 46281 separately by 5, 50, 500.
2. 7824645 „ „ 50, 500, 5000.
3. 942816 „ „ 25, 75, and 150
4. 2468145 „ „ 125, 375, and 875.
5. 82946 by 328 and 548714 by 7846.

79. Averages, Shares, &c.

Proceed as in the following Examples.

Ex. 1. I got 12 mangoes on the first day, 20 on the second, 28 on the third, 11 on the fourth and 14 on the fifth. How many mangoes did I get daily on the average?

In 1 day I got 12 mangoes
„ 1 „ 20 „
„ 1 „ 28 „
„ 1 „ 11 „
„ 1 „ 14 „
∴ „ 5 days „ 85 „

$85 \div 5 = 17$.

∴ the average was 17 mangoes.

Ex. 2. My annual income was *Rs* 400 in each of the first five years, *Rs* 500 in each of the next two, and *Rs* 600 in each of the following three. What was my average annual income?

In 5 years I earned ($Rs.400 \times 5$) or $Rs.2000$.
„ 2 „ „ („ 500×2) „ „ 1000.
„ 3 „ „ („ 600×3) „ „ 1800.
∴ in 10 years „ „ *Rs*.4800.

10|*Rs*.4800
Rs 480.

∴ the average was *Rs*.480. *Ans.*

Ex. 3. Divide 9000 rupees among *A*, *B*, *C* so that for every rupee given to *A*, *B* may get 2 rupees and *C*, 3 rupees.

$1 + 2 + 3 = 6$, $Rs\ 9000 \div 6 = Rs.1500$.

∴ *A*'s share is *Rs*. $1500 \times 1 = Rs\ 1500$ }
B's „ „ „ $1500 \times 2 =$ „ 3000 } *Ans.*
C's „ „ „ $1500 \times 3 =$ „ 4500 }

Ex. 4. Divide 3750 apples amongst four boys, so that for every 6 apples given to the first, the second boy may get 4, the third 3 and the fourth 2

$6 + 4 + 3 + 2 = 15$; $3750 \div 15 = 250$.

∴ the first boy gets 250×6 or 1500 apples, }
second „ „ 250×4 „ 1000 „ } *Ans.*
third „ „ 250×3 „ 750 „ }
fourth „ „ 250×2 „ 500 „ }

Ex. 5. A house and a garden together cost 52836 rupees; the house cost 5 times the garden: what is the cost of each?

If the garden costs 1 rupee, the house costs 5 rupees; also $1+5=6$.

$\left.\begin{array}{l}\therefore \text{ the cost of the garden} = Rs.52836 \div 6 = Rs.8806 \\ \therefore \quad ,, \quad ,, \quad ,, \quad ,, \text{ house } = Rs.\ 8806 \times 5 = Rs.44030\end{array}\right\} Ans.$

Ex. 6. Two persons *A* and *B* set to walk at the same time, *A* from Calcutta towards Krishnagar at the rate of 4 miles an hour and *B* from Krishnagar to Calcutta at the rate of 5 miles an hour. If the distance between the two places be 72 miles, when will they meet, and how many miles will each of them have walked?

For every 4 miles that *A* walks, *B* walks 5. Also they will meet when they have walked 72 miles between them.

$4+5=9$; $72 \div 9 = 8$. ∴ they meet after 8 hours.

$\left.\begin{array}{l}\text{Again, } A \text{ will have walked } 4 \times 8 \text{ or } 32 \text{ miles,} \\ \text{and} \quad B \quad ,, \quad ,, \quad ,, \quad 5 \times 8 \text{ or } 40 \quad ,, \quad .\end{array}\right\} Ans.$

Examples 25.

1. In a school there are 6 classes, in the first there are 97 boys, in the second 63, in the third 67, in the fourth 59, in the fifth 47 and in the sixth 39. Find the average number of boys in each class

***2.** A cloth-merchant bought 250 yards of silk at R5 per yard, 300 yards of velvet at R7 per yard, and 50 yards of brocade at R17 per yard. Find the average price per yard.

***3.** The yearly expenditure of a person during the first 4 years was Rs. 700, during the next 6 years Rs 800, and during the following 5 years Rs.400. What was his average expenditure?

4. A trader gained Rs.3420 in January, Rs.3450 in February, Rs.4650 in March, Rs 720 in April, Rs 8100 in May, and Rs.6300 in June; find his average monthly gain.

5. Divide 1200 rupees among *A*, *B*, *C*, so that for every 2 rupees given to *A*, *B* may get 4 rupees and *C*, 6 rupees.

6. Divide 6250 apples among *A*, *B*, *C*, so that for every 5 apples given to *A*, *B* may get 9 apples and *C*, 11.

7. A table and a chair together cost 75 rupees; the table cost four times as much as the chair; find the cost of the table.

8. A horse and its saddle are together worth 750 rupees; if the horse be worth nine times the saddle, find the cost of the horse.

***9.** Two trains start at the same time from the opposite ends of a railroad, one running with a speed of 25 miles and the other of 20 miles an hour. If the length of the road be 495 miles, when will they meet, and how many miles will each of them have travelled?

***10.** The distance between Calcutta and Burdwan is 66 miles. A man starts from Calcutta for Burdwan walking at the rate of 5 miles an hour; at the same time another leaves Burdwan for Calcutta walking at the rate of 6 miles an hour. When and where will they meet?

Examples worked out.

Ex. 1. Ram had *Rs.*7 more than Shyam, Shyam *Rs.* 21 more than Hem, Hem *Rs.*12 less than Gopal who had *Rs.*25. What sum had Ram ?

Gopal had *Rs.*25 ;

$$\left.\begin{array}{l}\therefore \text{ Hem had } Rs.25 - Rs.12 = Rs.13, \\ \therefore \text{ Shyam ,, ,, } 13 + \text{ ,, } 21 = \text{ ,, } 34, \\ \therefore \text{ Ram ,, ,, } 34 + \text{ ,, } 7 = \text{ ,, } 41.\end{array}\right\} \textit{Ans.}$$

Ex. 2. What is that number which if I divide by 4, to the quotient add 15, from the sum take 19, to the remainder add 9, the sum is 30 ?

The required number $= (30 - 9 + 19 - 15) \times 4 = 25 \times 4 = 100$. *Ans.*

Note that, in examples like the foregoing, we go backwards from the last number, and change Addition into Subtraction, Subtraction into Addition, multiplication into Division, and Division into multiplication.

Ex. 3. I lose *Rs.*40 on the first day, gain *Rs.*56 on the second, gain *Rs* 32 on the third, lose *Rs.*120 on the fourth, and have then only *Rs.*24 left. What sum had I at first ?

The required sum in rupees $= 24 + 120 - 32 - 56 + 40$
$= 184 - 88 = 96$. *Ans.*

Ex. 4. In doing a piece of work, how many children should be employed instead of 16 men and 24 women, if one man can do as much work as two women and one woman as much as three children ?

1 man $= 2$ women $= 2 \times 3$ or 6 children ;
$\therefore$ 16 men $= 16 \times 6$ or 96 children.
Again, 24 women $= 24 \times 3$ or 72 ,, ;
$\therefore$ the number of children required $= 96 + 72 = 168$. *Ans.*

Ex. 5. A man spends Rs. 100 a month for three months and saves some money. He increases his monthly expenses to *Rs.* 160 for the next 9 months and has nothing left. What was his monthly income ?

His expenditure in the 12 mos. $=$ *Rs.*$100 \times 3 +$ *Rs.*$160 \times 9 =$ *Rs.*1740.

Now at the end of the year he had nothing left ; therefore his income in the 12 mos. also must have been exactly the same, *viz.* *Rs* 1740 ;

$\therefore$ his monthly income $=$ *Rs.*$1740 \div 12 =$ *Rs.*145. *Ans.*

MISCELLANEOUS EXAMPLES 1.

1 (*a*) What number must be added to 834256 to make 1093263 ?

(*b*) What number must be subtracted from 852932 to leave 312511 as remainder ?

2. The difference of two numbers is 31256, and the smaller of them is 13453 ; find the greater number.

3. The sum of two numbers is 932854, and the greater of them is 621353 ; find the smaller.

4. The difference of two numbers is 1132468, and the greater of them is 63243l5 ; find the smaller.

5 The sum of three numbers is 34462; the first is 19842, and the second is 7250 less than the first; what is the third number?

6. What number must be added to 715 × 4231 to make it equal (1) to the least number of 8 digits, (2) to the greatest number of 8 digits?

7 Ram and Shyam together possess 89234 rupees. If Ram's share be 62345 rupees, find Shyam's share.

8 A house costs 9275 rupees more than its furniture; what is the house worth, if the value of the furniture is 17225 rupees?

9. A boy had 27 pice, another had 21 more than the first, and the third 15 more than the second; find the number of pice they had together.

10. A man had 5 sons; the age of the youngest son was 3 years, the fourth was 4 years older than the youngest, the third 5 years older than the fourth, the second 2 years older than the third; also the age of the first was 7 years more than the sum of the ages of the last two, and the father's age 3 years more than the joint ages of the sons, find the age of the father and that of the eldest son.

11. Buying a bigha of land I sold it back for 250 rupees, losing thereby 175 rupees, what did it cost me?

12. By how much does the sum of 17 and 25 exceed their difference?

13 *A* has **Rs.** 17 with him. He gives *B* **Rs.** 5, and then *B*'s money becomes the same as his, what had *B* at first?

14. I give 54 rupees to my servant to buy two shawls, one of which costs 12 rupees more than the other. Find the price of each shawl.

15 In a meeting composed of 217 members, Hindus and Mahomedans, there were 95 Mahomedans; how many more Hindus than Mahomedans were there?

16. Kutub died in 1210 and Baber in 1530, how many years after the death of Kutub did Baber die?

17. *A*'s age is 5 years less than *B*'s, and *B*'s 2 years less than *C*'s, *C* is 12 years under 48; find the ages of *A* and *B*.

18. From a certain place *A* goes 245 miles due east, and *B* 379 miles due west; find the distance between them.

19. Ram says to Hem, I have **Rs.** 87. Hem replies, I have just **Rs.** 37 less than you. Then Gopal says, I have as much as both of you, less **Rs.** 5. How much has Gopal?

20. *A*, *B* and *C* together had **Rs.** 375; *B* and *C* together **Rs.** 293, and *A* and *C* together **Rs.** 256. Find what they respectively had.

21. Bengal was conquered in 1203. How many years before the first battle of Panipat, which took place in 1526, was Bengal conquered?

22. *A* first takes 17 rupees out of a box which contains 100 rupees, then puts in 7, then takes out 15, then puts in 5, then puts in 15, then takes out 25, again takes out 15, and then puts in 5. How many rupees under 100, are now in the box?

23. (*a*) What number multiplied by 93 gives 20078, as the product?

(*b*) Find shortly the value of 35827 — 8 times 4281.

24. The quotient arising from the division of a certain number by 252 is 777, what is the dividend?

25. A certain number, when divided by 1313, gives 2645 as quotient and 718 as remainder; find the dividend.

26. If, in a division sum, the divisor be 8 times and the quotient 7 times the remainder; what is the dividend, when the remainder is 452?

27. If the dividend is 441, the remainder 4, and the quotient 23, what is the divisor?

28. *A* has Rs.2402 more than *B*, and *A* and *B* have Rs.4734 between them. Find what *A* and *B* respectively have.

29. The weight of two heaps of rice together is 1205 maunds, and one heap contains 21 maunds more than the other. What is the weight of the smaller heap?

30. What number is contained 3169 times in 128991450, with 474 over?

31. What least number must be added to 476254 to make it exactly divisible by 2405?

32. What is the smallest number which you should subtract from 2460325 to make it divisible by 9264?

33 How many times may 3795 be subtracted from 48040776, and what number will remain over?

34. What is the least number which added to 3473567608 makes the sum divisible by 468?

35. What is the least number which, when taken from 23450065, leaves a remainder divisible by 9?

36. A merchant bought 8975 yards of cloth at Rs 6 a yard, and retailed them at Rs.8 a yard; how much did he gain?

37. In a box there are sixteen divisions, each division contains three Currency Notes, one of Rs.20, another of Rs.10, and a third of Rs 5; what sum is there in the box?

38. There are 25 chests of drawers; in each chest there are 15 drawers; in each drawer there are 12 divisions; and in each division there are deposited 125 rupees. Find how much money there is in the chests.

***39**. Two men start from the same place, one going due north and the other due south: if they walk 5 and 6 miles per hour respectively, how far apart will they be when they have walked 51 hours?

***40**. A man who can walk 15 miles a day left Calcutta for Raneegunge, and another man who can walk 10 miles a day left Raneegunge for Calcutta at the same time as the first. The distance between Calcutta and Raneegunge being 125 miles, when and where will they meet?

41. A farmer sold 438 horses at Rs.80 a head and 529 cows at Rs.35 a head; how much more did the horses fetch than the cows?

42. A merchant who had 2350 maunds of sugar sold 1620 maunds at Rs.15 a maund, and the remainder at Rs.13 a maund; how much did he take in all?

43. A merchant bought 8485 maunds of rice for Rs.48119, and sold 5638 maunds at Rs.5 a maund. At what price per maund must he sell the remainder so as not to gain or lose by the transaction ?

44 Wishing to give 66 beggars Rs.5 each, I find I am Rs. 2 short ; what is the sum I have ?

45. There are two pipes attached to a cistern : by the first 130 *kalsis* of water pour in every minute, and by the second 180 *kalsis.* If the cistern when empty, is filled in 72 minutes, both pipes being opened, how much can the cistern hold ?

46. A tank has three pipes attached to it. By two of these 586 and 425 maunds of water respectively enter into it every hour, while by the third 1210 maunds go out in the same time. If the tank, when full, is emptied in 312 hours when all the pipes are opened together, how many maunds of water can the tank hold ?

***47.** In a workshop there are employed 25 men, 50 women and 150 children. A man can work as much as 2 women, and a woman as much as 4 children. If, instead of men and women, only children were employed, how many children would have to be engaged ?

***48.** In constructing a bridge there are employed 35 men, 62 women and 568 boys. A man can do as much work as three women, and a woman as much as four boys. Find how many women will be able to perform the same quantity of work.

49. A ship sailing at the rate of 9 miles per hour, leaves Calcutta and reaches a certain port after 15 days. How far is the port from Calcutta ? (A day=24 hours).

***50.** Two persons *A* and *B* started together from the same place in the same direction, *A* walking 5 miles, and *B*, 6 miles an hour. How many miles did *A* fall behind *B*, in 15 hours ?

***51.** A boy having 525 mangoes divided some equally among his six friends and himself, and gave the rest to his sister. The sister, however, got 3 mangoes less than he ; how many did she get ?

52. Divide Rs.1000 between *A* and *B*, giving *B* Rs 126 less than *A*.

53. Find the number, which if I multiply by 5, then add 15, then divide the result by 4, then subtract 25, and then divide the result by 8 and 11 successively, the result is 15 ?

54. How much greater is the product of 21 and 19 than the product of their sum and difference ?

55. What is the dividend, the remainder being 119, the quotient 32, and the divisor 151 more than the difference of the two ?

56. What number is that which being divided by 14, the quotient multiplied by 12, the product divided by 18, and the result increased by 78, the sum is 20 under 100 ?

57. If 45 men can earn Rs.270 every day, and if a man can earn as much as 3 boys in the same time ; what can each boy earn daily ?

58. Of what number is 2670 both divisor and quotient ?

***59.** Calcutta to Benares is 525 miles. Two railway trains which run 21 and 15 miles per hour respectively, start together from Calcutta. How far behind will the second train be, when the first reaches Benares?

60. What number added to itself 514 times, is 100541 more than 42×2842?

61. The cost of cutting a road is Rs.38889 and that of constructing a bridge is Rs.19062 more. If the expenses are to be borne equally by 9 villages, what amount is to be paid by each village?

62. Two books containing 3432 and 2675 pages respectively, are divided into chapters. A chapter of the first contains 22 pages; and a chapter of the second 25. How many chapters more are there in the first than in the second?

63. A quantity of blue serge, contained in 98 pieces of 69 yards each, is given out to make up into clothing for 729 sailors. If the cloth is divided equally and the sailors get as much as possible, how many yards of serge will remain?

***64.** In a class of 24 boys, 1 marble is given to one boy, 2 marbles to a second, 3 to a third, and so on Find how many marbles are given altogether.

***65.** A man left Rs. 26650 to be divided among his four sons and five daughters. It was directed in his will that each of his son should get twice as much as each of his daughters. Find the share of a son.

66. A man earns 240 rupees in the first year, 300 rupees in the second, and 450 rupees in the third. Find his average annual income.

67. Ram's annual income was 500 rupees in the first four years, and 650 rupees in the next six. What was his average income?

***68.** If my income be Rs.1440 in one year and Rs.3600 in the next two, how much should I spend every month that I might neither borrow nor save? (A year = 12 months).

***69.** A man spends 400 rupees a year for 3 years, and saves some money; he raises his expenditure during the next 5 years to 480 rupees a year, and finds all his savings spent. What was his yearly income?

***70.** A man spends 1200 rupees annually for 7 years and gets into debt. He then reduces his expenses to 880 rupees a year, and in 9 years just manages to clear off his debts. What did he earn every year?

71. In a game of cricket, A and B together scored 120 runs, A and C 99 runs, and B and C 71 runs. How much did they score each?

72. The ages of a boy and his father are together 36 years, those of the boy and his grandfather are 62 years, and those of the father and the grandfather 86 years. Find the boy's age.

73. Simplify

(a) $84-2[86-\overline{42+2\times 15}+\{4+10+(7-2)\}+4]$.

(b) $65^2+(496\times 27-\overline{86\times 25+67803}\div 13)$.

74. Divide 4850 marbles among A, B, C, D, so that for every 4 marbles given to A, 6 may be given to B, 7 to C, and 8 to D.

*75. A man sold a certain number of mangoes for 2800 pice which had cost him 2275 pice, thereby gaining 3 pice for each mango. What did each mango cost him.

76. Shew that the product of any three consecutive numbers is divisible by 6 without any remainder.

77. Arrange the 9 digits in 3 lines, 3 digits being in each line, so that the sum of the digits in every possible direction is the same.

78. Sixteen natural numbers *i. e.* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 are to be arranged in 4 lines, 4 numbers in each line, in such a way that the sum of the numbers taken in every possible direction be 34.

79. Add all the numbers that can be formed from the digits 3, 6 and 9 two digits being taken at a time.

80. Add all the numbers that can be formed from the digits 1, 5, 0, and 6 three digits being taken at a time.

81. When a number is divided by 7 the remainder is 5, the quotient being divided by 13 leaves the remainder 8, the second quotient being divided by 6, the remainder left is 4. If the number be divided by 546 ($=7\times13\times6$), what is the remainder? Verify your answer.

CHAP. XII. COMPOUND QUANTITIES.

80. It has already been seen that all quantities are expressed as *so many* times a certain quantity of the same kind, called the **unit.** Although it is always *possible* to use the same unit in dealing with all quantities of the same kind, it is, in practice, never *convenient* to do so. For instance, though the length of a rod may conveniently be given as ten **yards,** the measure of the distance between Calcutta and Benares, if expressed in yards, would be so inconveniently large that it is necessary to use another unit for the purpose, namely, a **mile.** Hence has arisen the practice of using large units for large quantities and small units for small quantities. Thus the price of a big book is usually given as 5 **rupees**; that of a slate as 4 **annas**; and that of a pen as 2 **pice.**

81. When different units are used in the same country for expressing different quantities of the same kind, some suitable quantity is selected as the principal or **standard** unit and the minor or **auxiliary** units are obtained either by dividing this unit into a certain number of equal parts or by multiplying it a certain number of times. The various **Tables** given in the following pages will furnish the student with the relations between the *standard* and the corresponding *auxiliary* units.

TABLE I. MONEY.

British Indian Money.

82.	3 Pies (*p*) or 2 half-pice	make	1 Pice (*ps.*)
	2 pice	,,	1 Half-anna.
	4 pice or 12 pies	,,	1 Anna (1*a.*)
	16 annas	,,	1 **Rupee** (*Re.* 1 or 1/.)
	15 rupees	,,	1 Sovereign (£1).

83. In Bengal, accounts are kept by the following Table.

4 Couries	make	1 Gandá	4 pans	make	1 Chouk
5 gandás	,,	1 Búri (Paisa)	4 chouks	,,	1 Kahán or Rupee.
4 búris	,,	1 Pan (Anna)			

Also 1 *Couri* or *Bat*=3 *Krántis*=4 *Kágs*=5 *Ta'ls*=7 *Dweeps* = 9 *Dantees*=27 *Jabs*=80 *Tils*; and 320 *Bindus*=16 *Ghúns*=1 *Til.*

84. The following Tables are in use in other parts of India.

IN BEHAR AND N.-W. P.

2 Addhis	make	1 Dámri
2 dámris	,,	1 Chhadám
2 chhadáms	,,	1 Adhelá
2 adhelás	,,	1 Paisá
2 paisás	,,	1 Táká
2 tákás	,,	1 Anna

IN BOMBAY.

100 Raes	make	1 Quarter
4 Quarters	,,	1 Rupee

IN CEYLON.

100 Cents make 1 Rupee.

IN MADRAS.

*Rs.*3. 8*a.*=1 Pagoda.

The gold *Mohur* which is an obsolete coin of the same weight (1 Tolá or 180 grains) as the Rupee. consists of 11 parts of gold and 1 of alloy. The *couri* is a shell imported from the Maldive and Laccadive Islands

Couri as a shell is now going to be out of use, while *couri* (generally known as karas) is in use in keeping accounts.

15 *Sicca* rupees=16 rupees. The *Lawyer's* Gold mohur=17 rupees; the *Doctor's* Gold mohur=16 rupees.

Silver coins (*current*): Two-anna piece; Four-anna piece or Quarter rupee; Eight-anna piece or Half-rupee; Rupee.

Copper coins (*current*): Pie; Half-pice; Pice or Paisá; Double Paisá or Half-anna.

The weight of a pice is 100 grains; that of a half-anna bit 200 grains.

English Money.

85.	4 Farthings (*q.*)	make	1 Penny (*d.*)*
	12 pence	,,	1 Shilling (1*s.* or 1/.)*
	20 shillings	,,	1 **Pound** (£1.)*

* The letter *d* stands for *denarius*, *s* for *solidus*; and the symbol £ for *libra*. These words were the names of certain Roman coins or sums of money.

[1, 2, 3, farthings are usually written a $\frac{1}{4}d.$, $\frac{1}{2}d.$, $\frac{3}{4}d.$, respectively.]

The gold coin corresponding to the *pound* is called a **sovereign**

Gold coins (*in circulation*): Half-sovereign: Sovereign.

" " (*obsolete*): Noble (6*s* 8*d*.); Angel (10*s*.); Half-guinea (10*s*.6*d*); Merk or Mark (13*s*. 4*d*.); Guinea (21*s*.); Carolus (23*s*.); Jacobus (25*s*.); Moidore (27*s*)

Silver coins (*in circulation*): Three-penny piece; Four-penny piece; Six-pence; Shilling; Florin (2*s*.); Half-crown (2*s*. 6*d*.); Crown (5*s*).

Silver coins (*obsolete*): Groat (4*d*.); Tester (6*d*.).

Copper coins: Farthing; Half-penny ($\frac{1}{2}d.$); Penny.

The standard of **gold coin** in England is 11 parts or 22 *carats* of pure gold and 1 part or 2 *carats* of copper. 40 pounds Troy or 40×5760 grains of standard gold, are coined into 1869 sovereigns.

The standard of **silver coin** is 37 parts of pure silver and 3 parts of copper. A pound Troy of standard silver is coined into 66 shillings.

In the **copper coinage,** 24 pennies are coined from 1 pound Avoirdupois or 7000 grains of copper.

In England gold coinage is the standard; more than 40*s*. in silver, more than 12*d*. in pence or half-pence or more than 6*d*. in farthings are not legal payments there.

REDUCTION.

86. When a quantity is expressed by means of one unit, it is called a **simple quantity**; as 5 rupees, 16 pounds.

When a quantity is expressed by means of more than one unit of the same kind, it is called a **compound quantity**; as *Rs*. 7. 7*a*. 8*p*.

87. **Reduction** is the method of expressing, (1) a simple or a compound quantity in terms of any of the lower units or denominations, or (2) a simple quantity in higher denominations.

(1) *To express a quantity in terms of a lower denomination.* **(Descending Reduction.)**

RULE. Multiply the number in the highest denomination by the number of times that the next lower unit is contained in one unit of the highest, and add to this product the number of this lower denomination, if there be any; repeat the same process for each succeeding denomination till the required denomination is reached.

Ex. 1. Reduce *Rs*. 75 to *pice*.

$$\begin{array}{r} Rs.\ 75 \\ \underline{16} \\ 1200a. \\ \underline{4} \\ \underline{4800ps.} \end{array}$$

Re. $1 = 16a.$;

$\therefore$ *Rs*. $75 = (75 \times 16)a. = 1200a.$

Again, $1a. = 4ps.$;

$\therefore 1200a. = 1200 \times 4ps = 4800ps.$

$\therefore$ *Rs*. $75 = 4800ps.$

Ex. 2. Reduce *Rs.* 24. 12*a.* 4*p.* to *pies.*

Rs. 24. 12*a.* 4*p.*
16
396*a.* (24 × 16 + 12)
12
4756*p.* (396 × 12 + 4)

*Rs.*24 = 24 × 16*a.* = 384*a.*
∴ *Rs.*24. 12*a.* = 396*a.*;
396*a.* = 396 × 12*p.* = 4752*p.*
∴ *Rs.*24. 12*a.* 4*p.* = 396*a.*4*p.*
= 4752*p.* + 4*p.* = 4756*p.*

Ex. 3. Reduce £15 to *farthings.*

£15
20
300*s.*
12
3600*d.*
4
14400*q.*

£1 = 20*s.*
∴ £15 = 15 × 20*s.* = 300*s.*
1*s.* = 12*d.*;
∴ 300*s.* = 300 × 12*d.* = 3600*d.*
1*d.* = 4*q.*;
∴ 3600*d.* = 3600 × 4*q.* = 14400*q.*

Ex. 4. Reduce £18. 15*s.* 9*d.* to *pence.*

£18. 15*s.* 9*d.*
20
375*s.*
12
4509*d.*

£18 = 360*s.*
∴ £18. 15*s.* = 375*s.*
375*s.* = 4500*d.*
∴ £18. 15*s.* 9*d.* = 4509*d.*

Examples 26.

[8 two-anna pieces = 4 quarter-rupees = 2 half-rupees = Re.1, and 2 double paisás = 1 anna. Also, 4 three-penny pieces = 3 four-penny pieces = 2 six-pence = 1 shilling; and 2 half-pence = 1 penny.]

1. Reduce to *annas*:

(1) R32; R47; R63; R82; R100; R260.

(2) R12. 8*a.*; R32. 6*a.*; R84. 13*a.*; R282. 5*a.*

2. Reduce (i) to *paisa's* and (ii) to *pies*:

(1) R24; R37; R99; R241; R450; R730.

(2) R45. 15*a.*; R102. 10*a.*; R265. 8*a.*; R904. 13*a.* 1*ps.*; R1001. 14*a.*; R9. 7*a.* 1*ps.*; R13. 13*a.* 2*ps.*; R23. 2*a.* 3*ps.*; R52. 1*a.* 3*ps.*; R20. 12*a.* 2*ps.*; R94. 0*a.* 1*ps.*; R141. 13*a.* 3*ps.*

3. Reduce (i) to *gandas* and (ii) to *couries*:

R25; R37; R75; R106; R15. 13*a.*; R141. 9*a.*; R207. 14*a.*; R408. 15*a.*; R902. 11*a.*; R7. 4*a.* 2*ps.*; R19. 13*a.* 1*ps.*; R102. 15*a.* 2*ps.*

4. Reduce (i) to *pice* and (ii) to *pies*:

250 half-rupees; 460 quarter-rupees; 744 two-anna pieces; 425 double-paisás; 1260 quarter-rupees; 1416 two-anna pieces; 19624 half-rupees; 72460 double-paisás.

5. Reduce (i) to *half-rupees*, (ii) to *quarter-rupees* and (iii) to *two-anna pieces* :

R4675; R7620; R1673; R4862; R2456; R26051; R46298; R91246.

6. Reduce (i) to *half-annas* and (ii) to *half-pice* :

R47. 5*a*.; R94. 13*a*.; R191. 8*a*.; R1400. 3*a*.; R704. 7*a*.; R1416. 10*a*.; R140. 14*a*. 2*ps*.; R947. 5*a*. 2*ps*.

7. Reduce to *shillings* : £425; £892; £4061; £924. 18*s*.; £1119. 13*s*.; £1892. 15*s*.; £1452. 12*s*.

8. Reduce to *pence* :

(1) £146; £167; £203; £1695; £7460; £9824.

(2) £154. 17*s*.; £180. 15*s*.; £217. 11*s*.; £416. 13*s*.; £809. 15*s*.

(3) £35. 11*s*. 7*d*.; £94. 12*s*. 11*d*.; £115. 16*s*. 7*d*.; £712. 0*s*. 8*d*.

9. Reduce to *farthings* :

£706. 0*s*. 9*d*.; £8064. 19*s*. 11¼*d*.; £1761. 13*s*. 5½*d*.; £8620. 0*s*. 11¾*d*.

10. Reduce (i) to *half-pence* and (ii) to *farthings* :

(1) £1346; £12680; £132. 19*s*.; £616. 12*s*.; £1126. 18*s*.

(2) £45. 7*s*. 8*d*.; £59. 11*s* 9*d*.; £744 0*s*. 11*d*.; £1402. 0*s*. 4½*d*.; £1560. 16*s*. 10½*d*.; £8126. 13*s*. 4½*d*.; £1210. 15*s*. 5½*d*.

(3). 18 three-penny pieces; 126 four penny pieces; 1256 six-pence; 2641 half-crowns; 1450 crowns; 446 half-guineas; 1140 guineas; and 1460 moidores.

11. Reduce (i) to *three-penny pieces*, (ii) to *four-penny pieces*, and (iii) to *six-pence* :

(1) £75; £220; £304; £740; £1112; £1912.

(2) £11. 14*s*.; £120. 11*s*.; £120. 14*s*.; £1560. 8*s*.; £4621. 17*s*.

12. If a pear cost an anna, how many can be bought for Rs.145. 4*a*.?

13. How many two-anna articles can be bought for Rs.715. 8*a*.?

14. If I give to a beggar a six-pence every day, in how many days will he receive £7. 4*s*. 6*d*. from me?

15. I distributed a certain number of penny-dictionaries, which cost me £16. 4*s* altogether, among the boys of a school, giving one copy to each; find the number of boys in the school.

88. *To reduce a simple quantity in terms of its higher denomination.* **(Ascending Reduction.)**

RULE. Divide the simple quantity by the number of units which are equivalent to one unit of the next higher denomination and put down the remainder, if any, as the surplus units of that lower denomination; and repeat the process until the required highest denomination is arrived at.

Ex. 1. Reduce 1920*ps* to *rupees.*

4|1920*ps.*
16|480*a.*
30*Rs.*

∵ 4 pice = 1 anna.
∵ 16 annas = 1 rupee.

Ex. 2. Reduce 2461*p*, to *rupees, annas and pies.*

12)2461*p.*
16)205—1*p.*
Rs. 12—13*a.*

∴ the result is *Rs.* 12. 13*a.* 1*p.*

Ex. 3. Reduce 4653285*q.* to *pounds, shillings,* &c. :

4|4653285*q*
12|1163321—1*q*
2,0|9694,3—5*d*
£4847—3*s.*

∵ 4*q.* = 1*d.*
∵ 12*d* = 1*s.*
∵ 20*s.* = £1

∴ the result is £4847. 3*s.* 5¼*d.* ∵ 1*q* = ¼*d.*

Examples 27.

1. Reduce to *rupees, annas* and *paisa's* :
59000 1*ps.*; 39161*ps.*; 17003*ps.*; 9605*ps*; 17111*ps.*; 70391*ps.*

2 Reduce to *rupees, annas* and *pies*
(1) 1037*p.*: 92861*p.*: 71596*p*; 30621*p.*: 54362*p*; and 70213*p.*
(2) 69446*p.*, 30622*p.*; 72131*p.*, 68324*p.*, 40316*p.*; and 338162*p.*

3 Reduce to *rupees, annas* &c.
(1) 10000 gandás: 37094 bûris: 787878 couries: 35353 gandás.
(2) 47147 bûris; 116002 couries; 30822 half-paisás; 26452 couries.
(3) 3784 half-paisás, 47641 paisás; 3082 double-paisás; 17308 bûris.

4. Reduce to *pounds, shillings, pence* ·
(1) 92861*d.*; 71596*l.*; 30621*l.*: 54362*d*; 70213*d.*; 69446*d*
(2) 721312*q.*; 68345*q.*: 50426*q.*; 304064*q.*; 590004*q.*: 9606*q.*
(3) 71341*q.*: 96419*q.*; 121210*q.*; 20463*q.*: 95527*q*; 17345*q.*

5 Reduce to £. *s. d.* :
96002 guineas: 456021 half-guineas; 17345 crowns: 475621 half-crowns; 42601 moidores: 43625 six-pence; 34201 four-penny pieces: 73614 three-penny pieces; 131596 six-pence.

6. To each of 2644 men, I gave a double-paisá: what sum did I give in all?

7. What sum is to be paid for 346257 penny postage stamps?

8. What sum is to be given for 24678 articles at a half-crown each?

89. *To reduce from one system of units to another system.*

RULE. Reduce the given quantity, as in ART. 87, to the lowest necessary denomination; and reduce the simple quantity thus obtained to the other system as in Art. 88.

Ex. 1. Reduce £16. 7*s*. 6*d*. to *half-crowns*.

£16. 7*s*. 6*d*.
20
327*s*.
12
3930*d*.

A half-crown = 2*s*. 6*d*. = 30*d*.
3,0)393,0(131 half-crowns.

Ex. 2. Reduce £13. 8*s*. 8*d*. to *Rs*., &c., when 1*d*. = 11*p*.

£13. 8*s*. 8*d*.
20
268*s*.
12
3224*d*.

3224*d*.
11
35464*p*.

12)35464*p*.
16)2955—4*p*.
Rs.184—11*a*.

∴ £13. 8*s*. 8*d*. = *Rs*. 184. 11*a*. 4*p*.

Examples 28.

1. Reduce (i) to *crown* and (ii) to *half-crowns*:
£126. 15*s*.; £256. 10*s*.; £315. 5*s*.; £1210; £616. 15*s*.

2. Reduce to *half-crowns*:
£75. 12*s*. 6*d*.; £110. 17*s*. 6*d*.; £80. 7*s*. 6*d*.; £21. 2*s*. 6*d*.

3. Reduce (i) to *guineas* and (ii) to *half-guineas*:
£21; £84; £252; £1008; £5040; £10080; £30240.

4. Reduce to *half-guineas*:
£21. 10*s*. 6*d*.; £252. 10*s* 6*d*.; £5040. 10*s*. 6*d*.

5. Reduce to *Rs*. *a*. *p*. (1*q*. = 1*ps*.):
£27. 13*s*. 4*d*.; £47. 16*s*. 8*d*; 24 *half-crowns*; £56. 17*s*. 8*d*.

6. Reduce to £. *s*. *d*. (1*d*. = 10*p*.):
₨2234. 10*a*. 2*p*.; ₨110. 15*a*. 10*p*.; ₨57. 14*a*. 8*p*.

7. A person exchanges £2661 for rupees. A rupee being equivalent to 1*s*. 4*d*., find the number of rupees he should get.

8. Convert 525 Pagodas to *Rupees*, &c.

COMPOUND ADDITION.

90. **Compound Addition** is the method of finding the sum of several simple and compound quantities of the same kind.

RULE. Place the quantities one under another so that *rupees* fall under *rupees*, *annas* under *annas*, *pies* under *pies*;

and so on. Add the numbers in the column of the lowest denomination; reduce this sum to the next higher denomination; set down the remainder under the column thus added, and carry the quotient to the column of the next higher denomination. Proceed in this way through all the columns.

Ex. 1. Add together *Rs.*72. 4*a.* 6*p.*, *Rs.*112. 14*a.* 9*p.*, *Rs.*314. 11*a.* 8*p.*, and *Rs.*200. 8*a.* 4*p.*

Rs. 72	4*a.*	6*p.*
112	14	9
314	11	8
200	8	4
Rs. 700	7*a.*	3*p.*

6*p.*, 9*p.*, 8*p.*, 4*p.*; 27*p.* = 2*a.* 3*p.*
Carry 2*a.*, 4*a.*, 14*a.*, 11*a.*, 8*a.*; 39*a.* = *Rs.*2. 7*a.*
*Carry Rs.*2, *Rs.*72, *Rs.*112, *Rs.*314, *Rs.* 200 = *Rs.*700.

Ex. 2. Add together £750. 15*s.* $6\frac{1}{4}$*d.*, £25. 7*s.* $9\frac{3}{4}$*d.*, £103. 9*s.* $11\frac{1}{2}$*d.*, and £807. 13*s.* $4\frac{3}{4}$*d.*

£750.	15*s.*	$6\frac{1}{4}$*d.*
25	7	$9\frac{3}{4}$
103	9	$11\frac{1}{2}$
807	13	$4\frac{3}{4}$
£1687.	6*s.*	$8\frac{1}{4}$*d.*

1*q.*, 3*q.*, 2*q.*, 3*q.*; 9*q.* = $2\frac{1}{4}$*d.*
Carry 2*d.*, 6*d.*, 9*d.*, 11*d.*, 4*d.*; 32*d.* = 2*s.* 8*d.*
Carry 2*s.*, 15*s.*, 7*s.*, 9*s.*, 13*s.*; 46*s.* = £2. 6*s.*
&c. &c.

Examples 29.

1. Add together:

(1)

a	*p.*
3	2
4	3
13	1
15	3

(2)

a.	*p.*
13	7
11	2
15	8
9	3

(3)

a.	*p.*
7	3
13	6
9	5
7	4

(4)

a.	*p.*
3	3
12	9
10	2
14	8

(5)

a.	*p.*
9	11
15	2
14	10
9	7

(6)

a.	*p.*
13	8
12	10
14	2
7	10

(7)

Rs.	*a.*	*p.*
13	7	3
19	10	10
15	15	6
3	14	9
8	12	9

(8)

Rs.	*a.*	*p.*
36	10	8
17	9	6
9	14	7
7	15	6
8	11	6

(9)

Rs.	*a.*	*p.*
75	15	11
32	15	0
84	13	7
98	0	6
30	1	11

(10)

Rs.	*a.*	*p.*
17	12	11
3	15	6
18	10	7
9	14	11
7	11	10

(11)

706	13	6
19	15	7
18	12	6
99	11	7
44	14	4
176	15	8
49	10	10

(12)

30	0	3
76	13	0
18	15	2
92	14	7
69	15	9
47	15	5
92	12	6

(13)

290	15	11
209	14	10
72	13	9
97	12	8
54	11	7
10	10	6
8	9	5

(14)

250	0	6
360	1	5
75	2	4
112	3	2
206	4	1
209	5	8
216	6	10

2. In this Example, add both in columns and in rows, and shew that the sum of the totals under the columns *tallies* with that of the rows.

1356	15	7	1202	5	8	225	11	11	211	15	1
808	11	9	613	8	11	336	13	10	333	14	2
35	9	8	916	10	6	801	12	9	444	13	3
196	13	11	2510	8	10	707	11	8	555	12	4
872	10	0	169	0	5	911	10	7	666	11	5
1210	11	8	20	5	0	516	9	6	777	10	6
8965	13	3	156	13	0	780	8	5	888	9	7
134	8	0	265	15	8	591	7	4	999	8	8
72	0	8	72	6	3	853	6	3	222	7	9
120	6	4	65	11	8	930	5	2	111	6	10
216	8	9	76	6	9	456	4	1	700	5	11
1304	9	8	250	11	11	340	3	0	800	4	0

3. Add together:

(1)			(2)			(3)			(4)		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
12	7	9	17	10	6	13	19	11	96	17	6
11	16	7	19	13	7	98	13	7	88	8	8
11	10	6	29	16	8	17	12	10	43	3	4
9	1	4	32	14	7	20	0	5	13	16	2
7	2	6	16	13	8	13	11	0	29	15	6
11	5	8	75	11	5	15	10	6	66	13	4

(5)			(6)			(7)			(8)		
103	17	$7\frac{1}{2}$	294	12	$7\frac{1}{4}$	903	17	$6\frac{1}{2}$	172	14	$2\frac{1}{2}$
597	13	$8\frac{1}{4}$	110	10	8	365	0	9	304	6	8
274	12	$6\frac{1}{4}$	864	9	11	898	2	$6\frac{3}{4}$	167	17	$9\frac{3}{4}$
916	9	$7\frac{1}{4}$	924	8	$0\frac{1}{4}$	362	12	$11\frac{1}{4}$	205	19	$11\frac{1}{4}$
762	6	$9\frac{3}{4}$	316	13	$9\frac{3}{4}$	138	11	8	806	18	$7\frac{1}{2}$
123	6	$11\frac{1}{2}$	175	17	6	255	16	$3\frac{1}{2}$	412	13	$0\frac{3}{4}$

4. In this example, add both in co
that the sum of the totals under the colun

£	s.	d.	£	s.	d	£	s.	d.
10368	12	$10\frac{1}{2}$	56702	0	$11\frac{1}{2}$	1401	12	$4\frac{1}{2}$
6482	19	$8\frac{3}{4}$	84115	19	$10\frac{1}{4}$	5301	11	0
5021	13	4	3261	18	$0\frac{3}{4}$	4444	10	$6\frac{3}{4}$
412	17	$2\frac{1}{4}$	5412	17	9	5666	9	8
504	11	$9\frac{3}{4}$	810	16	$8\frac{1}{2}$	7333	8	$7\frac{1}{2}$
411	10	5	910	15	$7\frac{1}{2}$	8666	7	6
1205	16	$7\frac{1}{2}$	811	14	0	7222	6	$8\frac{3}{4}$
410	11	8	1054	13	$4\frac{3}{4}$	9888	5	8

5. A merchant bought 17 bales of cloth for R1760. 14*a.* 10*p.*; 5 bales of paper for R275. 10*a.* 4*p.*; 5 dozen chairs for R516. 12*a.* 6*p.*; 47 clocks for R940. 15*a.* 9*p.*; 27 chests of iron for R1217. 9*a.* 6*p.*; and 125 boxes of tea for R7265. 12*a.* 9*p.* Find the total cost.

6. Add together : a moidore, a guinea, a noble, a crown, a sovereign, a mark, a half-guinea, a half-crown, a half-sovereign, a florin, a shilling, a sixpence, a penny, a half-penny, and a farthing.

7. I paid R15. 12*a.* to my tailor and R20. 4*a.* 6*p.* to my cook and had still R14. 15*a.* 6*p.* left : what sum had I?

COMPOUND SUBTRACTION.

91. **Compound Subtraction** is the method of finding the difference between two compound quantities or between a simple and a compound quantity, both quantities being of the same kind.

RULE. Set down the smaller quantity under the larger so that the numbers of the same denomination may stand one under the other, and draw a straight line under them. Beginning from the lowest denomination, subtract each number of the lower line from that which stands directly above it, and place the remainders below. But if, in any case, the number in the lower line be greater than that which is above it, add to the number in the upper line as many units as are equivalent to one unit of the next higher one, and then subtract, taking care *to add* 1 to the number of this higher denomination in the lower line. Proceed thus until the last number is subtracted.

Ex. 1. Subtract *Rs.* 47. 11*a.* 6*p.* from *Rs.* 87. 4*a.* 9*p.*

Rs. 87.	4*a.*	9*p.*
47	11	6
Rs. 39.	9*a.*	3*p.*

9*p.* − 6*p.*, 3*p.*;
4*a.* is less than 11*a.*, so add 16*a.* to 4*a.* and *Re.* 1 to *Rs.* 47; 20*a.* − 11*a.*, 9*a.*
Rs. 87 − *Rs.* 48 = *Rs.* 39.

Ex. 2. Subtract £17. 13*s.* 4¾*d.* from £20. 9*s.* 3½*d.*

£20.	9*s.*	3½*d.*
17	13	4¾
£2.	15*s.*	10¾*d.*

2*q.* is less than 3*q.*; so add 4*q.* to 2*q.*, and 1*d.* to 4*d.* 6*q.* − 3*q.*, 3*q.* or ¾*d.*

3*d.* is less than (4 + 1) or 5*d.*; so add 12*d.* to 3*d.* and 1*s.* to 13*s.* 15*d.* − 5*d.* = 10*d.*

9*s.* is less than 14*s.*; so add 20*s.* to 9*s.* and £1. to £17. 29*s.* − 14*s.* = 15*s.*
£20. − £18 = £2.

Examples 30.

1. Subtract :—

(1) Rs.	a.	p.	(2) Rs.	a.	p.	(3) Rs.	a.	p.	(4) Rs.	a.	p.
55	12	3	117	12	3	120	11	3	98	7	2
41	8	2	106	10	2	113	11	1	77	4	1

(5)			(6)			(7)			(8)		
119	7	2	160	15	11	300	10	7	520	12	6
112	4	1	99	13	8	218	12	9	498	12	9

(9)			(10)			(11)			(12)		
241	15	6	212	6	6	56	8	3	72	11	8
238	10	10	209	8	8	48	10	6	68	12	10

(13)			(14)			(15)			(16)		
98	12	3	100	0	0	216	0	9	170	1	9
88	14	8	99	10	8	205	6	10	160	2	10

2. Subtract :

(1) £.	s.	d.	(2) £.	s.	d.	(3) £.	s.	d.	(4) £.	s.	d.
112	12	8	306	4	4	111	11	11	200	0	0
99	10	6	212	5	8	101	13	9	199	7	8

(5)			(6)			(7)			(8)		
300	6	6	190	7	6½	121	19	9¼	119	5	1¼
212	6	10	90	19	5¼	120	19	9¾	37	16	9¾

(9)			(10)			(11)			(12)		
310	13	7½	114	3	8½	900	17	2	365	10	6
308	15	8¾	106	16	9¾	174	19	8	272	13	7¼

(13)			(14)			(15)			(16)		
313	17	6½	8001	1	10½	125	12	10¼	227	14	7½
280	18	7¾	73	2	11¼	119	18	7¾	113	14	7¾

3. Subtract :

(1) R9. 12*a.* 3*p.* from R12. 13*a.* 7*p.* (2) R39. 10*a.* 7*p.* from R57. 6*a.* 4*p.* (3) R309. 8*a.* 6*p.* from R400. 5*a.* 3*p.* (4) £19. 19*s.* from £25. 17*s.* 8*d.* (5) £210. 12*s.* 8¾*d.* from £311. 10*s.* 6½*d.* (6) £508. 16*s.* from £517. 13*s.* 4*d.*

4. By how much is R27. 13*a*. 8*p*. greater than R21. 14*a*. 10*p*.? By how much is R47. 11*a*. 6*p*. less than R100. 7*a*. 4*p*.?

5. If I pay R207. 13*a*. 9*p*., out of a debt of R400; what sum will remain unpaid?

6. What must be added to £713. 10*s*. 8*d*. to make the sum equal to £1200?

7. Find the value of R19. 7*a*. 1*ps*.+R10. 2*a*. 3*ps*.−R21. 14*a*. −R17. 4*a*. 1*ps*.+R47. 10*a*. 3*ps*.−R10. 11*a*. 2*ps*.

8. Find the value of £103. 16*s*. 6¼*d*.−£29. 19*s*. 9¾*d*.+£73. 12*s*. −£19. 12*s*. 6¼*d*.+£205. 12*s*. 7½*d*.−£180. 19*s*. 11¾*d*.

9. Subtract the sum of £4. 10*s*. 6*d*., £21. 14*s*. 5*d*. and £32. 12*s*. 7*d*. from £60.

10. What sum must be added to twice 13*s*. 6½*d*. to make 2 guineas?

11. A house and a garden together cost R17521. 14*a*. 11*p*.; if the house be worth R14720. 15*a*. 8*p*., find the value of the garden.

12. A house when furnished is worth R19461. 10*a*. 8*p*., and when unfurnished it is worth R16326. 12*a*. 10*p*.; by how much does the value of the house exceed that of the furniture?

13. By selling goods for £1515. 17*s*. 7½*d*., I gain £125. 10*s*. 10½*d*. For how much did I buy them?

14. What sum will remain, when three bills amounting to £5. 7*s*. 4½*d*., £10. 14*s*. 9¼*d*., and £13. 17*s*. 10¼*d*. have been paid out of £37?

COMPOUND MULTIPLICATION.

92. **Compound Multiplication** is the method of finding the sum of any given compound quantity repeated a given number of times.

RULE. Multiply the number of the lowest denomination of the given compound quantity by the given multiplier; reduce this product to the next higher denomination and set down the remainder; and carry and repeat the process through all the denominations, till the last is reached.

Ex. 1. Multiply *Rs*. 58. 7*a*. 5*p*. by 9.

Rs. 58. 7*a*. 5*p*.	5*p*.×9=45*p*.; 45*p*.=3*a*. 9*p*.
9	Carry 3*a*., 7*a*.×9=63*a*.; 66*a*.=R4. 2*a*.;
Rs.526. 2*a*. 9*p*.	„ R4., R58×9=R522; R526.

Ex. 2. Multiply £5. 16*s*. 7½*d*. by 11.

£5. 16*s*. 7½*d*.	2*q*.×11=22*q*.; 5*d*. 2*q*., 5½*d*.
11	Carry 5*d*., 7*d*.×11=77*d*.; 82*d*.=6*s*. 10*d*.
£64. 2*s*. 10½*d*.	„ 6*s*., 16*s*.×11=176*s*.; 182*s*.=£9. 2*s*.
	„ £9, £5×11=£55; £64.

93 When the multiplier is greater than 20 and can be resolved into factors none of which exceeds 20, it is more convenient to multiply *by parts, i. e.*, multiply by way of continued product, by the factors of the given multiplier.

Ex. Multiply *Rs.* 7. 4*a.* 5*p.* by 54, and £3. 5*s.* $3\frac{3}{4}$*d.* by 36.

54=6×9	36=4×9.
Rs. 7. 4*a.* 5*p.*	£3. 5*s.* $3\frac{3}{4}$*d.*
6	4
*Rs.*43. 10*a.* 6*p.*	£13. 1*s.* 3*d.*
9	9
*Rs.*392. 14*a.* 6*p.*	£117. 11*s.* 3*d.*

94. The following methods will sometimes help the student to multiply by large numbers, with comparative ease.

Ex. 1. Multiply *Rs.*4. 7*a.* 7*p.* by 23 and £10. 11*s.* $4\frac{1}{2}$*d.* by 29.

23=24−1=4×6−1.	29=28+1=4×7+1.
*Rs.*4. 7*a.* 7*p.*	£10. 11*s.* $4\frac{1}{2}$*d.*
4	4
Rs. 17. 14*a.* 4*p.*	£42. 5*s.* 6*d.*
6	7
Rs. 107. 6*a.* 0*p.*	£295. 18*s.* 6*d.*
4 7 7	10 11 $4\frac{1}{2}$
Rs. 102. 14*a.*5*p.*	£306. 9*s.* $10\frac{1}{2}$*d.*

Ex. 2. Multiply £5. 8*s.* $3\frac{1}{2}$*d.* by 337.

337=7+10×3+10×10×3.

£5. 8*s.* $3\frac{1}{2}$*d.*×7=£37. 18*s.* $0\frac{1}{2}$*d.*

10

£54. 2*s.* 11*d.*×3=£162. 8*s.* 9*d.*

10

£541. 9*s.* 2*d.*×3=£1624. 7*s.* 6*d.*

£1824. 14*s* $3\frac{1}{2}$*d.*

Examples 31.

1. Multiply :

(1)	R2. 3*a.* 3*ps.*	separately by	2,	5,	11,	13.
(2)	R7. 12*a.* 2*ps.*	,, ,,	4,	6,	12,	15.
(3)	R4. 11*a.* 9*p.*	,, ,,	7,	8,	16,	18.
(4)	R16. 5*a.* 10*p.*	,, ,,	8,	12,	15,	20.
(5)	£8. 16*s.* 9*d.*	,, ,,	8,	12,	14,	18.
(6)	£15. 13*s.* $8\frac{1}{2}$*d.*	,, ,,	15,	16,	18,	20.
(7)	R4. 13*a.* 4*p.*	,, ,,	31,	67,	97,	109.
(8)	R7. 14*a.* 10*p.*	,, ,,	726,	841,	963.	
(9)	£7. 13*s.* $4\frac{1}{2}$*d.*	,, ,,	53,	107,	111.	
(10)	£13. 17*s.* 6*d.*	,, ,,	4321,	3798,	3908.	

2. Multiply (by factors):

(1) R24. 14a. 3ps. separately by 32, 44, 56, 72.

(2) R45. 11a. 8p. „ „ 84, 96, 104, 112.

(3) £9. 10s. 7½d. „ „ 99, 108, 156, 180.

(4) £11. 11s. 11¾d. „ „ 216, 512, 800, 960.

3. Multiply, as in Examples, ART. 94:

(1) R12. 8a. 3p. separately by 37, 73, 103, 113.

(2) £5. 11s. 7¾d. „ „ 41, 71, 83, 157.

(3) R14. 12a. 8p. „ „ 3724, 7142, 3658.

(4) £8. 14s. 7½d. „ „ 5303, 4008, 7065.

4. Find the value of:

(1) 36 articles at 12a. 9p. each. (2) 84 articles at 10a. 6p. each.

(3) 96 „ „ 6a. 11p. „ (4) 120 „ „ 7a. 6p. „

(5) 307 „ „ 14a. 11p. „ (6) 408 „ „ R4. 3a. „

(7) 456 „ „ 7s. 9d. „ (8) 761 „ „ 7s 6d. „

(9) 797 „ „ 16s. 10½d. „ (10) 844 „ „ 15s. 6d. „

(11) 902 „ „ £1. 13s. 4d. „ (12) 1009 „ „ £5. 10s. „

5. Make out the following bills:

(1) 45 yards of carpet at R3. 2a. 6p. a yard; 60 yards of damask at 14a. 9p. a yard; 253 yards of sheeting at 7a. 6p. a yard; 507 yards of edging at 3a. 2p. a yard; 206 yards of calico at 4a. 9p. a yard; and 218 yards of longcloth at 5a. 9p. a yard.

(2) 50 goats at R2. 7a. each; 63 lambs at 14a. 9p. each; 97 sheep at R2. 2a. 6p. each; 42 oxen at R40. 12a. 9p. each; 35 cows at R51. 14a. 9p. each; 100 mules at R37. 4a. 4p. each; and 42 horses at R97. 6a. 9p. each.

6. There are 17 chests of drawers; in each chest there are 9 drawers, in each drawer 12 compartments, and in each compartment R37. 8a. 6p.; find the amount of money in the chests.

7. If the weekly earnings of a person be £7. 6s. 4½d., what will be the earnings of 320 persons in twice that time?

8. If a man be paid 5a. 3p. an hour, how much should he get in 10 days? (A day = 24 hours.)

9. A certain mill requires on the average 27 tons of coal every day; what would it cost in coal in 105 days at R7. 0a. 2p. per ton?

COMPOUND DIVISION.

95. **Compound Division** is (1) the method of dividing a compound quantity by an abstract number, *i. e.*, of dividing it into a given number of *equal parts*; (2) the method of dividing one compound quantity by another of the same kind, *i. e.*,

of finding *how often* the latter is contained in the former. The first method is called **Partition**, and the second **Quotation.**

96. *To divide a compound quantity by an abstract number.*

RULE. Place the dividend and divisor as in simple Division. Divide the number of the highest denomination by the divisor, set down the quotient, and reduce the remainder, if any, to the next lower denomination. Add to this the number of that denomination, and repeat the division. Continue the process until the lowest denomination is reached.

When the divisor does not exceed 20, the division should be performed by mental subtraction and reduction.

Ex. 1. Divide *Rs.*55. 7*a.* 6*p.* by 15.

```
15)Rs.55. 7a. 6p.        Rs.55 ÷ 15 is Rs. 3, with Rs.10 over.
   Rs 3. 11a. 2p.        Rs.10 = 160a., 167a. ÷ 15 is 11a. and 2a. over.
                         2a. is 24p., 30p. ÷ 15 is 2p.
```

Ex. 2. Divide *Rs.*18. 7*a.* 10*p.* by 25.

```
25)Rs. 18. 7a. 10p.(        Or thus : 25 = 5 × 5.
        16                  5)Rs.18.  7a.  10p.
       295(11a.             5)Rs.3.  11a.   2p.
       275                            11a.  10p.
        20
        12
       250(10p.        ∴ the required quotient is 11a. 10p.
```

Ex. 3. If 53 pieces of cloth cost £252. 3*s.* 7*d.*; find the cost of each piece, to the **nearest penny**.

```
53)£252  3  7 (£4
   212
    40
    20
   803(15s.
   795
     8
    12
   103(1d.
    53
    50
```

If we take 1*d.*, we *leave out* 50*d.*; but if we take 2*d.*, we take 53*d.* − 50*d.* or only 3*d.* *too much*; therefore the cost to the *nearest penny*, is £4. 15*s.* 2*d.*

The usual RULE in such cases is this :

If the remainder is *less* than the divisor divided by 2, neglect it : *otherwise*, add 1*d.* to the quotient.

97. *To divide a compound quantity by 10, 100, 1000, &c.*

RULE. Cut off, at each step, as many figures from the right of the dividend as there are cyphers in the divisor; the figures cut off give the remainder, and the remaining figures give the quotient.

Ex. Divide *Rs.* 9. 14*a.* 4*p.* by 10, and *Rs.* 121 3*a.* 8*p.* by 100.

10) *Rs.* 9. 14*a.* 4*p.* (	100) *Rs.* 1,21. 3*a.* 8*p.* (
16	16
*a.*15,8	*a.*3,41
12	12
*p.*10,0	*p.*5,00
∴ the quotient is 15*a.* 10*p.*	∴ the quotient is *Re.* 1. 3*a.* 5*p.*

Examples 32.

1. Divide:

(1) R. 5. 11*a.* 2*ps.* by 2.
(2) „ 8. 9*a.* 1*ps.* by 3.
(3) „ 14. 4*a.* 3*ps.* by 5.
(4) „ 22. 13*a.* 4*p.* by 8.
(5) „ 34. 5*a.* by 12.
(6) „ 46. 14*a.* 8*p.* by 16.
(7) „ 52. 4*a.* 3*p.* by 15.
(8) „ 159. 4*a.* by 48.
(9) „ 148. 5*a.* 9*p.* by 45.
(10) „ 164. 13*a.* 6*p.* by 50.
(11) „ 197. 13*a.* by 60.
(12) „ 237. 6*a.* by 72.
(13) „ 3939. 14*a.* 3*p.* by 99.
(14) „ 3979. 11*a.* by 100.
(15) „ 5911. 5*a.* 8*p.* by 121.
(16) „ 7014. by 144.
(17) £ 49. 16*s.* 8*d.* by 10.
(18) „ 54. 18*s.* 2*d.* by 11.
(19) „ 19. 19*s.* 9*d.* by 12.
(20) „ 125. 12*s.* by 24.
(21) „ 240. 10*s.* 8*d.* by 32.
(22) „ 543. 11*s.* by 42.
(23) „ 574. 16*s.* by 48.
(24) „ 244. 9*s.* 0¾*d.* by 63.
(25) „ 229. 11*s.* 3¼*d.* by 19.
(26) „ 569. 2*s.* 9¾*d.* by 47.
(27) „ 619. 19*s.* 4¼*d.* by 31.
(28) „ 165. 15*s.* 8¾*d.* by 139.
(29) „ 1288. 1*s.* 8*d.* by 754.
(30) „ 162. 3*s.* 6*d.* by 156.

2. Divide by the *short* method:

(1) R. 10. 10*a.* 1*ps.* by 3.
(2) R. 17. 11*a.* 3*ps.* by 5.
(3) „ 24. 13*a.* 3*p.* by 7.
(4) „ 88. 0*a.* 6*p.* by 9.
(5) „ 439. 4*a.* 1*p.* by 11.
(6) „ 319. 1*a.* 11*p.* by 13.
(7) £ 226. 12*s.* 4½*d.* by 6.
(8) £ 302. 3*s.* 2*d.* by 8.
(9) „ 491. 0*s.* 1½*d.* by 14.
(10) „ 3503 0*s.* 6*d.* by 18.

3. Divide, by using factors:

(1) R. 158. 4*a.* by 48.
(2) „ 148. 5*a.* 3*ps.* by 45.
(3) „ 263. 5*a.* 4*p.* by 80.
(4) „ 306. 8*a.* 8*p.* by 56.
(5) £ 543. 14*s.* 6*d.* by 42.
(6) „ 1639. 9*s.* 0¾*d.* by 75.
(7) „ 78009. 15*s.* by 81.
(8) „ 49526. 13*s.* 11½*d.* by 110.

4. Divide, by applying ART. 97:

(1) R. 31. 4*a.* 10*p.* by 10.
(2) „ 220. 13*a.* 4*p.* by 100.
(3) „ 5276. 0*a.* 8*p.* by 1000.
(4) £ 53. 6*s.* 8*d.* by 10.
(5) „ 731. 13*s.* 4*d.* by 100.
(6) „ 5495. 16*s.* 8*d.* by 1000.

5. If the price of 7 yards of silk be R35. 14*a.* 5*p.*, what is the cost per yard to the nearest pie ?

6. If I buy 99 maunds of rice for R311. 7*a.*, at what price per maund must I sell out, if I do not wish to gain or lose by the bargain ?

7. The rent of a house is £66. 1*s.* 8*d.* a year; what is the rate per week ? (A year consists of 52 weeks.)

8. The annual income of a gentleman is £860. 0*s.* 7½*d.*; find his daily income. (A year = 365 days.)

9. If 325 shawls cost R4981. 10*a.* 3*p.*; what is the cost of each shawl ?

10. A gentleman spends R16230. 6*a.* in 3 years; how much does he spend every month ? (A year = 12 months.)

98. *To divide one quantity by another of the same kind.*

RULE. Reduce both the dividend and the divisor to the same denomination, and proceed as in Simple Division.

Ex. 1. Divide R24. 14*a.* 3*p.* by R2. 12*a.* 3*p.*

R24. 14*a.* 3*p.* = 4779*p.*; R2. 12*a.* 3*p.* = 531*p.*

∴ the required quotient = 4779 ÷ 531 = 9. *Ans.*

Ex. 2. I bought some bales of cloth for £637. 13*s.* 1½*d.* at £30. 7*s.* 3½*d.* per bale; how many bales did I buy ?

£637. 13*s.* 1½*d.* = 612150*q.*; £30. 7*s.* 3½*d.* = 29150*q.*

∴ the number of bales = 612150 ÷ 29150 = 21. *Ans.*

Examples 33.

1. Divide :

(1) R43. 3*a.* 8*p.* by 4*a.* 2*p.*; R1424. 8*a.* 6*p.* by R31. 10*a.* 6*p.*

(2) R362. 4*a.* by R4. 5*a.*; R333. 5*a.* 9*p.* by R3. 14*a.* 9*p.*

(3) R935. 2*a.* 6*p.* by R62. 5*a.* 6*p.*; R10305 by R53. 10*a.* 9*p.*

(4) £149. 7*s.* by £24. 17*s.* 10*d.*; £131. 16*s.* 3*d.* by £2. 6*s.* 3*d.*

(6) £155. 13*s.* 3½*d.* by £5. 7*s.* 4½*d.*; £367. 12*s.* 1½*d.* by £12. 13*s.* 6½*d.*

2. How often is

(1) R 5. 10*a.* 9*p.* contained in R 113. 7*a.* ?

(2) „ 12. 6*a.* 8*p.* „ „ „ 310. 6*a.* 8*p.* ?

(3) £ 3. 10*s.* 7*d.* „ „ £ 264. 13*s.* 9*d.* ?

(4) „ 2. 12*s.* 6½*d.* „ „ „ 236. 6*s.* 10½*d.* ?

(5) „ 6. 10*s.* 3*d.* „ „ „ 32262. 18*s.* 6*d.* ?

3. How many yards of cloth can be purchased for R19. 15*a.* 6*p.*, when the price per yard is 4*a.* 6*p.* ?

4. If a rupee be worth 1*s.* 4½*d.*, find how many rupees must be given for £343. 15*s.*

5. If a score of oranges can be purchased for 3*a*. 4*p*., how many score can I buy for R15. 10*a*. 8*p*. ?

6. If I lay out £43. 9*s*. 4*d*. in the purchase of shawls at 5*s*. 4*d*. a piece, how many pieces shall I have ?

7. Rice is selling in the bazar at R4. 13*a*. 6*p*. per maund; how many maunds can a merchant buy, if he lays out R4867. 15*a*. 6*p*. ?

8. How many carpenters can I engage for a day for R72. 10*a*. 8*p*., if the daily wages of each be 10*a*. 8*p*. ?

11. MEASURES OF WEIGHT.

99. **Indian Bazar Weight** (used in Bengal)

4	Sikis	make	1 **Tola'**
5	sikis	„	1 Kanchá
4	kanchás or 5 tolás	„	1 Chhaták (*ch*.)
4	chhatáks	„	1 Póá
4	póás or 16 chhatáks	„	1 **Seer** (*sr*.)
5	seers	„	1 Pasuri
8	pasuris or 40 seers	„	1 **Maund** (*md*.)

49. Indian maunds = 54 Factory maunds. A seer = 80 tolás.

MADRAS LOCAL WEIGHT.

3	Tolás	make	1 Palam
8	palams	„	1 Seer
5	seers	„	1 Vis
8	vis	„	1 Maund
20	maunds	„	1 Kandi.

BOMBAY LOCAL WEIGHT.

4	Dhans	make	1 Ratika
8	ratikas	„	1 Masha
4	mashas	„	1 Tank
72	tanks	„	1 Seer
40	seers	„	1 Maund
20	maunds	„	1 Kandi.

175 Indian maunds = 576 Madras maunds; 49 Indian maunds = 144 Bombay maunds. 28 Madras maunds = 25 Bombay maunds.

100 **English Standard Weight.** (*Avoirdupois*).

16	Drams (dr.)	make	1 Ounce (oz.)
16	ounces	„	1 **Pound** (lb.)*
28	pounds	„	1 Quarter (qr.)
4	quarters	„	1 Hundredweight (cwt.)†
20	hundredweights	„	1 Ton.

* The symbol lb is a contracted form of the Latin word Libra.

† The letter c in cwt. stands for the Latin word *centum*—a hundred. In the United States a hundredweight contains 100 lbs.

14lbs. = 1 Stone ; 100lbs. = 1 Cental.

A stone of butcher's meat	= 8lbs.	A sack of flour	= 280lbs. (10 qr.)
A Firkin of Butter	= 56lbs.	A barrel of ,,	= 196lbs. (7 qr.)
A sack of coal	= 2 cwt.	A peck of ,,	= 14lbs. (1/8 qr.)
A barrel of gunpowder	= 100lbs.	A Pocket of Hops	= 168 lbs.
A pack of wool	= 240lbs.	Two Fodders of Lead	= 39 cwt.
A great Pound of silk	= 24 oz.	A quartern loaf	= 4lbs.

A pound (Avoirdupois) = 7000 grains. (Troy) : 7 Bengal maunds = 576lbs. (Avoir.) : 1 Madras maund = 25lbs. (Avoir.) : 1 Bombay maund = 28lbs. or 1 qr. (Avoir) : 3 Factory maunds = 224lbs. or 2 cwt.

The Jeweller's Tables.

INDIAN JEWELLER'S WEIGHT.		
4 Dhans	=	1 Rati (*ra.*)
6 ratis	=	1 Anna (*a.*)
8 ratis	=	1 Masha (*ma.*)
12 mashas or 16 annas	=	1 **Tola** or Bhari.

ENGLISH TROY WEIGHT.		
24 Grains (gr.)	=	1 Pennyweight (dwt.)
20 pennyweights	=	1 Ounce (oz. Tr.)
12 ounces or 5760 grains	=	1 Pound (lb. Tr.)

1 Tola = 180 grs. Troy; a *carat* = $3\frac{1}{6}$ grs. nearly (for weighing diamonds.) [The Troy measure is used for weighing diamonds, gold, &c.]

Note —Carat is a unit of weight for precious stones divided by jewellers into 4 grains called *diamond-grains*, sometimes called *carat-grains*. But it equals to about $3\frac{1}{6}$ troy grains. In 1877 the weight of the carat was fixed by a syndicate of London, Paris and Amsterdam jewellers at 205 milligrams or 151·76 carats to the troy ounce (=480 grains troy). Pearls are sold by the diamond-grains and not by the carat, while small baroque pearls, coral, rough garnets and inferior kinds of stones are sold by the ounce troy.

Carat is also used to mean a twenty-fourth part, specifically used in expressing the fineness of gold when used as jewellery. A mass of gold is said to be so many carats fine according to the number of twenty-fourths of pure gold which it contains ; as 22 carats fine = 22 parts gold and 2 parts copper and silver.

Standard gold is 22 carats fine. Jeweller's gold is generally 18 carats fine.

Measures of Weight for Medicines.

INDIAN KAVIRAJ'S WEIGHT.		
4 Dhans	make	1 Rati
10 ratis	,,	1 Masha
8 mashas	,,	1 **Tola**

ENGLISH APOTHECARIES' WEIGHT.		
20 Grains	make	1 Scruple (℈)
3 scruples	,,	1 Dram (ʒ)
8 drams	,,	1 Ounce (℥)
12 ounces	,,	1 Pound (lb.)

[The Avoirdupois weight is now used in selling medicines.]

1 Indian maund = 100lb. Troy ; 144lb. (Avoir.) = 175lb. (Troy or Apo.). 1lb. (Troy or Apo.) = 5760 grs. 1lb. (Avoir.) = 7000 grs. 35 sr. = 72lbs. 1lb. (Avoir.) + the weight of a pice = 7200 grains = half-a-seer.

Ex. 1. Reduce 2 mds. 7 sr. 8 ch. to *lbs. &c.* (Avoir).

2 mds. 7 sr. 8 ch. = 1400 ch. = 7000 tolás.

= 7000 × 180 grains = 1260000 grs.

∴ no. of lbs. (Avoir.) = 1260000 ÷ 7000 = 180. *Ans.*

Ex. 2. Reduce 2 cwt. 1 qr. 18 lb. to *Indian weight.*

2 cwt. 1 qr. 18 lb. = 270 lb. = 270 × 7000 grs. = 1890000 grs.

Also, 1890000 grs. = (1890000 ÷ 180) tolás = 10500 tolás

= 3 mds. 11 sr. 4 ch. *Ans.*

Examples 34.

1. Reduce (i) to *tolás* and (ii) to *kanchás*: 29 mds. 7 sr. 5 ch.; 5 mds. 9 sr. 5 ch.; 17 mds. 35 sr. 13 ch.; 19 mds. 33 sr. 11 ch.

2. Reduce to *maunds, &c.*: 246441 tolás; 3348921 kan.; 946489 ch.; 12467521 poás; 456281 ch.

3. Reduce 5 kan. 3 mds. 7 vis. 3 sr. 5 palams, to *tolás*; and 29648012 tolás, to kandis, *&c.*

4. Reduce 2 kan. 14 mds. 28 sr. 65 ta. 3 ma. (Bombay) to *dhans*, and 4628129748 dhans to *kandis, &c.*

5. Reduce 72 tolás 10 ma. 7 ra. (Indian Jewellers' weight) to *dhans*; and 15601 dhans to *tolás &c.*

6. Reduce 16 tolás 1 ma. 6 ratis (Kaviraj's weight) to *dhans*; and 56204 dhans to *tolás, &c.*

7 Reduce to *drams*: 125 tons; 16 tons 15 cwt.; 10 tons 4 cwt. 3 qr.; 19 tons 14 cwt. 2 qr. 23 lbs.; 7 tons 14 cwt. 3 qr. 15 lb. 12 oz.; 33 tons 19 cwt. 2 qr. 20 lbs. 5 oz. 14 dr.; 15 tons 4 cwt. 2 qr. 9 lbs. 14 oz.

8. Reduce to *tons, &c.*: 4561841 dr.; 56294142 dr.; 579214 oz.; 56241421 lbs. 14621058 dr.

9. Reduce to *grains*: 34 lbs. 6 oz. (Troy); 27 lbs. 8 oz. 19 dwt.; 33 lbs. 10 oz. 16 dwt.; 47 lbs. 9 oz. 15 dwt. 17 grs.

10. Reduce to *lbs., &c.* (Troy): 1260 oz.; 12346 dwt.; 57620 grs.; 96412 grs.; 7421 grs.

11. Reduce to *grains*: 15 lbs.; 19 lbs. 7℥. 5ʒ; 16 lbs. 11℥. 6ʒ. 2℈.

12. Reduce to *lbs.* (*Apo.*): 125678 grs; 245789℈.; 946211 grs.

13. Reduce to *grains*: 5 sr. 8 ch.; 19 sr. 15 ch. 3 tolás.

14. Reduce to *tons, cwt*, &c.: 17 sr. 8 ch.; 1 md. 12 sr. 8 ch.; 6 mds. 22 sr. 8 ch.; 13 mds. 5 sr.

15. Reduce to *maunds, seers, &c.*; 10 cwt. 1 qr. 4 lbs.; 2 tons 11 cwt. 1 qr. 20 lbs.

16. Reduce 11 cwt. 2 qr. to *Bombay maunds*; 6 tons 5 cwt. to *Madras maunds*; 117 Madras Maunds and 219 Bombay Maunds to *tons, &c.*

17. Add together :

(1)

mds.	sr.	ch.
141	37	15
244	31	4
206	19	2
561	0	8

(2)

mds.	sr.	ch.	kan.
17	33	7	2
11	15	6	1
19	8	7	3
44	27	1	2

(3)

oz.	dwt.	gr.
11	19	23
10	18	22
9	17	21
8	16	20

(4)

lbs.	oz.	dwt.	gr.
52	11	13	21
47	9	18	12
94	8	17	20
41	0	12	8

(5)

Tons.	cwt.	qr.	lb.	oz.
20	18	2	14	8
31	14	0	18	7
54	12	1	20	12
12	6	3	10	13

(6)

Tons.	cwt.	qr.	lb.	oz.	dr.
113	16	2	15	12	8
15	15	1	12	0	3
20	11	0	13	6	2
72	10	2	11	13	1

18. Subtract :

(1) 16 mds. 13 sr. 5 ch. from 29 mds. 9 sr. 11 ch. ; 201 mds. 28 sr. 10 ch. from 417 mds. 7 sr. 8 ch.

(2) 8 oz. 17 dwt. 19 grs. from 10 oz. 16 dwt. 16 grs. ; 18 lbs. 11 oz. 15 dwt. from 32 lbs. 8 oz. 12 dwt.

(3) 15 lbs. 13 oz. 9 dr. from 17 lbs. 14 oz. 11 dr. ; 3 qr. 22 lbs. 12 oz. from 1 cwt. 20 lbs. 14 oz. ; 15 cwt. 2 qr. 19 lbs. from 18 cwt. 1 qr. 21 lbs. ; 106 tons 14 cwt. 3 qr. from 271 tons 15 cwt. 2 qr.

19. 752 water-carts are employed to fill a tank ; each cart can bring 17 mds. 12 sr. 10 ch. of water at a time : what is the capacity of the tank if it becomes full when all the carts have been used 5 times ?

20. A merchant bought 746 bags of rice, each bag weighing 4 mds. 15 sr. 12 ch. ; find the whole quantity of rice.

21. If 225 mds. 0 sr. 12 ch. of rice be distributed among 44 persons, how much should each get ?

22. How many bags will be required to hold 305 mds. 11 sr. 8 ch. 3 kan. of sugar, if one bag can hold 5 sr. 10 ch. 3 kan. ?

23. How many plates, weighing 2 sr. 8 ch. each, can be made from 5 mds. 20 sr. of silver ?

24. I distributed 65 mds. 35 sr. 8 ch. of rice amongst a number of beggars, giving 10 sr. 8 ch. to each ; how many beggars were there ?

25. 36 kandies of sugar are to be packed up in bags of equal size. How many bags will be wanted if each hold 1 vis. 2 sr. 4 pal. ?

26. If 3 mds. 5 sr. 25 tanks of coal can be bought for a rupee, what quantity can be bought for 235 rupees ?

27. What would be the weight of 75 silver tea-services, if each weigh 1 lb. 7 oz. 14 dwt. 20 grs. ?

28. If a gold ring weighs 3 dwt. 18 grs. how many such can be made of a pound of gold ?

29. Find the weight of 720 iron-rollers, each weighing 5 cwt. 1 qr. 14 lbs.

30. If 157 iron beams weigh 7 tons 2 cwt. 2 qr. 24 lbs. 9 oz., what is the weight of each beam ?

31. How many bales, each weighing 2 qr. 7 lbs. 8 oz., can be made up out of goods weighing 39 tons 9 cwt. 24 lbs. ?

32. If a truck contain 15 tons 15 cwt. 3 qr. 15 lbs. 15 oz. 15 dr. of coal, how much will 256 trucks contain ?

33. If 311 bags of flour weigh 24 tons 5 cwt. 3 qr. 21 lbs., find the weight of each.

34. How often is 1 qr. 8 lbs. contained in 2 tons 5 cwt. 3 qr. 24 lbs. ?

35. How many poor men can be given 1 lb. 1 oz. 8 dr. each, out of 3 qr. 25 lbs. 6 oz. of flour ?

36. If a merchant buys 31 cwt. of jute at £2. 11*s.* per cwt., and exchange it for 11 tons 15 cwt. of soda at 4*s.* 3*d.* per cwt., what cash payment will be required in addition ?

37. Which is heavier and by how much,—a pound of gold or a pound of cork ?

III. MEASURES OF LENGTH.

101. **Bengal Lineal Measure.**

3	Yabs	make	1 Anguli
4	angulis	,,	1 Mushti
3	mushtis	,,	1 Bighat *(span)*
2	bighats or 24 angulis	,,	1 Hát or **Cubit**
4	cubits	,,	1 Dhanu or Danda
2000	dhanus or 8000 háts	,,	1 Kros
4	kros	,,	1 Yojan.

102. **English Lineal Measure.**

3	Barley-corns	make	1 Inch (*in.* or 1″)
12	inches	,,	Foot (*ft.* 1′)
3	feet	,,	**Yard** (*yd.*)
5½	yards	,,	Rod, Pole (*po.*), or Perch
40	poles	,,	Furlong (*fur.*)
8	fur. or 1760 yds.	,,	Mile (*mi.*)
3	miles	,,	

1 yard = 2 cubits. 1 kros = 4000 yds. 1 Ilahi gaj (N.-W. P.) = 33 in. 1 Karam (Mad.) = 3 cubits. 1 Kathi (Bom.) = 9·4 ft. 1 half-yard = 1 ft. 6 in.

Cloth Measure.

BENGAL.			ENGLISH.		
3 Angulis	make	1 Girah	2¼ Inches	make	1 Nail (*nl.*)
8 girahs	,,	1 Hát	4 nails	,,	1 Quarter (*qr.*)
2 háts or 16 girahs	,,	1 Gaj.	4 quarters	,,	1 Yard
BOMBAY.			3 quarters	,,	1 Flemish ell
2 Angulis	make	1 Tasu	5 quarters	,,	1 English ell
24 tasus	,,	1 Gaj.	6 quarters	,,	1 French ell.

1 Nail = 1 Giráh. 1 Bombay gaj (cloth measure) = 27 in. 1 Bengal gaj = 36 in. = 1 yard.

Land Measure.

BENGAL.			ENGLISH.		
4 Háts	make	1 Katha	25 Links	make	1 Pole or Rod.
20 Kathas	„	1 Bigha.	100 Links	„	1 Chain.
80 Háths	„	1 Rashi.	10 chains	„	1 Furlong.

In the N.-W. P., 3 Iláhi gaj = 1 Bans and 26 bans = 1 Jarib.

1 Hand = 4 in. (for measuring horses); 1 Palm = 6 in.: 1 Span = 1 bighat = 9 in.; 1 Pace = $2\frac{1}{2}$ft. (military); 1 Fathom = 6ft.; 1 cable's length = 120 fa.; 1 Knot (nautical mile) = 6080 ft. 60 knots = 1 degree of latitude 80 chains = 1 mi.; 1 ch. = 4 po. = 22 yds.

In *carpentry*, 12 lines = 1 inch (English); 8 shutas = 1 in. (Bengal).

103. In reducing poles to yards, multiply by 11 and divide the product by 2; and in reducing yards to poles, multiply by 2 and divide the product by 11. Also, in reducing miles and furlongs to yards, multiply by 1760 and 220, respectively.

Ex. 1. Reduce 3 mi. 5 fur. 25 po. 5 yds. 2 ft. to *inches*.

3 mi. 5 fur. 25 po. 5 yds. 2 ft.
8
29 fur.
40
1185 po.
11
2|13035 half-yards
6517 yds. + 1 half-yard

or 6517 yds. 1 ft. 6 in.
5 2 ft.
6523 yds. 0 ft. 6 in.
3
19569 ft.
12
234834 in.

Ex. 2. Reduce 246892411 inches to *miles*, &c

12)246892411 in.
3)20574367 ft. + 7 in.
6858122 yds. + 1 ft.

11)13716244 half-yards.
40)1246931 po. + 3 half-yards.
8)31173 fur. + 11 po.
3896 mi. + 5 fur.

the result = 3896 mi. 5 fur. 11 po. 3 half-yards 1 ft. 7 in.
= 3896 „ 5 „ 11 „ 1 yd. 2 ft. 13 in.
= 3896 „ 5 „ 11 „ 2 „ 0 „ 1 „

Examples 35.

1. (1) Reduce to *cubits*: 10 kros 800 dhanus; 10 kros 250 dhanus; 24 bi. 15 kat.; 30 bi. 18 kat. 3 háts.

(2) Reduce to *gaj*, &c.: 45631 angulis; 7264 girahs; 94628 girahs.

2. Reduce to *inches*:

(1) 5 fur. 23 po. 3 yds. 2 ft.; 3 mi. 7 fur. 15 po. 2 ft.; 7 mi. 4 fur. 17 po. 4 yds. 1 ft. 6 in.

(2) 24 mi. 7 fur. 31 po. 5 yds.; 57 mi. 6 fur. 35 po.; 57 mi. 2 fur. 39 po. 4 yds. 2 ft. 7 in.

3. Reduce to *miles*, &c.: 274 yds.; 946 ft.; 74441 in.; 462141 in.; 9462 ft.; 789211 in.; 2460210 in.; 7214102 in.; 9214614 in.; 72620046 in.

4. Reduce

(1) 15 kros to *miles* and *yards*. (2) 57 miles to *kros* and *hats*.
(3) 57 Ilahi gaj to *yds.*, &c. (4) 15 gaj 1 hat 4 girahs to *ft.* &c.
(5) 22 yds. 3 qr. to *nails*. (6) 27 yds. 3 qr to *nails*.
(7) 7 cubits 1 span to *inches*. (8) 7 miles to *links*.

5. Add together:

(1)

yds.	ft.	in.
24	2	9
13	1	10
9	0	11
20	2	3

(2)

po.	yds.	ft.	in.
40	5	1	9
51	4	2	11
62	3	0	4
23	2	2	6

(3)

mi.	fur.	po.	yds.	ft.	in.
74	7	37	5	1	6
27	6	28	4	0	9
19	5	19	2	2	4
112	4	24	3	1	7

6. Subtract:

(1)

yds.	ft.	in.
44	1	9
27	2	11

(2)

po.	yds.	ft.	in.
99	4	2	7
64	5	1	10

(3)

mi.	fur.	po.	yds.	in.
224	5	28	3	3
219	4	37	4	9

7. Multiply 2 mi. 5 fur. 4 po. 1 yd. 2 ft. by 28, by 80, and by 112.

8. Divide 1 mi. 297 yds. 1 ft. 2 in. by 29; 37 mi. 4 fur. 16 yds. 8 in. by 56; 109 mi. 2 fur. 17 po. 2 yds. 1 ft. 8 in. by 59.

9. A person bought 27 yds. 3 qr. 3 n., 15 yds. 2 qr., 18 yds. 3n. 1 in., 14 yds. 2n. 1 in., and 7 yds. 3 qr. 2n. of cloth; find the total length bought.

10. A ship has to go 200 leagues and sails 150 mi. 800 yds. on the first day, and 120 mi. 1200 yds. on the second. How far has she still to go?

11. What length of desks will be required to accommodate 350 scholars, allowing 2 ft. 4 in. to each?

12. How many sleepers placed 1 hát 9 angulis apart, would be required for a double line of rails 13 miles long?

IV. SQUARE MEASURE.

104
Bengal Land Measure.

20 Square cubits or Gandás	make	1 Chhaták
16 chhatáks	,,	1 Káthá
20 káthás	,,	1 Bigha

THE N.-W. P.

20 Kanchwansi	make	1 Biswansi
20 Biswansi	,,	1 Biswa
2 Biswas	,,	1 Bigha

THE PUNJAB.

9 Sarsai	make	1 Marla
20 mârlas	,,	1 Kanal
4 kanals	,,	1 Bigha
2 bighas	,,	1 Ghuma.

Bombay.			Madras.		
39¼	Square cubits make	Kathi	144 Sq. in. make	1 sq. ft.
20	kathis ,,	Pand	2400 sq. ft. ,,	1 Ground or Manai
20	pands ,,	Bigha		
6	bighas ,,	Rukeh	24 grounds ,,	1 Cawny
20	rukehs ,,	Chahur	484 cawnies ,,	1 sq. mile.

A Bengal Bigha = 1600 sq. yds. An N.-W. P. Bigha = 3025 sq. yds. A Punjab Bigha = 1620 sq. yds. A Bombay Bigha = 3997 sq. yds. Also 1 Madras Cawny = 6400 sq. yds. = 4 Bengal Bighas.

105. English Square Measure.

144	Square inches (sq. in.) make	1 **Square foot** (sq. ft.)
9	square feet	1 Square Yard (sq. yd.)
30¼	square yards	1 Square Pole (sq. po.)
40	square poles	1 Rood (ro.)
4	roods or 4840 sq. yds.	1 **Acre** (ac.)
640	acres	1 Square Mile.
Also, 484	square yards	1 Square Chain.
10	square chains	1 Acre
100.000	square links	1 Acre

A rod of brick work = 272¼ sq. ft.; a Rod of building = 36 sq. yds.; a square of Flooring = 100 sq. ft. A yard of land = 30 ac.; a hide of land = 100 ac.; 1 sq. chain = 10000 sq. links.

40 ac. = 121 Bengal Bighas; 5 ac. = 8 N.-W. P. Bighas; 81 ac. = 242 Punjab Bighas; 160 ac. = 121 Madras Cawnies. Also 1 sq. mi. = 1936 Bengal Bighas = 1024 N.-W. P. Bighas = 484 Madras Cawnies.

106. To reduce square poles to square yards, multiply by 121 and divide the product by 4. For remainders 1, 2, 3 take 2 sq. ft. 36 sq. in., 4 sq. ft. 72 sq. in., and 6 sq. ft. 108 sq. in.

Ex. Reduce 5 ac. 2 ro. 17 sq. po. 20 sq. yds. 6 sq. ft. to *sq. inches.*

5 ac. 2 ro. 17 sq. po. 20 sq. yds. 6 sq. ft.
4

4)108527
27134 sq. yds. with 1, remainder
= 27134 sq. yds. 2 sq. ft. 36 sq. in.
20 6

27154 sq. yds. 8 sq. ft. 36 sq. in.
9

244394 sq. ft. (144 = 12 × 12)
2932728

35192736 sq. in. + 36 sq. in. = 35192772 sq. in.

107. To reduce square yards to square poles, multiply by 4, and divide the product by 121. The remainder will be given in quarter square yards. See ART. 106.

Ex. Reduce 742169456 sq. in. to *acres.*

144 { 12|742169456
12|61847454—8 } ∴ true remainder=(6×12+8) or 80 sq. in.
9|5153954 sq. ft —6 }
572661 sq. yds.+5 sq. ft.
4

121 { 11|2290644 quarter sq. yds.
11|208240 —4 } ∴ true remainder=(10×11+4) =114 quarter sq. yds. =28 sq. yds. 4 sq. ft. 72 sq. in.
40|18930 sq. po—10 }
4|473 ro.—10 sq. po.
118 ac.—1 ro.

∴ the result=118 ac. 1 ro. 10 sq. po. 28 sq. yds. 4 sq. ft. 72 sq. in.
+ 5 „ „ 80 „ „
=118 ac. 1 ro. 10 sq. po. 29 sq. yds. 1 sq. ft. 8 sq. in.

108. To reduce acres and roods to square yards, multiply by 4840 and 1210, respectively. To reduce square yards to acres, divide by 4840.

Examples 36.

1. Reduce to *gandas* or *sq. cubits* : 27 bi. 17 kat.; 39 bi. 18 kat. 15 ch.; 50 bi. 15 kat. 14 ch. 15 ga.; 37 bi. 15 kat. 12 ch.; 20 bi. 19 kat. 15 ch. 17ga.

2. Reduce to *bighas* : 7461 ch.; 942146 ga. ; 4126180 sq. cubits ; 564128 ga.

3. Reduce to *kanchwansi* : 15 bi. 5 bisw.; 27 bi. 10 bisw. 16 biswansi; 142 bi. 17 bisw. 15 biswansi.

4. Reduce to *sq. karam or sarsai* : 25 ghm ; 20 ghm. 1 bi. ; 40 ghm. 1 bi. 3 kan. 5 marlas.

5. Reduce to *sq. inches* : 5 sq. mi. 280 caw. ; 7 sq. mi. 300 caw. 1 6 grounds ; 15 sq. mi. 480 caw. 22 grounds, 1000 sq. ft.

6. Reduce :

(1) 45601 kanchwansi to *bighas.* (2) 2468140 sq. sarsai to *ghumas.*

(3) 46281461 sq. in. to *cawnies.* (4) 462814201 kathis to *chahurs.*

7. Reduce to *sq. cubits* : 17 bi. ; 27 bi. 15 kat. ; 75 bi. 19 kat.

8. Reduce to *sq. inches* :

(1) 123 sq. yds. ; 45 sq. yds. 7 sq. ft. ; 19 sq. yds. 5 sq. ft. 114 sq. in. ; 12 sq. yds. 4 sq. ft. 18 sq. in.

(2) 214 sq. po. ; 244 ro. ; 1741 ro. ; 15 ac. ; 140 ac. ; 25 sq. mi.

(3) 17 ac. 3 ro. 4 po.; 42 ac. 2 ro. 17 po. 20 sq. yds. 8 sq. ft. 108 sq. in.; 104 ac. 1 ro. 24 sq. po. 25 sq. yds.

9. Reduce to *acres*: 4569 sq. yds.; 94621 sq. ft.; 462141 sq. in.; 4621 sq. yds.; 641211 sq. ft.

10. Reduce to *sq. inches*: 14 sq. chains; 24 sq. chains.

11. Reduce (*Bengal bighas*): 4840 bi. to *acres*; 2600 ac. to *bighas*; 1210 bi. to *acres*; 24200 bi. to *acres*; 48400 acres to *bighas.*

12. Reduce: 64800 Bengal bighas to *Punjab bighas*; 605000 Bengal bighas to *N.-W. P. bighas*; 2560000 Punjab bighas to *Bengal bighas*; and 5120000 N.-W. P. bighas to *Bengal bighas.*

13. Add together:

(1)

bi.	ka.	ch.
14	12	7
25	17	14
18	19	15
37	10	8

(2)

sq. yd.	sq. ft.	sq. in.
47	8	108
12	5	36
44	6	72
18	7	108

(3)

ac.	ro.	sq. po.	sq. yd.
99	2	20	12
41	3	15	14
12	1	18	19
101	2	17	28

14. Subtract:

(1)

bi.	ka.	ch.
14	7	8
11	12	10

(2)

ac.	ro.	sq. po.
124	2	15
109	3	24

(3)

ac.	ro.	sq. po.	sq. yd.
17	2	21	13
10	3	22	14

15. Multiply:

(1) 12 bi. 10 kat. 13 ch. by 25 and by 60.

(2) 5 ac. 2 ro. 14 po. 20 sq. yds. by 24.

16. Divide:

(1) 50 bi. 17 kat. 8 ch. separately by 20, 44, 148.

(2) 958 sq. yds. 3 sq. ft. 80 sq. in. by 80; 987 ac. 3 ro. 27 po. by 26.

17. How many bighas does the city of Calcutta contain, supposing its area to be 7 sq. miles? If London be thrice as large, what is its size in acres?

V. CUBIC MEASURE.

109. **Bengal Measure of Solidity.**

8 Cubic hâts	make	1	Cubic yard.
8 cubic yards	„	1	Chouka.

English Measure of Solidity.

1728 Cubic Inches	make	1	Cubic Foot (cub. ft.)
27 cubic feet	„	1	Cubic Yard (cub. yd.)

1 cub. hât=5832 cub. in. A load of rough Timber=40 cub. ft.; a load of squared Timber=50 cub. ft.; a ton of Shipping=42 cub. ft. A stack of wood=108 cub. ft. A cord of wood=128 cub. ft.

Examples 37.

Reduce :

1. to *cub. cubits* : 71 choukas 51 cub. cubits ; 51 choukas 63 cub. cubits ; 42 choukas 57 cub. cubits.

2. to *cub. in.* : 10 cub. yds. 24 cub. ft. 12 cub. in. ; 27 cub. yds. 20 cub. ft. 956 cub. in. ; 51 cub. yds. 26 cub. ft. 1100 cub. in.

3. to *choukas* : 567412 cub. cubits ; 674801 cub. cubits.

4. to *cub. yds.* : 4781246 cub. in. ; 94214611 cub. in.

110. **Measures of Capacity** (Dry.)

BENGAL MEASURE (corn).

5 Chhatáks	make	1 Kunka
2 kunkas		1 Khunchi
2 khunchis		1 Rek
2 reks		1 Pali
2 palis		1 Doan
2 doans		1 Kati
8 katis		1 Arhi
20 arhis		1 Bish
16 bishes		1 Kahan
16 pa. or 8 do.		1 Maund (md.)
20 doans		1 Sali.

MADRAS MEASURE (corn).

8 Ollaks	make	1 Puddi
8 puddis	„	1 Markal
5 markals	„	1 Phera.
80 Pheras	„	1 garce

[1 markal = 750 cub. in.]

10000 markals = 3466 imperial gallons

BOMBAY MEASURE (corn).

36 Tanks	make	1 Tipari
2 tiparis	„	1 Seer.
4 seers	„	1 Payli
16 paylis	„	1 Phera
8 pheras	„	1 Kandi
25 pheras	„	1 Muda.

In Bengal, lime is measured thus : 1 pherá = 27″ × 20″ × 9″ ; 6 pherás = 5 cub. háts. 80 pherás = 100 mds.

English Measure for Corn and Dry Goods.

2 **Gallons** (gal.)	make	1 Peck (pk.)
4 pecks	„	1 Bushel (bus.)
8 bushels	„	1 Quarter (qr.)
5 quarters	„	1 Load (ld.)
2 loads	„	1 Last.

For Coal.

4 Pecks	make	1 Bushel
3 bushels	„	1 Sack
12 sacks	„	1 Chaldron.

8 Pints = 4 Quarts = 2 Pottles = 1 gallon : 1 Strike = 2 bushels ; 1 Comb = 4 bushels. A gallon contains 277·274 cub. in. A bushel contains 8 × 277·274 or 2218·192 cub. in. **A pound (Avoir.)** = the weight of 27·727 cub. in. (or the *tenth* of a gallon) of distilled water at the temperature of 62° F. the barometer standing at 30 in.

Examples 38.

1. Reduce to *chhatáks* : 7 mds. 5 do. 1 pa. ; 15 mds. 7 do. ; 5 kah. 15 bis. ; 25 kah. 12 bis. 18 arh. ; 74 mds. 6 do. 1 pa. 1 rek

2. Reduce : 267489641 ch. to *maunds* ; 72167000 ch. to *ka'hans* ; 94256003 ch. to *maunds* ; 146521 reks to *káhans*.

3. Reduce : 215 pheras to *ollaks* : 150 pheras 4 markals to *ollaks* ; 7460 ollaks to *pheras* ; 754 puddis to *pheras* ; 12456 ollaks to *pheras.*

4. Reduce: 15 kandis to *tanks*; 40 mudas to *tiparis*; 15 kandis 6 pheras 15 paylis to *tanks*; 9468214 tanks to *kandis*; 462801 tiparis to *mudas.*

5. Reduce to *gallons*: 10 bus. 3 pk. 1 gal.; 17 lds. 4 qr. 7 bus.; 18 lasts 1 ld. 3 qr.; 56 lasts 1 ld. 3 qr. 5 bus 2 pk.

6. Reduce to *lasts*: 4678412 gal.; 67894 pk; 96214678 gal.; 462894 bus.

III. Measures of Capacity (Fluids).

Indian Fluid Measure.

4	Chhatáks	make	1	Póa
4	póas	,,	1	Seer
40	seers	,,	1	maund.

[A seer of water is supposed to weigh a seer of 80 tolas.]

English Wine Measure.

4	Gills	make	Pint (pt.)
2	pints	,,	uart
4	quarts	,,	allon (gal.)
63	gallons	,,	Hogshead(hhd.)
2	hogshead	,,	Pipe (pipe)
2	pipes		Tun

1 Anker = 10 gallons
1 Runlet = 18 gallons
1 Tierce = 42 gallons
1 Puncheon = 2 Tierces = 84 gal.

English Ale & Beer Measure.

2	Pints	make	1	Quart
4	quarts	,,	1	Gallon
36	gallons	,,	1	Barrel (bar.)
1½	barrels or 54 gallons	,,	1	Hogshead
2	hogshead	,,	1	Butt
2	Butts	,,	1	Tun.

1 Firkin or quarter-barrel = 9 gal.
1 Kilderkin or half-barrel = 18 gallons.

6 Tuns (Wine Measure) = 7 tuns (Ale Measure).

1 gallon of pure water weighs 10 lbs. (Avoir.); therefore a pint weighs 20 oz. (Avoir). 1 cub. ft. of water weighs 997 oz, or *about 1000 oz.*

Apothecaries' Measure (Fluids).

60	Minims (*m.*) or drops	make	1	Fluid Dram (ʒ)
8	fluid drams	,,	1	Fluid Ounce ( )
20	fluid ounces	,,	1	Fluid Pint (*O.*)
8	pints	,,	1	Gallon (*C.*)

A tea-spoonful = 1 fluid dram; a dessert-spoonful = 2½ fluid drams; a table-spoonful = 4 fluid drams. 1 fluid ounce = 1 ounce (Avoir).

Examples 39.

1. Reduce to *gills*. 25 gal. 3 qt. 1 pt.; 10 hhd. 52 gal.; 15 hhd. 60 gal.; 17 pipes 1 hhd. 52 gal.; 17 tuns 1 pipe 1 hhd. 44 gal. 2 qt. 1 pt.

2. Reduce to *pints*: 75 butts 1 hhd. 40 gal.; 22 tuns; 34 tuns 18 gal.

3. Reduce to *minims*: 15*C*.; 24*C*. 5*O*.; 15*C*. 7*D*. 15 .

4. Reduce : 842614 gills to *pipes* ; 7412681 pt. to *tuns* ; 489214 pt. to *pipes* ; 824241 pt. to *tuns*.

5. Reduce to *tuns*, &c. : 4281416 pt. ; 489214 qt. ; 8941 gal. ; 21461 pt.

6. Find the weight of 12 gal. 3 qt. of water in lbs., Avoir.

7. What is the weight of 14 cub. yds. 7 cub. ft. of water in lbs., Avoir. ?

112. VI. (*a*) Indian Measure of Time.

60	Anupals	make	1	Bipal (bip.)
60	bipals	,,	1	Pal (pal)
60	pals	,,	1	Danda (dan.)
$7\frac{1}{2}$	dandas or 3 hours	,,	1	Prahar (pr.)
60	dandas or 8 prahars	,,	1	**Din** or *day* (da.)
30	dins	,,	1	Másh or month (ma.)
12	máshes	,,	1	Batsar or year (ba.)
12	batsars	,,	1	Yuga.

$2\frac{1}{2}$ dandas = 1 Ghantá ; 1 danda = 24 minutes. 7 dins = 1 Saptáha ; 15 dins = 1 Paksha ; 2 pakshas = 1 month. A lunar month = $29\frac{1}{2}$ days.

113. (*b*) English Measure of Time.

60	Seconds (sec.)	make	1	Minute (min.)
60	minutes	,,	1	Hour (hr.)
24	hours	,,	1	**Day** (da.)
365	days	,,	1	**Year** (yr.)
100	years	,,	1	Century.

A month = 30 days. A year = 4 quarters = 12 months = 52 weeks. 7 days = 1 week ; 2 weeks = 1 fortnight. A **Leap Year** = 366 days.

1 minute and 1 second are sometimes written 1′ and 1″ respectively.

114. The following lines give the lengths of the twelve Calendar months :

Thirty days hath September,
April, June and November ;
February hath twenty-eight alone ;
All the rest have thirty-one ;
But a leap year comes once in four,
And February then has one day more.

Bengali Months.		English Months.		
1. Baisákh	(বৈশাখ)	1. January	=31	days
2. Jaistha	(জ্যৈষ্ঠ)	2. February	=28	,,
3. Ashárh	(আষাঢ়)	3. March	=31	,,
4. Srávan	(শ্রাবণ)	4. April	=30	,,
5. Bhádra	(ভাদ্র)	5. May	=31	,,
6. Aswin	(আশ্বিন)	6. June	=30	,,
7. Kártic	(কার্ত্তিক)	7. July	=31	,,
8. Agraháyan	(অগ্রহায়ণ)	8. August	=31	,,
9. Paus	(পৌষ)	9. September	=30	,,
10. Mágh	(মাঘ)	10. October	=31	,,
11. Phálgun	(ফাল্গুন)	11. November	=30	,,
12. Chaitra	(চৈত্র)	12. December	=31	,,

The Bengali month also is supposed to consist of 30 days. This however, is not strictly correct. Some months have 32 days, some 31, some 30, some 29, and some 28. As a rule, the summer months are long and the winter months short. Then, again, 12 months of 30 days would make the year consist of 360 days only, instead of 365 or 366.

Mahomedan months are :—

1. Maharrum (মহরম)	7. Rajab (রজব)
2. Safar (শফর)	8. Shaban (শাবান)
3. Rabial-auwal (রবিয়লআউয়ল)	9. Ramzan (রমজান)
4. Rabias-sani (রবিয়স্সানি)	10. Shawal (শওয়াল)
5. Jamadial-auwal (জমাদিয়লআউয়ল)	11. Zilkad (জেল্কদ)
6. Jamadias-sani (জমাদিয়স্সানি)	12. Zil-hijja (জেলহজ্জ)

115. Every *fourth* year is a **leap year. If the last two figures of a year can be divided by 4, with nothing over, the year is a leap year or bissextile. In the case of years ending with two o's, divide by 400 ; if no remainder is left, it is a leap year, otherwise not.**

Thus, 1892 is a leap year, but 1894 is not ; 1600 is a leap year, but 1900 is not. In every 400 years there are 97 leap years.

The mean solar year consists of 365·242218 days. Now 365·242218 $=365+{\cdot}25-{\cdot}0075-\ldots=365+\frac{1}{4}-\frac{3}{400}-,\ldots$ Thus we have the RULE for the intercalation of the Calendar : Add 1 day to every 4th year, but omit to do so *thrice* in 400 years. It will be seen that this would make

the year too long by [(365+·25−·0075)−365·242218...] or (365·2425−365·242218...) or ·000282 of a day. The error would, therefore, still be 1 day in 4000 years, approximately.

Note.—In Indian Calendars the day is reckonned from sun-rise to the next sun-rise while in English Calendar each day is considered to commence at midnight.

The period of revolution of the earth round the sun is called a solar year. As this period does not consist of an exact number of days, to make the average length of a civil year equal to this period, in B. C. 46 Julius Cæsar with the help of an astronomer made this arrangement: that in every 4 years, 3 years should consist of 365 days each, and the remaining year of 366 days, making the average length of each civil year = 365·25 days. This mode of reckoning is called the Julian Calendar or the old-style. In this method the difference between the solar and common year is therefore ·007782...of a day every year. Therefore in 400 years the error would amount to 3·1128 days. To rectify this, the omission of the extra day three times in 400 years was ordered in 1582 by Pope Gregory XIII and is called the Gregorian Correction. This mode of reckoning is called the New Style.

This New Style was adopted in Roman Catholic countries from 1582, but in England not until 1752. In that year the day after September 2nd was called 14th September. It was also arranged in this new style that the legal year should begin on the 1st. of January, instead of on the 25th of March. Thus the year 1751 began on 25th of March but ended on the 31st of December.

The New Style has not yet been adopted in Russia, so that since 1752 they have had two more leap years (1800 and 1900) than we have, and they are now 13 days behind us.

116. To reduce prahars to *dandas*, multiply by 15 and divide the product by 2 ; the remainder, if any, is a half-danda or 30 pals. To reduce dandas to *prahars*, multiply by 2 and divide the product by 15 ; the remainder, if any, is equal to so many half-dandas.

Examples 40.

1. Reduce to *anupals* :

(1) 6 dan. 50 pa. (2) 7 pr. 5 dan. 15 pa. 45 bip. (3) 5 days.
(4) 6 sap. 6 da. 5 dan. (5) 4 sap. 5 da. 5 pr. (6) 15 sap. 4 da.
(7) 6 sap. 4 da. 5 pr. 40 pa. 50 bip.

2. Reduce to *seconds* :

(1) 15 da. 17 hrs. 15 min. ; 365 da. 21 hrs. 45 min. ; 256 da. 16 hrs.
(2) 7 wks. 5 da. 18 hrs. 45 min. ; 13 wks. 6 da. 20 hrs. 45 min. 56 sec.

3. Reduce :

(1) 462105 anupals to *pals*. (2) ·7462148 bipals to *dandas*.
(3) 762140 bipals to *prahars*. (4) 9421601 pals to *days*.

4. Reduce :

(1) 4678901 sec. to *hours*. (2) 64121460 sec. to *days*.
(3) 412900 min. to *days*. (4) 4628146 min. to *weeks*.

5. Reduce : (1) 2462 hrs. to *dandas*. (2) 46250 dandas to *hours*.

6. Add together :

(1)

sap.	da.	pr.
15	6	7
16	4	2
4	5	4
19	2	1

(2)

dan.	pal.	bip.	anu.
18	45	56	20
20	30	28	40
16	15	42	50
6	50	30	10

(3)

da.	pr.	dan.
16	7	7
44	6	6
21	5	5
219	4	

(4)

hrs.	min.	sec.
15	45	52
21	54	18
16	12	35
14	21	25

(5)

da.	hrs.	min.	sec.
172	15	45	50
84	16	35	40
14	12	55	30
116	10	15	20

(6)

wk.	da.	hrs.	min.
17	6	20	40
12	5	12	30
52	2	10	50
44	1	16	12

7. Subtract :

(1)

dan.	pal.	bip.	anu.
41	41	52	45
18	41	52	56

(2)

sap.	da.	pr.
416	4	6
415	4	7

(3)

da.	pr.	dan.
117	5	5
94	5	6

(4)

hrs.	min.	sec.
52	48	36
49	54	40

(5)

da.	hrs.	min.	sec.
112	21	40	30
67	22	48	36

(6)

wks.	da.	hrs.	min.
17	4	15	30
16	4	16	26

8. Multiply :

(1) 21 da. 5 pr. 5 dan. 30 pals separately by 12, 20, and 32.
(2) 5 da. 20 hrs. 15 min. separately by 15, 27, and 48.

9. Divide :

(1) 14 sap. 3 da. 4 pr. separately by 4, 14, and 28.

(2) 1265 wks. 3 da. 1 hr. 20 min. separately by 20, 33, and 44.

117. **Measure of Angles.**

60 Seconds (60″)	= 1 Minute (1′)
60 minutes	= 1 Degree (1°)
90 degrees	= 1 **Right Angle.**

Examples 41.

1. Reduce to *seconds* :

(1) 15° 40′ 15.″ (2) 80° 45′ 30.″ (3) 72° 48′ 24.″

2. Reduce to *right angles, degrees, &c.* :

(1) 74211′. (2) 6214021″. (3) 4621′. (4) 4512′. (5) 678124216″.

118. Measures of Numbers.

BENGALI TABLE.		ENGLISH TABLE.	
4 Units	= 1 Ganda	12 Units	= 1 Dozen.
5 gandas	= 1 Kuri	12 dozen	= 1 Gross.
4 kuris	= 1 Pan	12 gross	= 1 Great Gross.
16 Pans	= 1 Kahan	20 units	= 1 Score.

FOR PAPER.

24 Sheets = 1 Quire ; 20 quires = 1 Ream ; 10 reams = 1 Bale.

For printing paper, there are 500 sheets, instead of 24 × 20 or 480 sheets, to the ream. The number of reams in a bale is also different for different kinds, and varies from 10 to 25.

CHAP. XIII. MISCELLANEOUS PROPOSITIONS.

119. The Unitary Method. (Simple Cases.)

(i) The price, weight, &c. of **one** article being known, that of a given number of articles is found by *multiplying* the given price, weight, &c. by the number of articles.

Ex. The price of a maund of rice is *Rs.*3. 12*a.* 6*p.* ; find the price of 20 maunds.

*Rs.*3. 12*a.* 6*p.*	The price of 1 maund = *Rs.*3. 12*a.* 6*p.*
20	∴ the price of 20 mds. = *Rs.*3. 12*a.* 6*p.* × 20
*Rs.*75. 10*a.*	= *Rs.*75. 10*a.*

(ii) The price, weight, &c. of **any number** of articles being given, the price, weight, &c. of one article is found by *dividing* the given price by the number of articles.

Ex. 24 yards of silk cost £2. 18*s.* ; find the price of a yard.

The price of 24 yds. = £2. 18*s.* ;
∴ the price of a yard = £2. 18*s.* ÷ 24 = 2*s.* 5*d.*

(iii) The price, weight, &c. of a certain number of articles being given, the price, weight, &c. of a certain other number of articles may be found thus :

Ex. 1. If 12 cricket balls cost £3. 10*s.* how much will 7 balls cost ?

12)£3. 10*s.*	12 balls cost £3. 10*s.* ;
5*s.* 10*d.*	∴ 1 ball costs £3. 10*s.* ÷ 12 = 5*s.* 10*d.* ;
7	∴ 7 balls cost 5*s.* 10*d.* × 7 or £2. 0*s.* 10*d.*
£2. 0*s.* 10*d.*	

Ex. 2. A man earns *Rs.* 6. 3*a.* 2*p.* in 5 weeks ; how much will he earn in 6 weeks ?

5)*Rs.* 6. 3*a.* 2*p.*	He earns in 5 weeks *Rs.* 6. 3*a.* 2*p.* ;
Re. 1. 3*a.* 10*p.*	∴ " " " 1 week *Rs.* 6. 3*a.* 2*p.* ÷ 5
6	= *Re.* 1. 3*a.* 10*p.*
Rs. 7. 7*a.*	∴ " " " 6 weeks *Re.* 1. 3*a.* 10*p.* × 6
	= *Rs.* 7. 7*a.*

(iv) The price, weight, &c. of a certain number of articles being known, the number of articles corresponding to some other price, weight, &c. may be found thus :

Ex. 1. If 36 bighas of land can be bought for *Rs.* 1200, how many bighas will be bought for *Rs.* 900 ?

36)*Rs.* 1200
Rs. 33. 5*a.* 4*p.*
= the price of a bigha.

Rs. 900 = 172800*p.* : *Rs.* 33. 5*a.* 4*p.* = 6400*p.*
∴ the reqd. no. of bighas = 172800 ÷ 6400
= 27. *Ans.*

Ex. 2. If £567 will buy 126 cows, how many can be bought for £972 ?

126)£567
£4. 10*s.*

£972 ÷ £4. 10*s.* = 216.
∴ the required number of cows = 216. *Ans.*

Examples 42.

1. If 15 yds. of broad cloth cost £3, how much will 7 yds. cost ?

2. If 11 carts hold 225 mds. 9 sr., what weight will 3 carts carry ?

3. If a cwt. of coffee cost £28, what is the price of a 3 lb. packet ?

4. If I can buy 10 yards of ribbon for 15*d.*, what length of ribbon can I buy for 21*d.* ?

5. If the Railway fare for 10 mi. be 5*a.*, what is the fare for 170 mi. ?

6. If I spend R104 in 16 weeks, how much do I spend in a year ?

7. If 11 sacks of coal weigh 8 cwt. 6 lbs., how many sacks can be filled with 3 cwt. 74 lbs. ?

8. If the Railway fare for 105 miles is 13*s.* 1½*d.*, how far should you be carried for £1. 0*s.* 9*d.* ?

9. If 100 dinners can be paid for with £6. 15*s.* 5*d.*, how many can be paid for with £16. 18*s.* 6½*d.* ?

10. If I earn £3. 9*s.* 2¼*d.* in 27 days, how long will it take to earn £18. 6*s.* 5¼*d.* ?

11. A trader sold 45 mds. of rice for R148. 5*a.* 9*p.* ; for how much should he sell 237 mds. of rice of the same quality ?

12. A man bought 47 cwt. of tea for £569. 2*s.* 9¾*d.* ; for how much did he buy 93 cwt. of the same quality ?

13. If 100 ponies cost R3979. 11*a.*, how much will 337 ponies cost ?

14. A man spends £1288. 1*s.* 8*d.* in 754 days : how much does he spend in 536 days ?

15. The carriage of 48 mds. costs R159. 4*a.* ; what will the carriage of 127 mds. cost ?

16. If the weekly wages of 63 carpenters amount to £244. 9*s.* 0¾*d.* ; find the weekly wages of 583 carpenters.

17. If 225 mds. 0 sr. 12 ch. of rice be distributed equally among 44 persons; how much will 39 of them get?

18. If 311 bags of flour weigh 24 tons 5 cwt. 3 qr. 21 lbs.; find the weight of 47 bags.

19. If 85 srs. of sugar cost R22. 2*a*. 2*p*., what quantity can be bought for R43. 3*a*. 8*p*. at the same price?

20. If 91 horses are worth £2265. 2*s*. 10*d*., how many horses are worth £149. 7*s*., the price of a horse being the same in both the cases?

21. 85 seers of tea are sold for R333. 5*a*. 9*p*.; what quantity can be bought for R1000. 1*a*. 3*p*.?

22. If R10305 will buy 192 cows, how many will R1663. 13*a*. 3*p*. buy?

23. If 60 bags can hold 8 mds. 20 srs. 5 ch. of sugar, how many bags will hold 305 mds. 11 sr. 8 ch. 3 kan. of sugar?

24. If 720 iron rollers weigh 193 tons 10 cwt, how many will weigh 301 tons?

25. If 96 gold rings can be made from 1 ℔ 6 oz. of gold, how many can be made from 10 ℔s.?

26. If 170 persons can be seated on benches extending over a length of 132 yards 0 ft. 8 in, how many persons can be accommodated on benches 272 yards 0 ft. 8 in. long?

27. 19758 ac. 1 ro. 20 po. of land were divided amongst 520 tenants in equal portions, how many of them got 14818 ac. 3 ro. 5 po. together?

28. The weekly wages of 4953 persons amount to £32256. 8*s*. 3*d*.; find the weekly wages of 793 persons.

29. If £32262 18*s*. 6*d*. can keep 4954 soldiers for a certain time, how many soldiers can be kept for £15630 for the same time?

30. 53 horses cost £406. 8*s*. 10½*d*., 27 oxen cost £119. 6*s* 1½*d*., and 761 sheep cost £285. 7*s*. 6*d*.; what is the total cost of 20 horses, 40 oxen and 80 sheep?

120. **Revolution of Wheels.** A **wheel** in one **revolution** passes over a length of ground equal to the length of its **circumference.** In all questions on revolutions of wheels we should, therefore, consider each revolution as a strip of length equal to the circumference. Hence,

(1) to find how far a carriage will have gone after a given number of revolutions of one of its wheels, we should multiply the circumference of the wheel by the number of revolutions;

(2) to find the circumference of a wheel, we should divide the distance passed over by the number of revolutions;

(3) to find the number of revolutions, we should divide the distance passed over by the circumference of the wheel.

Ex. 1. A carriage wheel, 3 yds. 2 ft. 3 in. in circumference, makes 1386 revolutions on a journey. What is the length of the journey ?

$1386 = 11 \times 9 \times 14$.

(220 yds. = 1 furlong.)

	mi.	fur.	yds.	ft.	in.
			3	2	3
					11
			41	0	9
					9
		1	151	0	9
					14
	2	7	137	1	6

∴ the carriage has passed over 2 mi. 7 fur. 137 yds. 1 ft. 6 in.

Ex. 2. In passing over 2 mi. 934 yds. 2 revolutions ; what is its circumference ?

The circumference = 2 mi. 934 yds. 2 ft. ÷ 1542 = 8 ft. 8 in.

Ex. 3. How many revolutions will the driving wheel of an engine, 23 ft. 7 in. in circumference, make on a journey of 30 mi. 5 fur. 153 yds. ?

23 ft. 7 in.
12
283 in.

30 mi. 5 fur. 153 yds.
8
245
220
54053 yds.
36
1945908 in.

283)1945908(6876
1698
2479
2264
2150
1981
1698
1698

∴ the required number of revolutions = 6876. *Ans.*

Examples 43.

1. If a wheel 3 yds. 2 ft. 9 in. in circumference makes 840 revolutions on a journey, how far will the carriage go ?

2. On a journey a wheel makes 1247 revolutions : if the circumference of the wheel is 4 yds. 2 ft. 11 in., what is the length of the journey ?

3. A wheel makes 3080 revolutions on a journey of 9 mi. 3 fur. 183 yds. 1 ft. ; what is the circumference of the wheel ?

4. The driving wheel of an engine revolves 4620 times in passing over 13 mi. 5 fur. 18 yds. 1 ft. What is the circumference of the wheel ?

5. If the circumference of a coach-wheel is 3 yds. 1 ft., how often will it turn in a distance of 6 mi. 4 fur. ?

6. How many revolutions will a wheel 5 yds. 2 ft. in circumference make on a journey of 8 mi. 880 yds. ?

7. A carriage goes at the rate of 3 mi. 4 fur. an hour ; how often will its forewheel, the circumference of which is 6 ft. 5 in., turn in 5 hours ?

8. The fore and the hind wheels of a carriage are 10 ft. and 15 ft. round respectively ; how often will each of them revolve in a journey of 7 mi. 880 yds. ?

9. In passing over a distance of 100 miles the forewheel revolves 52800 times and the hindwheel 33000 times. How much longer is the circumference of the latter than that of the former?

10. The hindwheel and the forewheel of a carriage are 18 ft. and 12ft. in circumference, respectively; how many revolutions will the forewheel make more than the other in passing over 4 mi. 4 fur.?

11. The hindwheel of a carriage which is 16 ft. round, makes 550 turns less than the forewheel in passing over 5 miles: find the circumference of the forewheel.

***12.** *A* and *B* ride bicycles round a tract of 440 yds. The wheel of *A's* bicycle is 10 ft. in circumference and turns twice in a second; that of *B's* bicycle is 8 ft. in circumference, and turns 3 times in a second: how long after the one should the other start, so that at the end of 10 rounds the two men may be exactly level with one another?

121. **Gain or Loss.** The price at which a thing is bought is called its **cost** price; that at which it is sold, its **selling** price. When the selling price is greater than the cost price, there is **gain**; if less, there is **loss.** The difference between the two prices is the *gain* or *loss.*

(i) The cost and selling prices being given, and also the quantity sold, to find the gain or loss.

Ex. 1. A trader bought 462 yds. of cloth at *Rs.* 3. 5*a.* 4*p.* per yard, and retailed them at *Rs.* 3. 10*a.* 8*p.* per yard: how much did he gain?

Selling price per yd.	= *Rs.* 3.	10*a.* 8*p.*
Cost ,, ,, ,,	= *Rs.* 3.	5*a.* 4*p.*
∴ gain per yd.	=	5*a.* 4*p.*
∴ gain on 462 yds.	=	5*a.* 4*p.* × 462 = *Rs.* 154. *Ans.*

Ex. 2. A person bought 1000 yds. of silk at 16*s.* 3*d.* a yard and sold them at 15*s.* 9*d.* a yard; what did he lose?

Cost price per yard	= 16*s.*	3*d.*
Selling ,, ,,	= 15*s.*	9*d.*
∴ loss per yard	=	6*d.*
∴ loss on 1000 yards	=	6*d.* × 1000 = £25. *Ans.*

(ii) The cost and selling prices being given, and also the gain or loss, to find the quantity sold.

Ex. 1. A merchant bought rice at *Rs.*3. 12*a.* per maund and sold at *Rs.*4. 1*a.* 6*p.* per maund, gaining thereby *Rs.*257. 2*a.* on his outlay; find the quantity bought.

Selling price per maund	= *Rs.*4.	1*a.* 6*p.*
Cost ,, ,,	= *Rs.*3.	12*a.*
∴ gain per maund	=	5*a.* 6*p.* = 66*p.*
Now the whole gain	= *Rs.*257 2*a.* = 49368*p.*;	

∴ the quantity bought = (49368 ÷ 66) or 748 mds. *Ans.*

Examples 44.

1. A trader bought 2175 maunds of rice at R3. 14*a*. 9*p*. per maund, and sold them at R4. 2*a*. 6*p*. per maund : find his gain.

2. A farmer bought 45 oxen at R24. 8*a*. 6*p*. each and 30 oxen at R30. 7*a*. each, and sold all of them at an average price of R29. 15*a*. 9*p*. each ; what did he gain or lose ?

3. A man lost 10*a*. 8*p*. by selling an article for R7. 5*a*. 4*p*. ; what would he have gained had it been sold for R8. 4*a*. 9*p*.

4. I bought 216 pieces of cloth at R5. 7*a*. 6*p*. per piece, and sold 96 pieces at R5. 10*a*. 6*p*. per piece and the rest at R5. 1*a*. 3*p*. per piece ; what did I gain or lose ?

5. A grocer buys 200 maunds of sugar at R15. 10*a* 8*p*. per maund, and sells the whole for R3212. 8*p*. ; find the gain per maund.

6. A man bought 163 gallons of spirits at 5*s*. 4*d*. per gallon, and after 19 gallons had leaked out, sold the remainder at 7*s*. 6*d*. per gallon ; did he gain or lose, and by how much ?

7. A farmer bought a flock of sheep consisting of 320 sheep at R2. 4*a*. 6*p*. per head, and after 20 of them had died sold the remainder at R2. 10*a*. per head ; what was his gain or loss ?

8. A merchant bought 60 cwt. of sugar, and lost £10 by selling them for £130 ; at what rate per lb. did he buy ?

9. A paper merchant bought 8 bales of paper at £1. 8*s*. 4*d*. per ream ; 160 quires, being damaged, were sold at 1*s*. per quire : at what price per quire must he sell the remainder so as to gain £26. 13*s*. 4*d*. on the whole ?

10. A dealer in cloth bought a certain number of yards of silk at 6*s*. 2½*d*. per yard, and retailed them at 7*s*. 10*d*. per yard ; he gained £143. 4*s*. 10½*d*. on the whole ; how many yards were bought ?

11. A trader bought rice at R3. 11*a*. 6*p*. per maund, and selling at R4 a maund, gained R126. 9*a*. on the outlay ; what quantity did he buy ?

12. A merchant bought tea at R2. 7*a*. 6*p*. per seer, and retailed it at R2. 13*a*. 2*p*. per seer ; supposing that he gained R663. 11*a*. 4*p*., find the quantity bought.

122. **Mixtures.** To find the price of a mixture, when the quantity and price of each of the component parts are given.

Ex. 1. A man bought 130 lbs. of tea at 1*s*. 4*d*. a lb. and 110 lbs. at 10½*d*. a lb. At what price per pound must he sell the mixture, in order that he may gain £1. 10*s*. 5*d*. by the transaction ?

The cost of 130 lbs. = 1*s*. 4*d*. × 130 = £8. 13*s*. 4*d*.
„ 110 „ = 10½*d*. × 110 = £4. 16 3
∴ „ 240 „ = £13. 9*s*. 7*d*.
Profit = 1 10 5
∴ the selling price of 240 lbs. = 15. 0*s*. 0*d*.
∴ the selling price of 1 lb. = £15 ÷ 240 = 1*s*. 3*d*. *Ans.*

Ex. 2. In the above Example, if he sells the mixture at 1*s*. per lb., how much does he gain or lose?

The cost price of 240 lbs.	=£13 9*s*. 7*d*.
The selling price of 240 lbs. at 1*s*. per lb.	=£12 0*s*. 0*d*.
∴ the loss	=£1 9*s*. 7*d*. *Ans.*

Ex. 3. A wine merchant buys 214 gallons of wine for 70 guineas; how much water must he mix with it that he may gain 7 half-guineas by selling the mixture at 5*s*. 3*d*. a gallon?

70 guineas = 70 × 21 × 12*d*. = 17640*d*. = cost price;
7 half-guineas = 10*s*.6*d*. × 7 = 882*d*. = profit;
∴ 17640*d*. + 882*d*. = 18522*d*. = selling price of the whole.
Also, 5*s*. 3*d*. = 63*d*. = selling price per gallon;
∴ the quantity sold = (18522 ÷ 63) or 294 gallons;
∴ the " of water added = (294 − 214) or 80 gallons. *Ans.*

Examples 45.

1. A grocer buys 15 maunds of sugar at R10. 10*a*. 8*p*. per maund, and 25 maunds more at R14. 5*a*. 4*p*. per maund: he mixes them and sells the mixed sugar at R14. 13*a*. 9*p*. per maund; find his gain.

2. A grocer bought 2 cwt. 15 lbs. of tea at 5*s*. 5*d*. per lb. and 4 cwt. 1 qr. 2 lbs. at 4*s*. 2*d*. per lb., and mixed the two together; find the cost price per lb. of the mixture. At what price per lb. must he sell so as to gain £29. 17*s*. 6*d*. on his outlay?

3. A grocer mixes 40 lbs. of black tea with 10 lbs. of green tea at 5*s*. 1*d*. per lb.; if, by selling the mixture at 4*s*. 5*d*. per lb., nothing is gained or lost, what did the black tea cost per lb.?

4. A grocer buys 14 sr. of black tea at R2. 4*a*. per seer and mixes them with 18 sr. of green tea; what is the cost price of the green tea per seer, if he gain R20. 12*a*. by selling the mixture at R3. 4*a*. per seer?

5. A market-woman buys 24 dozen of eggs at 2*a*. per dozen, and 30 dozen more at 1*a*. 9*p*. per dozen; at what price per dozen must she sell the whole so as to gain 12*a*. by the bargain?

6. A grocer buys 250 sr. of green tea at R3. 4*a*. per seer; what quantity of tea dust must he mix, so that he may sell the mixture at R2. 8*a*. per seer?

7. What quantity of water must a milk-maid mix with 92 sr. of milk costing 2*a*. 6*p*. per seer to reduce the price to 2*a*. a seer?

8. A wine merchant buys 256 gal. of spirits at 12*s*. 6*d*. per gallon; what quantity of water must he add that, by selling at 7*s*. 6*d*. per gallon, he may gain £20 on his outlay?

9. A person lays out £54 in buying wine at 7*s*. 6*d*. a gallon; how much water must he mix with it that he may sell at 6*s*. a gallon?

10 A wine merchant mixes together two pipes of wine at £80 per pipe, two pipes of another at £90 per pipe and two pipes of a third at £100 per pipe, and sells one-third of the mixture at 13*s*. 4*d*. a gallon; at what price per gallon must he sell the remainder so as to gain £69 by the transaction?

11. A grocer mixes together 173 seers of black tea at R2. 3*a*. 6*p*. per seer, 239 seers of green tea at R2. 8*a*. 9*p*. per seer and 191 seers of tea dust at 10*a*. 6*p*. per seer. If he sells the mixture at R1. 15*a*. 6*p*. per seer, find his gain or loss.

12. A *mudi* mixes 276 mds. of ghee at R41. 12*a*. per maund with 76 mds. of oil at R17. 8*a*. per maund, 50 mds. are sold at R39. 6*a*. a maund; at what price per maund must he sell the remainder so as to gain R1208. 4*a*. by the transaction?

123. **Exchange** and **Barter**. People **barter** when they **exchange** one sort of goods for an equal value of goods of a different sort. Thus, to find how much of the second sort must be given in exchange, we must *first* find the money value of the goods to be exchanged, and *then* find how much of the second sort are worth that value.

Ex. 1. How many slates worth 3*a*. 6*p*. each, should be given in exchange for 420 books at *Re*. 1. 4*a*. each?

Re. 1. 4*a*.	240*p*.	3*a*. 6*p*.	42 { 7)100800
16	420	12	6)14400
20*a*.	48	42*p*.	2400
12	96	$=7\times 6p$.	
240*p*.	100800*p*.		

∴ the required number of slates = 2400. *Ans.*

Ex. 2. How many tons of coal at 10*s*. 3*d*. a ton must be given in exchange for 270 yds. of silk at 3*s*. 5*d*. a yard?

3*s*. 5*d*.	41*d*.	10*s*. 3*d*.	123)11070(90
12	270	12	11070
41*d*.	11070*d*.	123*d*.	

∴ the number of tons of coal = 90. *Ans.*

Examples 46.

1. How many rupees (1*s*. $4\frac{1}{2}$*d*. each) must be given up for £396?

2. Find in £. *s*. *d*. the value of R2560 at 1*s*. $5\frac{1}{4}$*d*. for the rupee.

3. How many seers of sugar at 5*a*. 4*p*. per seer, must be given in exchange for 128 yards of silk at R3. 10*a*. 8*p*. per yard?

4. A farmer who has 248 sheep worth R2. 7*a*. each, gives them in exchange for a certain number of cows at R7. 12*a*. each; find the number of cows.

5. A grocer gives 116 lbs. of tea at 3*s*. 6*d*. per lb. in exchange for 100 yds. of cloth at 4*s*. 7*d*. per yard; does he gain or lose, and how much?

6. *A* has 30 oxen worth R30. 12*a*. each, and *B* has 12 horses worth R80 each: should they exchange, which of them ought to give money also and how much?

7. If 60 yds. of silk at Rs.1. 10*a.* a yard be exchanged for 126 lbs. of tea, find the price of tea per lb.

8. A grocer exchanges 40 cwt. of green tea at 4*s.* 6*d.* per lb. and 36 cwt. of black tea at 4*s.* per lb. for 214 horses at £54. 10*s.* per head; does he receive or pay money, and how much?

9. If 1771 Roubles be given for 378 Nepoleons at 15*s.* 9½*d.* each, find the value of a Rouble.

10. A grocer exchanges 23 maunds of ghee at 13*a.* 4*p.* per seer, for 32 maunds of sugar at 5*a.* 4*p.* per seer: does he gain or lose by the bargain, and by how much?

124. **Allotment** or **Shares.** When a given sum of money or a given quantity of goods is allotted in a certain way among a number of persons, the amounts or quantities received by the several persons are called their respective **shares.**

Ex. 1 Divide *Rs.*25. 8*a.* 4*p.* among *A*, *B* and *C*, in such a way that *A* may get *Rs.*5. 6*a.* 4*p.* more than *B*, and *B* *Rs.*2. 4*a.* 3*p.* more than *C*.

$$\begin{array}{llrrrll}
 & A \text{ gets} & Rs.5. & 6a. & 4p. & \text{more than} & B; \\
\text{and } B & \text{,,} & 2 & 4 & 3 & \text{,,} & C; \\
\hline
\therefore A & \text{,,} & 7 & 10 & 7 & \text{,,} & C.
\end{array}$$

We may *first pay* *Rs.*7. 10*a.* 7*p.* to *A* and *Rs.*2. 4*a.* 3*p.* to *B*, and then divide the remainder equally among *A*, *B*, *C*.

$$\begin{array}{rrr}
Rs. & a. & p. \\
7 & 10 & 7 \\
2 & 4 & 3 \\
\hline
9 & 14 & 10
\end{array}
\qquad
\begin{array}{rrr}
Rs. & a. & p. \\
25 & 8 & 4 \\
9 & 14 & 10 \\
\hline
3)15 & 9 & 6 \\
\hline
5 & 3 & 2
\end{array}
\qquad
\begin{array}{rrr}
Rs. & a. & p. \\
5 & 3 & 2 \\
7 & 10 & 7 \\
\hline
12 & 13 & 9
\end{array}
\qquad
\begin{array}{rrr}
Rs. & a. & p. \\
5 & 3 & 2 \\
2 & 4 & 3 \\
\hline
7 & 7 & 5
\end{array}$$

∴ *A*'s share = <u>*Rs.* 12. 13*a.* 9*p.*</u>, *B*'s share = <u>*Rs.* 7. 7*a.* 5*p.*</u>, and *C*'s share = <u>*Rs.* 5. 3*a.* 2*p.*</u>

Ex. 2. Divide *Rs.*5535 1*a.* 11*p.* among *A*, *B* and *C*, giving *B* four times and *C* six times that given to *A*.

If *A*'s share is 1, *B*'s share is 4 and *C*'s share is 6.

$$1+4+6=11. \qquad \begin{array}{l} 11)Rs.5535.\ 1a.\ 11p. \\ \hline \quad Rs.\ 503.\ 3a.\ 1p. \end{array}$$

$$\left.\begin{array}{lll}
\therefore A\text{'s share} & & = Rs.\ 503.\ 3a.\ 1p., \\
B\text{'s} \ \text{,,} & = Rs.\ 503\ 3a.\ 1p. \times 4 = & \text{,,}\ 2012.\ 12a.\ 4p., \\
\text{and } C\text{'s} \ \text{,,} & = \text{,,}\ 503.\ 3a.\ 1p. \times 6 = & \text{,,}\ 3019.\ 2a.\ 6p.
\end{array}\right\} Ans.$$

Ex. 3. Divide £1800 among *A*, *B* and *C* in such a manner that as often as *A* may get £2, *B* shall get £3, and *C* shall get £4.

$$2+3+4=9. \qquad \begin{array}{l} 9)£1800 \\ \hline \quad £200 \end{array}$$

$$\left.\begin{array}{ll}
\therefore A\text{'s share} & = £\,200 \times 2 = £\,400, \\
B\text{'s} \ \text{,,} & = \text{,,}\ 200 \times 3 = \text{,,}\ 600, \\
\text{and } C\text{'s} \ \text{,,} & = \text{,,}\ 200 \times 4 = \text{,,}\ 800.
\end{array}\right\} Ans.$$

Ex. 4. An equal number of rupees, half-rupees, quarter-rupees and two-anna pieces amount to *Rs.*330. Find the number of each.

If they be taken 1 each, they will together $=16a.+8a.+4a.+2a.$ $=30a.$ Also *Rs.*$330=5280a.$; and $5280\div30=176$.

$\therefore$ the number of each $=176$ *Ans.*

Examples 47.

1. Divide R764. 8*a.* 11*p.* between *A* and *B*, so that *A* may get R242. 6*a.* 5*p.* more than *B*.

2. Divide £4749. 16*s.* 7*d* between *A* and *B*, so that *A* may receive £671. 18*s.* 5*d.* less than *B*.

3. Divide R2270. 10*a.* among 10 men and 12 women, so that each man may get R7. 1*a.* more than what each woman receives.

4. If out of 17745 yds. of silk *A* buys 225 yds. 2 ft. 8 in. more than *B*, how much does each buy ?

5. Divide R826. 13*a.* 4*p.* among *A*, *B* and *C*, so that *A* may receive R25 4*a.* 2*p.* more than *B*, and *B* R22. 6*a.* 4*p.* more than *C*.

6. Divide 39 tons 16 cwt. of rice among *A*, *B* and *C*, so that *A* may get 2 tons 17 cwt. 1 qr. more than *B*, and *B* 1 ton 11 cwt. 2 qr. more than *C*.

7. Divide £8243. 1*s.* 2*d.* among *A*, *B* and *C*, so that *A* may get £462. 10*s.* 8*d.* more than *B*, and *B* £516. 11*s.* 9*d.* less than *C*.

8. In a factory there are employed 20 men, 15 women, and 12 boys, and their weekly wages amount to R107. 12*a.* ; divide the money so that each man may get 4*a.* more than each woman, and each woman 4*a.* more than each boy.

9. How should 1412 tons 10 cwt. of coal be divided among 40 men, 45 women and 50 children, so that each man may get 10 cwt. more than each woman and each woman 10 cwt. more than each child ?

10. Divide 500 mds. of wheat between *A* and *B*, in such a manner that for every 2 mds. given to *A*, *B* shall get 3 mds.

11. How should 65 mds. 6 sr. 12 ch. of rice be divided between 2 *mudis*, so that one may receive 5 mds. more than twice what the other gets ?

12. Divide R256. 12*a.* 8*p.* among *A*, *B* and *C*, so that *A* may have 4 times, and *B* 3 times what *C* receives.

13. Divide £1452 among *A*, *B* and *C*, in such a manner that as often as *A* gets £5, *B* shall get £8 and *C* shall get £7.

14. Divide £724. 7*s.* 6*d.* among *A*, *B* and *C*, so that *A* may receive three times as much as each of *B* and *C*.

***15.** How many boys must be taken to help as many men to do a piece of work in 15 days for R468. 12*a.*, the daily wages of a man being 4*a.* and a man doing three times as much work as a boy in the same time and getting four times as much per day ?

16. An equal number of guineas, sovereigns, half-guineas, crowns, and half-crowns together amount to £695 ; how many of each are there ?

17. An equal number of rupees, half-rupees, quarter-rupees, two-anna pieces, double-paisás and paisás amount to R240. 3*a*. 3*ps*.; find the number of each.

18. A certain number of rupees, three times as many half-rupees, four times as many quarter-rupees, five times as many two-anna pieces, amount to R412. 8*a*. ; find the number of each.

19. I distributed 242 lbs. of sugar among an equal number of men, women and boys ; to every man I gave 2 lbs. 8 oz., to every woman 1 lb. 12 oz , and to every boy 1 lb. 4 oz. : find the number of men.

20. A person lays out R5150 in the purchase of equal quantities of sugar at R16. 8*a*. per maund, rice at R4. 12*a*. per maund, and wheat at R4. 8*a*. per maund : how many maunds of each does he buy ?

MISCELLANEOUS EXAMPLES II.

1. How many nobles are equivalent to £66. 13*s*. 4*d*. ?

2. A boy can count 40 rupees per minute : how many will he count in 5 days, if he counts 10 hrs 45 min. every day ?

3. A rupee weighs 180 grains Troy ; how many rupees can be made of 27 lbs. Avoir. of silver ?

4. A train can travel 27 mi. 4 fur. 135 po. in an hour ; how far will it travel in 45 hours ?

5. R214 is divided equally to the nearest anna among 23 men, and the remainder distributed among a certain number of beggars, 2*ps*. being given to each ; how many beggars are there ?

6. The income of a gentleman in the year 1852 was £120. 5*s*. 4½*d*., out of which he saved £21. 18*s*. 1¼*d*. ; how much did he spend daily ?

7. A silk-worm produces 28 Troy grains of silk ; how many must be kept to produce a cwt. ?

8. A fruiterer sells 500 mangoes at R1. 8*a*. per hundred, 420 guavas at 1*a*. a *kuri*, 240 oranges at 2*a*. per doz., 860 apples at 1*a*. 2*ps*. each, and 720 lichies at 4*a*. per score ; how much does he get altogether ?

9. A grocer buys 10 chests of tea, each containing 420 lbs. for £630 and sells the tea at 3*s*. 7*d*. per lb. ; how much does he gain ?

10. A man earns R225 a week, and spends R2406 every quarter ; how much will he have saved at the end of 12 years ?

11. In marching, soldiers take 75 steps of 2 ft. 6 in. every minute : in quick march 108 steps : how far would a regiment go in 4 hrs., the last hour at quick march ?

12. A house and its furniture are worth R64872. 12*a*. ; if the house is worth 7 times as much as the furniture, what is the house worth ?

13. A certain Rajah is said to be worth R4 a minute ; what is his annual income ?

14. A gentleman's income in 1872 is R2464; how much can he spend daily, if he has to pay off a debt of R1245. 14*a*. 6*p*. in the year?

15. A jeweller sold jewels to the value of 4625 moidores, for which he received as many guineas; what sum remained due?

16. *A* and *B* have together R17. 13*a*. 8*p*., *A* and *C* together R14. 8*a*. 6*p*., and *B* and *C* together R20. 10*a*. 10*p*.; how much has each?

17. *A* and *B* together buy 123 cwt. 2 qr. 8 lb. of coal, *A* and *C* together 115 cwt. 3 qr. 18 lb., and *B* and *C* together 143 cwt. 1 qr. 14 lb.; find the quantity bought by each?

18. If one hour is misspent daily, what time is wasted in 10 yrs.?

19. There are two pieces of cloth of the same length, the prices of which are R30. 10*a*. 8*p*. and R43. 2*a*. respectively: if the price of the first be 5*a*. 4*p*. per yard, what is the price of the other per yard?

20. A trader buys 756 cwt. of sugar at £1. 18*s*. 11½*d*. per cwt., with which he mixes 1921 cwt. of a better quality, which cost him £2. 2*s*. per cwt.; at how much per lb. must he sell the mixed sugar in order that he may make a profit of £739. 12*s*. 2*d*.?

21. How often will a carriage-wheel 2 yds. 2 ft. 3 in. in circumference turn in passing over from Calcutta to Hooghly, a distance of 24 miles?

22. A stationer bought 150 reams of paper at R7. 4*a*. per ream, and 210 reams at R10. 8*a*. per ream: how much did he gain by selling the whole at an average price of 7*a*. 10*p*. per quire?

23. In a boarding school the annual cost of educating pupils amounts to R43537. 8*a*.; if the monthly cost of educating each pupil be R12. 1*a*. 6*p*., find the number of pupils in the school.

24. The rent of a taluk amounts to R1200, of which R400 is to be paid in cash, R600 in rice, and the rest in peas. If rice be R3. 12*a*. a maund, and peas R2. 8*a*. a maund, how much of each must be paid?

25. Looking at my watch I find it shews 28 minutes past 7 in the morning; what time will have elapsed if, on looking again, I find it indicates 15 minutes past 3 in the afternoon?

26. In a school of 500 pupils, 100 boys subscribed 4*a*. 6*p*. each, and bought mangoes at 9*a*. per score; supposing the mangoes were equally distributed among all the pupils, how many did each get?

27. A man buys a pipe of wine and bottles it into an equal number of quarts and pints; how many dozen of each shall he have?

28. Goods are bought at 1*s*. 10*d*. per lb., and the cost of carriage and other expenses amount to 2*d*. per lb; if they are sold at £12. 14*s*. 6*d*. per cwt., how much is gained or lost per cwt.?

29. A servant is engaged for R45. 12*a*. for the year 1880; what should he get if he go away after 15 weeks?

30. A man walks a certain distance and rides back in 3 hrs. 45 min. He could ride both ways in 2 hrs. 15 min. How long would it take him to walk both ways?

31. A wine-merchant bought wine for R225. at R1. 2*a.* per quart, and when some of it had leaked out, sold the remainder for R270 at R1. 8*a.* per quart. How much had leaked out ?

32. I was born on the 16th of June 1831. What was my age on the 27th of October 1852 ?

***33** The cost of 9 pipes of wine together with the duty thereon amounts to R8505 ; if the duty per gallon be a ninth part of the original cost of a gallon of wine, find it.

34. I buy an equal number of goats and sheep for R262. 8*a.* and R365 respectively ; if the sheep cost me R2. 4*a.* 6*p.* per head, what is the cost of a goat ?

35. In making 220 chairs, the cost of each for wood is R3. 7*a.*, for labour R1. 10*a.* 6*p.*, for cane-work R1 2*a.*, and for polish 7*a.* 6*p.* What is gained on each chair by selling the whole lot for R1581. 4*a.* ?

36. A man spends in four months as much as he earns in a quarter. What can he save from an annual income of R5614. 8*a.* ?

37. A grocer buys a chest of tea containing 2460 lbs. for £369 and retails them at 2*s.* 11*d.* per lb. ; does he gain or lose, and how much ?

38. Divide R1400. 7*a.* 4*p.* among 6 men, 8 women, and 16 boys, so that a man may get twice as much as a woman and a woman thrice as much as a boy.

39. A carriage-load weighs 11 tons 19 cwt. 7 lbs., and is packed up in 3150 equal packages ; find the weight of each.

40. An equal number of moidores, guineas, sovereigns, half-guineas, half-sovereigns, nobles, crowns, half-crowns, shillings, pennies and farthings, amount together to £4981 ; find the number of each.

41. A merchant sells to another 4761 yards of silk at 17*s.* 6*d.* per yard and gets in return £325 14*s.* 6*d.* in cash with 25601 lbs. of tea ; what is the value of the tea per pound ?

42. A cloth merchant bought cloth at R3. 7*a.* 9*p.* per yard, and sold it at R4. 3*a.* 6*p.* per yard ; it he gained R56. 8*a.* 9*p.* on his outlay, find the quantity bought.

43. The fore and the hind wheels of a carriage are 12 ft and 18 ft. in circumference respectively ; how many revolutions will one make more than the other in passing over 63 miles ?

44. A heap of cannon balls weighs 16 tons 6 cwt. 24 lbs. 8 oz ; find the number of balls in the heap, the weight of each being 3 lbs. 4 oz.

45. A loaded truck weighs 32 tons 1 cwt. 1 qr., and the load weighs twice as much as the truck itself. If the load be divided into 4560 equal packages, find the weight of each package.

46. A man's annual income is R4572. 10*a.* 3*p.*, but he spends twice as much as he saves ; find his annual expenditure.

47. Sound travels at the rate of 1140 ft. per second : if a gun be discharged at a distance of 4 miles 560 yds., how long will it be, after seeing the flash, that I shall hear the report ?

48. Convert 4560 lbs. Avoir. to lbs. Troy; and find how many spoons each weighing 8 oz. 15 dwt, can be made out of this weight of silver.

49. A man takes 108 steps of 2 ft. 6 in. every minute; find how far he will go in 7 hours 30 minutes.

50. The yard measure of a cloth merchant is an inch too short; find by how much his customers will be cheated, when 256 yards are sold.

51. The hindwheel of a carriage, which is 5 yds. 1 ft. in circumference, makes 27630 revolutions in passing from one place to another: how many revolutions will be forewheel, which is 4 yds in circumference, make in passing over the same distance?

52. The forewheel of a carriage makes 704 revolutions more than the hindwheel in passing over 4 mi.; what is the circumference of the hindwheel, that of the forewheel being 10 ft?

53 On the 5th of January 1880, a man started on a journey, and returned after 256 days; find the day and month of his return.

***54.** The 23rd August, 1885, was a Monday; what day of the week was the 15th February, 1886?

55 A vendor takes, on Monday £29 17*s.*; on Tuesday £39. 7*s.* 3¾*d.*; on Wednesday £29 2*s.* 6½*d.*; on Thursday £25 12*s* 9¼*d.*; on Friday £31. 4*s.*; on Saturday £47: what is his average daily takings?

56 The STANDARD of Monday, Aug. 31st 1885, was numbered 19073; supposing the paper to have been published every week-day, on what day was it numbered 18000?

***57.** A weekly newspaper was numbered 48 on the 14th August 1887, when was it numbered 112?

58. A train starts from Calcutta travelling at the rate of 30 mi. an hour: 3 hours later, another train starts travelling 35 mi. an hour. How long will it be before the latter overtakes the former?

59. A farmer invested R1460, 6*a.* 3*p.* in the purchase of oxen at R32 7*a.* 3*p.* per head; after some of them had died, he sold the remainder for R1626. 10*a.* 8*p.* at R40 10*a.* 8*p.* a head; how many died?

60 *A* was born when *B* was 22 yrs. 10 mo. old, and *C* was born when *B* was 30 yrs. 5 mo. old; how old was *A* when *C* was 15 yrs. 8 mo. old?

61. Your father was 24 yrs. 7 mo. old when you were born, and your sister was born when your father was 40 yrs 2 mo. of age; how old were you when your sister was 5 yrs. 3 mo. old?

62. How many times will a clock which chimes quarters, strike and chime in February 1902?

63 A gentleman spends in 4 months, as much as he earns in 3: how much can he save per annum out of an annual income of £2736. 18*s*?

***64.** A man's male labourers, of which he has 18, are each paid 2*s.* 4*d.* per day, and his female labourers are each paid 1*s.* 10*d.* per day; their average wages was 2*s.* per day; how many female labourers has he?

65. Light travels at the rate of 186500 miles per second. We receive light from the sun 8 minutes 18 seconds after he has risen, what is the distance of the earth from the sun?

66 At what height is a thunder-cloud situated from which the peals of thunder come 8 mins. 58 secs. after the flashes of lightning? Sound travels at the rate of 1140 ft. per second.

67. The eldest of three brothers is 30 years 5 months and 17 days old, the age of the youngest is 25 years 8 months and 21 days. The second brother is as much younger than the first as he is older than the third. What is his age?

68. Shew that in any year the same days of the month in (1) March and November, (2) September and December and (3) April and July will fall on the same days of the week.

69. In a leap year shew that the same days of the month in January and July will fall on the same days of the week.

70. 1000 pieces of half-rupees and quarter-rupees make up Rs. 419; what is the number of each?

71. A grocer had two kinds of sugar 256 maunds in all; first kind was Rs. 8. 8*a.* per maund and the second kind was R6. 12*a.* per maund. He mixed them and sold the mixture at R7. 8*a.* 3*p.* per maund, thereby having gained or lost nothing. How much of each kind of sugar had he?

72. In the previous example if the grocer gained R28, how much of each kind of sugar there should be?

PART III.

CHAPTER XIV. NUMBERS.

125. Numbers are called **consecutive** when they increase by 1. The consecutive numbers beginning with 1 are called **natural numbers.**

Thus 12, 13, 14, 15...... .are *consecutive*, and 1, 2, 3, 4, 5, 6.........are *natural* numbers.

126. Numbers are called **even** when they can be exactly divided by 2, and **odd** when they cannot be exactly divided by 2.

Thus 4, 10, 16, &c. are *even*, and 3, 7, 13, 17, &c. are *odd* numbers. All odd numbers, when divided by 2, leave 1 as remainder.

A number must be either even or odd.

127. When one number exactly divides, or more simply divides, another, the former is called a **factor** or **measure** of the latter, and the latter a **multiple** of the former. See Art. 60.

Thus 20, being exactly divisible, or more briefly, divisible by 5, 5 is a *factor* or *measure* of 20, and 20 a *multiple* of 5.

128. A **prime number,** or more simply **prime,** is a number which has no measure besides itself and unity. A **composite** number is one which has a measure greater than unity, but less than itself.

Thus, it is easily seen that 2, 3, 5, 7, 11, &c. are *primes*; and 4, 6, 9, 10, 12, &c. are *composite* numbers.

A number must be either prime or composite.

129. The following Rules are important, and should be carefully remembered :

(1) If one number is divisible by another, any multiple of the first is also divisible by the second.

Thus, 10 is divisible by 2, 5, and 10; therefore any number ending with 0, being a multiple of 10, is divisible by 2, 5, and 10; for instance, $150 = 15 \times 10$, and is, therefore, divisible by 2, 5, and 10.

100 is divisible by 4, and 1000 by 8; hence all numbers ending with two 0's are divisible by 4, and all numbers ending with three 0's by 8.

$1001 = 7 \times 11 \times 13$, and is therefore divisible by 7, 11, and 13; hence numbers like 8008 or (8×1001), 14014 or (14×1001), 231231 or (231×1001) are all divisible by 7, 11, and 13.

(2) If each of two numbers is divisible by a third, their sum or difference is also divisible by the third.

Thus, 2146 = 2140 + 6 and is divisible by 2, if *6* is;
3145 = 3140 + 5 " " " 5, if *5* is;
7244 = 7200 + 44 " " " 4, if *44* is;
31240 = 31000 + 240 " " " 8, if *240* is;
7322 = 7007 + 315 and is divisible by 7, if 315 or (*322* − *7*) is;
25542 = 25025 + 517 " " " 11. if 517 or (*542* − *25*) is;
837343 = 837837 − 494 " " " 13, if 494 or (*837* − *343*) is.

(3) When each of two numbers is divisible by a third, the sum or difference of any multiple of the first and of any multiple of the second is also divisible by the third.

Thus, $258 = 200 + 50 + 8 = 2\ (99 + 1) + 5\ (9 + 1) + 8$
$= 2 \times 99 + 5 \times 9 + 2 + 5 + 8$;
∴ 258 is divisible by 3, if $2 + 5 + 8$ is.
$6282 = 6000 + 200 + 80 + 2 = 6\ (999 + 1) + 2\ (99 + 1) + 8\ (9 + 1) + 2$
$= 6 \times 999 + 2 \times 99 + 8 \times 9 + 6 + 2 + 8 + 2$;
∴ 6282 is divisible by 9, if $6 + 2 + 8 + 2$ is.
$52844 = 50000 + 2000 + 800 + 40 + 4$
$= 5\ (9999 + 1) + 2\ (1001 - 1) + 8\ (99 + 1) + 4\ (11 - 1) + 4$
$= 5 \times 9999 + 2 \times 1001 + 8 \times 99 + 4 \times 11$
$+ 5 + 8 + 4 - 2 - 4$;
∴ 52844 is divisible by 11, if $(5 + 8 + 4) - (2 + 4)$ is.
(∵ 9999, 1001, 99, 11 are all divisible by 11)

(4) If a number is separately divisible by two numbers prime to each other, it is divisible by their product.

$6 = 2 \times 3$; therefore a number will be divisible by 6, if it can be divided separately by 2 and 3. Again, $12 = 4 \times 3$; therefore a number is divisible by 12, if it can be divided by 4 and 3 separately.

130. **Criteria of Divisibility.** A number is divided by

2, if its *last* figure is divisible by 2;

[Thus, *370*, *426*, *164*, are all divisible by 2; but *313* is *not*.]

3, if the *sum* of its **figures** is divisible by 3;

[Thus, 225 is divisible by 3, for $2 + 2 + 5 = 9$; but 241 is *not*.]

4, if its *last two* figures are divisible by 4;

[Thus, *532*, *420*, are divisible by 4; but *610* is *not*.]

5, if its *last* figure is 0 or 5;

[Thus, *320*, *415* are both divisible by 5; but *422* is *not*.]

6, if it is divisible both by 2 *and* 3;

8, if its *last three* figures are divisible by 8 ;

[Thus, 4120, 3248 are divisible by 8 ; but 4276 is *not.*]

9, if the *sum* of its **figures** is divisible by 9 ;

[Thus, 315 is divisible by 9, for 3+1+5=9 ; but 228 is *not.*]

10, if its *last* figure is 0 ;

11, if the *difference* of the sums of its figures in the *odd* and the *even* places respectively, either is 0 or is divisible by 11.

[Thus, 7315 and 32549 are divisible by 11 ; but 2315 is *not.*]

For **7** and **13**, see ART. 129 (2).

131. It might be of some use to the student to remember that the following numbers are primes.

1	5	13	23	37	47	61	73	89
2	7	17	29	41	53	67	79	97
3	11	19	31	43	59	71	83	101

132. **To resolve** a composite number into simple factors, is to find the several factors of which it is the product.

Ex Resolve 30 into factors.

$$30=5\times6=5\times3\times2.$$

Thus, 30 may be resolved into the factors 5 and 6, or into 5, 3, and 2.

133. When the factors obtained are all primes the number is said to be resolved into its **prime** or **elementary factors.** A number can be resolved into **only one** set of prime factors.

Ex. 1. Resolve 39270 into prime factors.

2)39270
3)19635
5)6545
11)1309
7)119
17

The last figure is 0 : and the sum of the figures $=3+9+2+7+0=21$. Also $(9+7)-(3+2+0)=11$. Hence the number is divisible by 2, 3, 5, 11.

Also we know that $119=7\times17$, and that 17 is a prime.

∴ the prime factors are 2, 3, 5, 7, 11, and 17.

Ex. 2. Resolve 1611225 into elementary factors.

5)1611225
9)322245
11)35805
3)3255
5)1085
7)217
31

The last figure is 5, the sum of the figures = 18, which is divisible by 9 $(1+1+2+5)-(6+1+2)=0$. Therefore the number is divisible by 5, 9 and 11.

Again in 3255 the last figure is 5, the sum of the figures is 15, $(3+5)-(2+5)=1$; ∴ 3255 is divisible by 5 and 3, but *not* by 11.

31 is a prime.

$$\therefore\ 1611225=5\times9\times11\times3\times5\times7\times31$$
$$=3\times3\times3\times5\times5\times7\times11\times31.$$

Examples 48.

1. Resolve the following, *mentally*, into elementary factors :

(1) 6; 10; 14; 35; 44; 56; 60; 75; 78; 81.

(2) 102; 105; 121; 132; 144; 176; 210; 252; 288; 315.

2. Resolve the following into elementary factors :

(1) 495; 5250; 612; 750; 1089; 1215; 1872; 6006; 9009.

(2) 15015; 34034; 57057; 68068; 74178; 98384; 189189.

(3) 4851; 6480; 7623; 8424; 13824; 42600; 11025; 14157.

(4) 4240236; 5290285; 6407830; 24624600; 85395310.

3. Determine which of the following numbers are prime, and find the prime factors of those amongst them which are composite :

(1) 29; 37; 81; 93; 101; 111; 113; 119; 161; 165; 171; 187.

(2) 207; 299; 503; 514; 671; 810; 1120; 2197; 4576; 2343.

4. State which of the following numbers are divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 13 respectively :

(1) 124; 134; 445; 704; 721; 793; 828; 9190; 7942; 99120.

(2) 336000; 987650; 433378; 123456; 444444; 777777; 4732101.

CHAP. XV. GREATEST COMMON MEASURE.

134. A number which divides two or more given numbers exactly, is called a **common measure** or **common factor** of those given numbers.

Thus, each of the numbers 2, 3, 4, 6, and 12 will divide both 24 and 36, and is therefore a *common measure* or *common factor* of them.

135. The *greatest* number that will divide two or more given numbers exactly, is called their **Greatest Common Measure (G.C.M.) or Highest Common Factor H.C.F.**

Thus, 12 is the *Greatest Common Measure* of 24, 36, and 60.

Examples 49.

Find, by inspection, the G. C. M. of :

1. 4 and 6. **2.** 6 and 8. **3.** 6 and 9. **4.** 8 and 12. **5.** 9 and 15.
6. 10, 15. **7.** 12, 16. **8.** 12, 18. **9.** 15, 25. **10.** 16, 24.
11. 18, 27. **12.** 20, 30. **13.** 22, 33. **14.** 24, 36. **15.** 25, 40.
16. 27, 45. **17.** 28, 35. **18.** 28, 42. **19.** 39, 52. **20.** 49, 63.

136. The G. C. M. of two or more given numbers is the product of **all** the prime factors common to the given numbers.

Ex. Find the G C. M. of 72 and 96.

$$72=2\times2\times2\times3\times3\ ;\ 96=2\times2\times2\times2\times2\times3.$$

Therefore the factors common to 72 and 96 are 2, 2, 2 and 3 ; hence the G. C. M. $=2\times2\times2\times3=24$. *Ans.*

137 To find the G.C.M. of two or more numbers resolve any one of them into prime factors, and then find *by trial* which of these factors are common to all the other given numbers ; the product of those common factors is the G.C.M. required.

Ex. Find the G. C. M. of 1435, 3535 and 7385.

The prime factors of 1435 are 5, 7 and 41 ; of these factors, both 5 and 7 divide 3535 and 7385 exactly, but 41 does not divide them ; therefore 5×7 or 35 is the G. C. M. required.

Examples 50.

Find, by factors, the G. C. M. of :

1. 45 and 75. **2.** 54 and 81. **3.** 64 and 80. **4.** 72 and 126.
5. 88 and 132. **6.** 165 and 275. **7.** 258 and 288. **8.** 480 and 720.
9. 216 and 258. **10** 272 and 425. **11.** 324 and 576. **12.** 390 and 663.
13. 775 and 1800. **14.** 616 and 1518. **15.** 856 and 936. **16.** 639 and 873.
17. 128, 136, 256. **18.** 256, 442, 940. **19.** 868, 3164, 4228.
20. 612, 816, 448. **21.** 102, 446, 428. **22.** 803, 363, 8107.

138. To find the G. C. M. of two numbers whose prime factors cannot be easily found, proceed as follows.

(i) Divide the greater number by the less ;
(ii) then divide the first divisor by the remainder, if any ;
(iii) then divide the second divisor by the second remainder, if any ;
(iv) continue the operation till no remainder is left ;
(v) the **last** divisor will be the G. C. M. required.

Ex. Find the H. C. F. of 3164 and 4228.

```
3164)4228(1
     3164
     ----
     1064)3164(2
          2128
          ----
          1036)1064(1
               1036
               ----
                 28)1036(37
                     84
                     ---
                     196
                     196
```

The first divisor is 3164, and the first remainder is 1064.

The second divisor is 1064, and the second remainder is 1036.

The third divisor is 1036, and the third remainder is 28.

The final divisor is 28.

∴ the reqd. H. C. F. is 28. *Ans.*

139. To find the G. C. M. of three or more numbers.

(i) Find the G. C. M. of the first two numbers ;

(ii) then find the G. C. M. of this G. C. M. and the third number ;

(iii) then find the G. C. M. of this G. C. M. and the fourth numbe

(iv) proceed similarly till the last of the numbers is disposed of ;

(v) the **final** G. C. M. will be the G. C. M. required.

Ex. Find the G. C. M. of 504, 5292, 1516.

```
504)5292(10                 252)1516(6
    5040                        1512
    ----                        ----
     252)504(2                    4)252(63
         504                        252
```

∴ the required G. C. M. is 4. *Ans.*

Examples 51.

Find the G. C. M. of :

1. 715, 884. **2.** 425, 527. **3.** 190, 399. **4.** 735, 1512.
5. 1848, 2808. **6.** 1485, 1809. **7.** 5456, 10075. **8.** 2720, 5152.
9. 1020, 11594. **10.** 47124, 44460. **11.** 5376, 12800. **12.** 5610, 10465.
13. 15453, 20907. **14.** 77143, 259481. **15.** 7670, 13878.
16. 313441, 111221. **17.** 12019, 44137. **18.** 805, 1311, 1978.
19. 432, 1134, 1347. **20.** 805, 1310, 1975. **21.** 1326, 3094, 4420, 5577.

140. Numbers are said to be *prime to each other* when they have no common factor, greater than 1.

Thus, 16 and 25 are prime to each other.

Ex. Are 313 and 421 prime to each other ?

```
313)421(1                        11)97(8
    313                             88
    ---                             --
    108)313(2                        9)11(1
        216                             9
        ---                            --
         97)108(1                       2)9(4
             97                           8
             --                           -
             11                           1
```

∴ 313 and 421 are prime to each other.

Examples 52.

1. Prove that the following numbers are prime to each other :

(1) 256, 305. (2) 440, 1269. (3) 992, 445. (4) 9021, 407.

2. Are the following numbers prime to each other ?

(1) 677 and 809. (2) 73030 and 29991. (3) 61655 and 70092.
(4) 94248 and 165336. (5) 72700 and 90901. (6) 34562 and 78568.

Examples worked out.

Ex. 1. Find the greatest number that will divide 39, 52 and 65, leaving the remainders 3, 4 and 5 respectively.

$$39-3=36;\ 52-4=48;\ 65-5=60.$$

The required number = the G. C. M. of 36, 48, and 60 = 12. *Ans.*

Ex. 2. The sum of two numbers is 1596, and their G. C. M. is 133: how many pairs of such numbers can be formed? Form these pairs.

$$1596 \div 133 = 12.$$

Now, $12 = 1+11 = 2+10 = 3+9 = 4+8 = 5+7 = 6+6.$

The only pairs of numbers that have no common factor greater than 1 are 1, 11 and 5, 7: therefore *two* pairs of numbers can be formed. *Ans.*

∴ 1st pair $= 1\times133$ and 11×133; or 133 and 1463;
and 2nd „ $= 5\times133$ „ 7×133; „ 665 „ 931. } *Ans.*

For the other pairs; 133 will be a common measure but not the G.C.M.

Ex. 3 The product of two numbers is 300 and their G. C. M. is 5; how many pairs of such numbers can be formed? Form them.

$300 \div 5^2 = 12$; and $12 = 1\times12$ or 3×4.

∴ only *two* pairs of numbers can be formed *Ans.*

Also, 1st pair $= 5\times1$ and 5×12, or 5 and 60;
and 2nd „ $= 5\times3$ „ 5×4, „ 15 „ 20. } *Ans.*

Examples 53.

1. Find the greatest number which is contained in 378, 462 and 693.

2. By what coin of greatest value can £371. 5*s.* and £508. 19*s.* be paid?

3. Find the greatest length, that will exactly measure both 1 mi. 5 fur. 110 yds. and 2 mi. 5 fur. 154 yds.

4. What is the highest sum of money that is exactly contained in R33. 12*a.* and R36. 9*a.*?

5. 20391 gallons of beer and 49287 gallons of stout have to be put into barrels, all of the same size. Find how much each barrel must hold, if the barrels used are to be the largest possible.

6. Two heaps of the same sized shot weigh respectively 15 cwt. 3 qr. and 2 tons 12 cwt. 2 qr.; find the greatest possible weight of each shot.

7. Two masses of silver weighing 5 sr. 3 ch. 2 tolas and 7 sr. 5 ch. 2 tolas respectively, are each to be made into coins of the same size; what is the weight of the largest possible coin?

8. From a heap of equal packages weighing 76011 seers in all, some, weighing 36556 seers, are removed. Shew that no package can weigh more than 13 seers.

9. What highest number will divide 229, 342 and 735, leaving the remainders 5, 6, and 7 respectively?

10. Find the greatest number which will divide 636, 761 and 1236, leaving 11 as remainder after each division.

11. In a school which has 30 boys and 24 girls, the boys and girls are separately divided into equal groups: find the smallest number of groups that can be thus formed.

12. The sum of two numbers is 28237, and their G. C. M. is 2567; form as many pairs of numbers as convenient.

***13.** *A* and *B* possess between them R60000 in Bank Notes of R5000 each: find the highest sum *A* can have, supposing the numbers of notes they have are prime to each other.

14. Three bills of £26. 5s., £32. 11s and £37. 5s. 6d. respectively, are to be paid off by means of one sort of coin; what coin of the greatest value can be used?

15. Find the greatest number of persons amongst whom 4755 rupees and 4121 seers of rice can be equally distributed.

***16.** A labourer is engaged for R6. 9*a.* for a certain number of days: he is absent for some days, and gets only R4. 4*a.* 3*p.*: shew that his daily wages cannot exceed 5*a.* 3*p.*

17. What is the greatest and least number of 6 digits that have 433 for their common measure.

18. What are the numbers nearest to a lac that have 277 for their G. C. M.?

CHAP. XVI. LEAST COMMON MULTIPLE.

141. A number which can be exactly divided by two or more numbers is called a **common multiple** of them.

Thus 48 is a *Common Multiple* of 2, 3, 4, 6, 8, and 12.

142. The *least* number which can be exactly divided by two or more given numbers, is called their **Least** or **Lowest Common Multiple (L.C.M.)**

Thus, 24 is the *Least Common Multiple* of 2, 3, 4, 6, 8 and 12.

143. Every Factor that divides any of the given numbers must divide their L.C.M.; therefore the L.C.M. of the numbers must always contain all the prime factors of the given numbers in their highest powers.

Ex. Find the L. C. M. of 12, 16, 18, 27 and 36.

$$12 = 2\times2\times3 = 2^2\times3;$$
$$16 = 2\times2\times2\times2 = 2^4;$$
$$18 = 2\times3\times3 = 2\times3^2;$$
$$27 = 3\times3\times3 = 3^3;$$
$$\text{and } 36 = 2\times2\times3\times3 = 2^2\times3^2.$$

In the given numbers, the highest power of 2 is 2^4, and that of 3 is 3^3;
$\therefore$ the L. C. M. $= 2^4\times3^3 = 16\times27 = 432$. *Ans.*

Examples 54.

1. Find *mentally* the L.C.M. of :

(1) 3, 6. (2) 4, 8. (3) 6, 9. (4) 5, 10.
(5) 8, 12. (6) 9, 12. (7) 10, 12. (8) 7, 14.
(9) 10, 15. (10) 12, 16. (11) 12, 18. (12) 12, 30.
(13) 2, 3, 4. (14) 2, 3, 5. (15) 4, 6, 8. (16) 4, 8, 12.
(17) 5, 7, 15. (18) 5, 10, 15. (19) 12, 16, 24. (20) 15, 20, 25.

2. Find, by resolving into factors, the L. C. M. of :

(1) 6, 14, 39. (2) 4, 18, 20. (3) 6, 16, 24.
(4) 15, 13, 39, 65. (5) 18, 36, 45, 81. (6) 16, 24, 28, 32, 39.
(7) 508, 889. (8) 3003, 7007. (9) 12012, 8008.
(10) 936, 2925. (11) 84, 6720, 448. (12) 625, 1225, 1575.
(13) 165, 855, 9500. (14) 504, 5292, 1520. (15) 28, 84, 154, 343.

144. To find the L.C.M. of two large numbers whose prime factors are not easily made out, find their G.C.M. as in ART. 138. ; divide one of the numbers by their G.C.M. ; and multiply the other number by the quotient thus obtained.

Ex. Find the L.C.M. of 2501 and 8651.
The G.C.M is 41. Also $2501 \div 41 = 61$.
∴ the L.C.M. $= 8651 \times 61 = 527711$. *Ans.*

145. The L.C.M. of more than two numbers is found thus :
First find the L.C.M. of the first two given numbers ; then find the L.C.M. of this L.C.M. and the third given number ; and so on : the final result will be the L.C.M. required.

Examples 55.

Find the L.C.M. of :

1. 791, 1017. **2.** 841, 1247. **3.** 1265, 440. **4.** 729, 4374.
5. 321, 1177. **6.** 7688, 4805. **7.** 2697, 3441. **8.** 9997, 7670.
9. 6241, 71416, 71854. **10.** 1482, 1938, 8398.

146. The following is the most **convenient** method of finding the L.C.M. of several numbers, whose prime factors can be ascertained by ART. 130.

Ex. Find the L.C.M. of (1) 12, 22, 55, and 20 ; (2) 4, 6, 8, 12, 18, 90.

$$\begin{array}{r|l} 2 & 12,\ 22,\ 55,\ 20 \\ 2 & 6,\ 11,\ 55,\ 10 \\ \hline 5 & 3,\ 11,\ 55,\ 5 \\ 11 & 3,\ 11,\ 11,\ 1 \\ \hline & 3,\ 1,\ 1,\ 1 \end{array} \qquad \begin{array}{r|l} 2 & 4,\ 6,\ 8,\ 12,\ 18,\ 90 \\ \hline 2 & 4,\ 6,\ 45 \\ \hline 3 & 2,\ 3,\ 45 \\ \hline & 2,\ 15 \end{array}$$

∴ the L.C.M. $= 2 \times 2 \times 5 \times 11 \times 3 = \underline{660}$.

∴ the L.C.M. $= 2 \times 2 \times 3 \times 2 \times 15 = \underline{360}$.

N.B. The work can always be shortened by omitting, at any stage, the numbers which are exactly contained in the others.

Thus, in the second of the above Examples, 4 is contained in 8, 6 in 12, and 18 in 90; it is therefore sufficient to find the L.C.M. of 8, 12, and 90.

Examples 56.

Find the L.C.M. of:

1. 7, 14, 21. **2.** 7, 6, 9. **3.** 8, 16, 14. **4.** 18, 20, 30.
5. 8, 12, 10, 6. **6.** 21, 63, 105. **7.** 4, 18, 21, 20. **8.** 44, 48, 52, 92.
9. 13, 38, 39, 19, 57. **10.** 32, 35, 48, 40. **11.** 14, 16, 18, 20, 22.
12. 13, 17, 91, 51, 273. **13.** 56, 70, 180, 125. **14.** 15, 35, 55, 75, 165.
15. 16, 90, 91, 280, 455. **16.** 24, 64, 12, 80. **17.** 20, 27, 28, 35, 45, 63.
18. 12, 18, 28, 35, 60, 84, 100. **19.** 12, 20, 24, 54, 81, 63, 144.
20. 24, 35, 52, 60, 91, 108, 126. **21.** 10, 15, 21, 24, 35, 45, 63, 70.
22. 7, 11, 21, 35, 44, 63, 88, 117, 135, 150, 195.
23. 18, 24, 28, 35, 42, 48, 54, 56, 60, 70, 90, 120, 150, 160.
24. 16, 20, 24, 32, 36, 40, 48, 56, 60, 64, 72, 80, 96, 120, 180, 200
25. 24, 28, 36, 45, 48, 54, 56, 63, 70, 81, 100, 112, 140, 160, 240.

147. (1) If two numbers are prime to each other, their L.C.M. is equal to their product.

Thus, the L.C.M. of 15 and 16 is $15 \times 16 = 240$. *Ans.*

(2) The *least* number that will contain each of two or more given numbers exactly, is their L.C.M.

Ex. 1. Find the smallest number, which is divisible by 36, 54, 72.

The required number = the L.C.M. of 36, 54, 72 = 216. *Ans.*

Ex. 2. Five bells chime at intervals of 8, 10, 24, 30, 48 minutes respectively. If they chime together at noon, when will they all chime together again?

The L.C.M. of 8, 10, 24, 30, 48, is 240:

∴ the required time = 240 min. or 4 hrs. after noon, *i.e.*, 4 P. M. *Ans.*

148. To find the *least* number which being divided by each of several given numbers will leave the same remainder, we should add the given remainder to the L.C.M. of the numbers.

Ex. Find the least number which when divided by 16, 24, and 32, leaves in each case a remainder 7.

The L.C.M. of 16, 24, 32 = 96;

∴ the required number = 96 + 7 = 103. *Ans.*

149. The product of two numbers is equal to the product of their G.C.M. and L.C.M. (Art. 144). When therefore the G.C.M., the L.C.M., and *one* of the two numbers are given,

we can find the other by multiplying the G.C.M. and L.C.M. together and dividing the product by the given number.

Ex. The G.C.M. and the L.C.M. of two numbers are 18 and 216 respectively, and one of them is 54; what is the other?

The product of the G.C.M. and the L.C.M. $= 18 \times 216 = 3888$;

$\therefore$ the required number $= 3888 \div 54 = 72$. *Ans.*

Examples 57.

1. Find the least number which when divided by each of the numbers 8, 12, 15, 18, 20 and 24 leaves no remainder.

2. Three persons, who can walk once round a town in 8, 9 and 12 hours respectively, start together from a place to walk continually round it. How soon will they be together again at the place of starting?

3. *A*, *B* and *C* begin to walk at the rates of 21, 28 and 35 miles per day respectively. What is the nearest place where they can all stop, after walking a complete number of days?

4. A cask is required to be exactly filled by any one of the following measures; 1 pint, 2 pints, 3 pints, 5 pints, 6 pints, 9 pints: find the smallest cask that may be used for the purpose.

5. What is the smallest sum which can be paid either in 5-rupee notes, in 10-rupee notes, in 20-rupee notes or in 50-rupee notes?

6. There is an island 24 miles round. Three persons *A*, *B* and *C* begin to walk continually round it, starting from the same place at the same time. They walk 3, 4, 6 miles per hour respectively. How soon will they all start together from the same place for the second time?

7. Five bells begin to toll at intervals of 24, 32, 36, 48, 60 seconds respectively, all going at the same moment. After how many seconds will they again toll together?

8. *What is the smallest sum of money that can be paid either in* moidores, guineas, pounds, half-guineas, crowns, or half-crowns?

9. The circumference of the hindwheel and the forewheel of a carriage are 15 ft. 4 in. and 10 ft. 8 in., respectively: what is the shortest distance in which both the wheels will make complete revolutions?

10. A book is divided into four parts, each part being divided into chapters. The number of pages in each part is the same. Each chapter in the first part contains 40 pages, each chapter in the second 60, each chapter in the third 80, and each chapter in the fourth 100. Find the number of pages and chapters in the book, the number of pages in the book not exceeding 5000.

11. Three friends during a walk take steps of 2 ft. 4 in., 2 ft. 6 in., and 2 ft. 8 in.: if they start in step, how far will they have walked before they are in step again, and how many steps will each have taken?

12. Three planets are seen together in the same part of the heavens, and they revolve round the Sun once in 130, 910 and 975 days, respectively. How soon will they be together in the same place again?

13. Three boys start together to run round a circular park 960 yds. round. They run at the rates of 240, 320 and 480 yards a minute respectively. When will they be together again at the starting point, and how many times will each have run round the park ?

14. Three rollers are 24 ft. 8 in., 18 ft. 6 in. and 30 ft. 10 in. round, respectively ; find the shortest chain that can be wrapped round each of them an exact number of times.

15. What smallest number, when divided separately by 25, 30, 40 and 45, will always leave 21 as remainder ?

16. What is the least number which when increased by 13, is divisible separately by 24, 32, 96 and 120 ?

***17.** A company of sepoys arrange themselves in a line 5 deep, and after a time re-arrange themselves in a line 7 deep. Find the least number above 1000 which the company may consist of.

18. The G.C.M. and the L.C.M. of two numbers are 44 and 2772 respectively, and one of the numbers is 308 ; find the other.

19. What is the least number which when diminished by 57, is exactly divisible by 64, 80, 96, 112, and 128 ?

***20.** A distance not greater than a mile is measured by three rods, 6, 7, and 9 yards long respectively, and each time there are 5 yards over. Find the greatest such distance.

***21.** Two rods can both be marked off into equal divisions not more than 6 in. long, and can both measure a length of at least 6 ft. : if one of the rods be 6 in. longer than the other, find the length of each.

22. What is the least number which when divided by 15, 17, 23, 27 and 31 leaves the remainders 5, 7, 13, 17 and 21 respectively ?

23. What is the greatest number of 4 digits that is divisible by 5, 7, and 9 ?

24. Find the least number of 6 digits that is exactly divisible by 11, 13, 15 and 17.

25. What are the least and the greatest numbers of 6 digits which when divided by 5, 9, 12, 15, 25, 42 and 168 have the remainders 3, 7, 10, 13, 23, 40 and 166 respectively ?

CHAP. XVII. FRACTIONS.

150. A **whole number** or **integer** is a number composed solely of complete or entire units.

Note. The numbers treated of in the previous Chapters were all *whole numbers* or *integers*.

151. Since in practice a unit or any other quantity has to be divided into several equal parts, there must be some mode of expressing the relation which one or more such parts bear to the whole. If a yard, for instance, be divided into 5 equal

parts, one such part would be called a *fifth*, and denoted by $\frac{1}{5}$; two of them would be called *two-fifths* and denoted by $\frac{2}{5}$; similarly all the parts, *i. e.*, the entire unit, would be denoted by $\frac{5}{5}$. Again, if a second yard be divided into 5 equal parts, 6 of the 10 equal parts thus obtained would be called *six-fifths*, and denoted by $\frac{6}{5}$; and so on. Such numbers are called *broken numbers* or *fractions*. Hence

A **Fraction** is a number which being applied to a unit indicates a part or parts of the unit.

As will appear from the above, a faction is usually denoted by two numbers placed one above the other with a line drawn between them. The number of equal parts into which the unit is divided is placed below the line, and is called the **denominator**; the number of such parts taken is placed above the line, and is called the **numerator**. Thus, $\frac{7}{8}$ denotes that the unit is divided into 8 equal parts, and that 7 of these parts are taken; also 8 is the *denominator* and 7 the *numerator* of the *fraction* $\frac{7}{8}$.

The numerator and denominator are called the **terms** of a fraction.

152. When fractions are expressed in the manner explained above, they are called **Vulgar Fractions.**

N. B The fraction $\frac{1}{2}$ is called **one-half**; and $\frac{1}{3}$ is called **one-third.**

VULGAR FRACTIONS

153. Vulgar fractions are usually divided into (1) SIMPLE FRACTIONS. (2) MIXED NUMBERS, (3) COMPOUND FRACTIONS, and (4) COMPLEX FRACTIONS.

154. A **Simple Fraction** is one whose numerator and denominator are both integers. Simple fractions are of two kinds, **proper** and **improper.**

A **proper fraction** is one whose numerator is less than the denominator; as, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{7}$.

An **improper fraction** is one whose numerator is either equal to or greater than the denominator; as, $\frac{4}{4}$, $\frac{9}{8}$, $\frac{14}{7}$.

If we divide 5 units into 4 equal parts, each such part $=5\div4$. Again if we divide a unit into 4 equal parts, each of these parts $=\frac{1}{4}$. Now one of the former equal parts is obviously 5 times one of the latter, *i. e.*, is equal to 5 of the latter equal parts, *i. e.*, to $\frac{5}{4}$. Hence $5\div4=\frac{5}{4}$.

Thus, a fraction may be considered as the *quotient of the numerator by the denominator.*

Similarly $\frac{6}{3}=6\div3=2$; $\frac{8}{8}=8\div8=1$; &c.

155. If the numerator and denominator of a fraction be both *multiplied* or both *divided* by the same number, the value of the fraction will remain unaltered.

Let any unit be divided into 4 equal parts, and let each such part be again divided into 3 equal parts, so that the unit is divided first into 4, and then into 12 equal parts. By what has already been said, each of the larger parts in one-*fourth* and each of the smaller parts one-*twelfth* of the unit. Now each of the larger parts contains 3 of the smaller; therefore 2 of them will contain 6 of the smaller parts, 3 of them 9, and so on. Therefore, 1 *fourth* = 3 *twelfths*, or $\frac{1}{4}=\frac{3}{12}$; 2 *fourths* = 6 *twelfths*, or $\frac{2}{4}=\frac{6}{12}$; 3 *fourths* = 9 *twelfths*, or $\frac{3}{4}=\frac{9}{12}$ and so on.

Thus, $\frac{3}{4}=\frac{9}{12}=\frac{3\times3}{4\times3}$ and $\frac{9}{12}=\frac{3}{4}=\frac{9\div3}{12\div3}$.

Hence a fraction with any given denominator may be converted into another, having as its denominator either a *multiple* or a *part* of the given denominator.

Ex. Convert $\frac{7}{8}$ into a fraction with 8×15 or 120 for its denominator, and reduce $\frac{32}{48}$ to the form $\frac{2}{3}$.

$$\frac{32\div16}{48\div16}=\frac{2}{3}$$

156. A whole number may be converted into a fraction with any number as denominator.

Thus $6=6\div1=\frac{6}{1}=\frac{6\times8}{1\times8}=\frac{48}{8}$; &c.

Examples 58.

1. Convert each of the whole numbers 4, 5, 7, 9, 11 and 14 into a fraction with the denominator 12.

2. Reduce 12, 15, 18, 19 to fractions with denominators 10, 16, 20, and 25, respectively.

3. Express 19, as fractions with denominators 2, 3, 5, 6, 9 separately.

4. Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{10}$, $\frac{8}{15}$ into fractions having 60 for their denominator.

5. Express $\frac{21}{49}$, $\frac{27}{63}$, $\frac{30}{35}$, $\frac{36}{42}$, and $\frac{48}{56}$ each as a fraction with denominator 7.

6. Convert $\frac{25}{45}$, $\frac{36}{48}$, $\frac{44}{99}$, and $\frac{75}{625}$ into equivalent fractions with denominators 9, 4, 9, and 25, respectively.

157. A fraction is said to be in its **lowest terms** or **simplest form** when its denominator is the *least possible*. This will evidently be the case when its numerator and denominator are prime to each other, *i. e.*, have no common factor.

To **reduce** a fraction to its lowest terms, or to **simplify** it, we should divide both its numerator and denominator by their G. C. M.

Ex. Reduce $\frac{125}{425}$ to its lowest terms.

The G. C. M. of 125 and 425 is 25.

$\therefore \frac{125}{425} = \frac{125 \div 25}{425 \div 25} = \frac{5}{17}$, which is the form required.

158. In simplifying a fraction, it is practically more convenient to split up both the numerator and the denominator into factors, and *cancel* those which are common to both. These common factors can be found either by inspection, or by employing the tests of divisibility given in ART. 130.

Ex. Reduce $\frac{495}{1210}$ and $\frac{21 \times 57}{70 \times 95}$ to their lowest terms.

$$\frac{495}{1210} = \frac{\cancel{5} \times \cancel{11} \times 9}{\cancel{5} \times \cancel{11} \times 12} = \frac{9}{22}. \quad \textit{Ans.}$$

$$\frac{21 \times 57}{70 \times 95} = \frac{3 \times \cancel{7} \times 3 \times \cancel{19}}{10 \times \cancel{7} \times 5 \times \cancel{19}} = \frac{3 \times 3}{10 \times 5} = \frac{9}{50}. \quad \textit{Ans.}$$

N. B. It should be noticed that when a factor is *cancelled*, it is replaced by 1 and **not by 0.**

Examples 59.

1. Reduce to their lowest terms (by inspection) :

(1) $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{6}{8}, \frac{2}{10}, \frac{5}{10}, \frac{3}{12}, \frac{4}{12}, \frac{8}{12}, \frac{2}{12}.$

(2) $\frac{7}{14}, \frac{5}{15}, \frac{9}{15}, \frac{4}{16}, \frac{12}{16}, \frac{12}{18}, \frac{56}{96}, \frac{21}{28}, \frac{9}{30}, \frac{24}{36}, \frac{33}{44}.$

(3) $\frac{24}{40}, \frac{14}{50}, \frac{39}{52}, \frac{44}{132}, \frac{15}{135}, \frac{38}{57}, \frac{36}{63}, \frac{42}{63}, \frac{26}{65}, \frac{25}{100}.$

(4) $\frac{57}{171}, \frac{144}{288}, \frac{65}{91}, \frac{117}{130}, \frac{52}{91}, \frac{48}{128}, \frac{76}{133}, \frac{144}{156}, \frac{132}{143}.$

2. Reduce each of the following to its lowest terms :

(1) $\frac{500}{3125}, \frac{132}{4158}, \frac{189}{729}, \frac{315}{924}, \frac{714}{918}, \frac{791}{1071}, \frac{825}{2709}, \frac{630}{936}, \frac{1125}{1350}.$

(2) $\frac{729}{810}; \frac{1763}{1414}; \frac{2021}{5035}; \frac{1260}{1647}; \frac{1210}{7788}; \frac{3913}{7239}; \frac{7016}{15508}; \frac{4484}{5605}.$

(3) $\frac{1428}{1530}; \frac{3100}{10925}; \frac{1144}{9900}; \frac{1008}{2520}; \frac{7845}{96780}; \frac{1395}{1488}; \frac{7125}{17250}; \frac{8880}{14430}.$

(4) $\frac{5184}{6912}$; $\frac{3444}{3556}$; $\frac{6339}{12963}$; $\frac{9110}{9910}$; $\frac{3366}{9680}$; $\frac{6816}{10656}$; $\frac{7497}{15729}$; $\frac{5490}{36105}$.

(5) $\frac{10265}{14371}$; $\frac{4301}{95887}$; $\frac{55247}{74841}$; $\frac{10812}{22800}$; $\frac{19968}{20480}$; $\frac{32436}{68400}$; $\frac{890274}{1213641}$.

(6) $\frac{12483}{13653}$; $\frac{13690}{99900}$; $\frac{114135}{220661}$; $\frac{128352}{238368}$; $\frac{1854432}{3171276}$; $\frac{542938}{1999998}$.

8. Reduce to their simplest forms :

(1) $\frac{15\times 72}{35\times 96}$; $\frac{18\times 21}{42\times 54}$; $\frac{12\times 13\times 16}{90\times 65\times 42}$; $\frac{10\times 14\times 18\times 24}{27\times 28\times 60}$.

(2) $\frac{11\times 13\times 144}{33\times 52\times 12}$; $\frac{17\times 21\times 36}{51\times 70\times 64}$; $\frac{12\times 12\times 18\times 10}{27\times 28\times 30\times 24}$; $\frac{25\times 64\times 49}{75\times 80\times 56}$.

(3) $\frac{33\times 60\times 19}{44\times 80\times 57}$; $\frac{15\times 38\times 36}{135\times 57\times 63}$; $\frac{36\times 18\times 42}{63\times 72\times 63}$; $\frac{26\times 25\times 75}{65\times 100\times 150}$.

159. **A mixed number** is one which is composed of a whole number and a fraction. The whole number and the fraction are respectively called the *integral* and *fractional* parts.

Thus $2+\frac{3}{4}$, *which is usually written* $2\frac{3}{4}$ is a *mixed number*, 2 being the *integral*, and $\frac{3}{4}$ the *fractional* part

Again, $2\frac{3}{4}=2+\frac{3}{4}=\frac{8}{4}+\frac{3}{4}=8$ *fourths* + 3 *fourths* = 11 *fourths* $=\frac{11}{4}$.

Now, if 11 be divided by 4, the quotient is 2 and the remainder 3; hence the following RULES are obtained.

160. (1) *To express an improper fraction as a whole or mixed number* :

RULE. Divide the numerator by the denominator : then the quotient is the integral part ; and the remainder, if any, is the *numerator*, while the denominator of the given fraction is the *denominator*, of the fractional part.

Ex. Express, $\frac{77}{7}$ and $\frac{49}{8}$ as whole or mixed numbers.

$$7)\underline{77} \qquad\qquad 8)\underline{49}$$
$$11 \qquad\qquad 6-1.$$

$\therefore \frac{77}{7}=11$. *Ans.* $\qquad \therefore \frac{49}{8}=6\frac{1}{8}$. *Ans.*

(2) *To reduce a mixed number to an improper fraction* :

RULE. Multiply the integral part by the denominator of the fractional part, and to the product add the numerator ; the sum so obtained will be the numerator of the required fraction, the denominator being the same as that of the fractional part.

Ex. Convert $5\frac{2}{7}$ into an improper fraction.

$$5\tfrac{2}{7}=\frac{5\times 7+2}{7}=\frac{37}{7}.\quad \textit{Ans.}$$

161. It will be seen from the above that we can now find the **complete quotient** of one number divided by another.

Thus, $49 \div 8 = \frac{49}{8} = 6\frac{1}{8}$; hence the *complete quotient* of 49 by 8 is the mixed number $6\frac{1}{8}$.

Examples 60.

1. Express *mentally* the following as mixed or whole numbers:

(1) $\frac{3}{2}$; $\frac{4}{3}$; $\frac{4}{2}$; $\frac{8}{6}$; $\frac{10}{3}$; $\frac{11}{5}$; $\frac{17}{5}$; $\frac{22}{7}$; $\frac{27}{8}$; $\frac{25}{2}$; $\frac{16}{8}$; $\frac{12}{7}$.

(2) $\frac{108}{13}$; $\frac{110}{17}$; $\frac{115}{16}$; $\frac{129}{16}$; $\frac{144}{16}$; $\frac{148}{12}$; $\frac{161}{10}$; $\frac{170}{10}$; $\frac{101}{20}$.

2. Reduce to the form of whole or mixed numbers:

(1) $\frac{1197}{17}$; $\frac{1521}{25}$; $\frac{549}{29}$; $\frac{561}{34}$; $\frac{365}{30}$; $\frac{105}{21}$; $\frac{8201}{365}$; $\frac{4271}{261}$; $\frac{6031}{73}$.

(2) $\frac{94125}{71}$; $\frac{8764321}{992}$; $\frac{1703121}{1517}$; $\frac{907133}{7816}$; $\frac{1235321}{1111}$; $\frac{1234567}{305}$.

3. Reduce *mentally* to improper fractions.

(1) $1\frac{1}{2}$; $1\frac{1}{3}$; $2\frac{1}{3}$; $3\frac{2}{7}$; $3\frac{4}{9}$; $5\frac{3}{8}$; $6\frac{4}{9}$; $7\frac{9}{10}$; $8\frac{9}{10}$.

(2) $7\frac{7}{12}$; $17\frac{2}{5}$; $19\frac{5}{6}$; $6\frac{7}{15}$; $14\frac{3}{5}$; $12\frac{11}{13}$; $17\frac{2}{7}$; $19\frac{4}{7}$.

(3) $20\frac{11}{12}$; $7\frac{3}{20}$; $9\frac{7}{20}$; $13\frac{7}{17}$; $15\frac{5}{16}$; $19\frac{9}{16}$; $18\frac{11}{15}$.

4. Convert into improper fractions.

(1) $171\frac{4}{15}$; $95\frac{11}{21}$; $12\frac{14}{26}$; $49\frac{7}{35}$; $85\frac{17}{62}$; $17\frac{17}{112}$.

(2) $371\frac{31}{211}$; $462\frac{57}{307}$; $191\frac{1}{71}$; $806\frac{109}{807}$; $206\frac{99}{97}$.

(3) $9999\frac{47}{99}$; $9999\frac{91}{999}$; $9999\frac{925}{9999}$; $99999\frac{4762}{99999}$.

162. **A Compound Fraction** is a fraction of a fraction.

Thus, $\frac{3}{5}$ of $\frac{2}{7}$, $\frac{4}{7}$ of $\frac{14}{25}$ of $\frac{24}{35}$ are *Compound Fractions*.

(i) $4 \times \frac{3}{8} = 4 \times 3$ *eighths* $= 12$ *eighths* $= \frac{12}{8} = \frac{4 \times 3}{8} = \frac{3}{2} = \frac{3}{8 \div 4}$.

The product of a simple fraction and a whole number is therefore obtained either by *multiplying the numerator*, or by *dividing the denominator*, of the fraction by the whole number.

(ii) $\frac{6}{7} \div 2 = 6$ *sevenths* $\div 2 = 3$ *sevenths* $= \frac{3}{7} = \frac{6 \div 2}{7}$;

also $\frac{6}{7} \div 2 = 12$ *fourteenths* $\div 2 = 6$ *fourteenths* $= \frac{6}{14} = \frac{6}{7 \times 2}$.

In dividing a simple fraction by a whole number we may therefore either *divide the numerator*, or *multiply the denominator*, of the given fraction by the whole number.

(iii) $\frac{4}{7}$ of $\frac{3}{5} = 4 \times \frac{1}{7}$ of $\frac{3}{5} = 4 \times \frac{3}{7 \times 5} = \frac{4 \times 3}{7 \times 5}$.

The following RULE is thus obtained.

163. *To reduce a compound fraction to a simple fraction.*

RULE. Multiply the numerators for the new numerator and the denominators for the new denominator; and reduce the resulting fraction to its lowest terms by cancelling the factors that are common, as in *Ex.* 2.

Ex. 1. Reduce $\frac{3}{4}$ of $\frac{5}{7}$ to its simplest form.

$$\frac{3}{4} \text{ of } \frac{5}{7} = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}. \quad \textit{Ans.}$$

Ex. 2. Reduce $\frac{3}{4}$ of $\frac{8}{15}$ to its simplest form.

$$\frac{3}{4} \text{ of } \frac{8}{15} = \frac{3 \times 8}{4 \times 15} = \frac{\cancel{3} \times 2 \times \cancel{4}}{\cancel{4} \times \cancel{3} \times 5} = \frac{2}{5}. \quad \textit{Ans.}$$

Examples 61.

Express as simple fractions:

1. $\frac{1}{2}$ of $\frac{2}{3}$; $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{1}{4}$ of $\frac{2}{3}$; $\frac{2}{3}$ of $\frac{6}{9}$; $\frac{11}{12}$ of $\frac{3}{11}$; $\frac{20}{51}$ of $\frac{34}{45}$

2. $\frac{2}{5}$ of $\frac{5}{12}$ of $\frac{6}{11}$; $\frac{9}{13}$ of $\frac{26}{3}$ of $\frac{5}{6}$; $\frac{11}{12}$ of $\frac{6}{7}$ of $\frac{5}{9}$; $\frac{7}{8}$ of $\frac{5}{6}$ of $\frac{24}{49}$.

3. $\frac{5}{6}$ of $\frac{3}{4}$ of $\frac{8}{3}$; $\frac{6}{11}$ of $\frac{110}{126}$ of $\frac{63}{80}$; $\frac{120}{11}$ of $\frac{481}{4}$ of $\frac{11}{40}$ of $\frac{9}{5}$.

4. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{15}$ of $\frac{25}{36}$. 5. $\frac{3}{2}$ of $\frac{5}{9}$ of $\frac{21}{9}$ of $\frac{36}{8}$.

6. $\frac{8}{9}$ of $\frac{25}{2}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{3}{8}$ of 9 7. $\frac{4}{7}$ of $\frac{3}{4}$ of $\frac{64}{7}$ of $\frac{7}{32}$ of $\frac{6}{7}$ of 7.

8. $\frac{21}{4}$ of $\frac{18}{19}$ of $\frac{19}{33}$ of $\frac{1}{6}$ of $\frac{11}{12}$ of $\frac{5}{3}$ of $\frac{31}{6}$ of $\frac{2}{5}$ of $\frac{17}{31}$ of $\frac{3}{2}$.

9. $\frac{1}{3}$ of $\frac{1}{19}$ of $\frac{38}{3}$ of $\frac{7}{13}$ of $\frac{91}{3}$ of $\frac{6}{7}$ of $\frac{1}{14}$ of $\frac{13}{51}$ of $\frac{17}{18}$ of 65.

10. $5\frac{5}{8}$ of $5\frac{1}{3}$ of $1\frac{3}{31}$ of $1\frac{8}{11}$ 11. $4\frac{2}{3}$ of $1\frac{1}{7}$ of $\frac{1}{6}$ of $1\frac{3}{10}$ of $1\frac{5}{8}$.

12. $13\frac{1}{7}$ of $5\frac{1}{4}$ of $4\frac{1}{2}$ of $2\frac{1}{520}$ of $1\frac{1}{5}$. 13. $25\frac{1}{4}$ of $17\frac{1}{3}$ of $3\frac{1}{4}$ of $1\frac{3}{103}$ of $1\frac{2}{5}$.

14. $90\frac{1}{6}$ of $2\frac{1}{2}$ of $7\frac{1}{4}$ of $\frac{3}{481}$ of $\frac{13}{32}$ of $2\frac{5}{23}$ of $7\frac{1}{2}$ of $3\frac{9}{31}$.

15. $4\frac{3}{8}$ of $\frac{4}{7}$ of $\frac{7}{8}$ of $\frac{3}{8}$ of $3\frac{3}{5}$ of $2\frac{1}{4}$ of $\frac{2}{7}$ of $70\frac{2}{3}$ of $\frac{4}{30}$ of $1\frac{5}{8}$.

164. Fractions with different denominators can be expressed as equivalent fractions with the same denominator.

Thus, if we take the fractions $\frac{5}{6}$ and $\frac{4}{15}$ it is evident that the denominators 6 and 15 will both be contained in their L.C.M. 30; also $30 \div 6 = 5$, and $30 \div 15 = 2$. Hence

$$\frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30}, \text{ and } \frac{4}{15} = \frac{4 \times 2}{15 \times 2} = \frac{8}{30}.$$

Thus the following RULE is obtained.

165. *To convert different given fractions to equivalent fractions with a least common denominator.*

Rule. Find the L.C.M. of the denominators; this L.C.M. will be the common denominator. Divide this L.C.M. by the denominator of each fraction successively, and multiply the quotients by the respective numerators of the fractions.

Ex. Reduce $\frac{2}{7}$, $\frac{5}{9}$ and $\frac{8}{21}$ to equivalent fractions having a least common denominator.

The L.C.M. of 7, 9, and 21 = 63.

$63 \div 7 = 9$; $63 \div 9 = 7$; $63 \div 21 = 3$.

$$\therefore \frac{2}{7} = \frac{2 \times 9}{7 \times 9} = \frac{18}{63}; \quad \frac{5}{9} = \frac{5 \times 7}{9 \times 7} = \frac{35}{63}; \quad \frac{8}{21} = \frac{8 \times 3}{21 \times 3} = \frac{24}{63}.$$

Hence the given fractions are $\frac{18}{63}$, $\frac{35}{63}$, $\frac{24}{63}$, respectively. *Ans.*

Examples 62.

Reduce each set of the following fractions to equivalent fractions having the same least common denominator:

1. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$. 2. $\frac{1}{2}$; $\frac{2}{3}$; $\frac{4}{5}$. 3. $\frac{3}{10}$; $\frac{7}{20}$; $\frac{11}{30}$. 4. $\frac{5}{6}$; $\frac{11}{12}$; $\frac{18}{45}$

5. $\frac{1}{11}$; $\frac{2}{33}$; $\frac{4}{11}$; $\frac{7}{22}$. 6. $\frac{5}{6}$; $\frac{4}{9}$; $\frac{7}{18}$; $\frac{11}{12}$. 7. $\frac{2}{3}$; $\frac{3}{15}$; $\frac{9}{10}$; $\frac{2}{5}$.

8. $\frac{5}{6}$; $\frac{7}{12}$; $\frac{11}{18}$; $\frac{3}{8}$ 9. $\frac{5}{9}$; $\frac{7}{18}$; $\frac{23}{54}$; $\frac{31}{72}$. 10. $\frac{1}{6}$; $\frac{7}{8}$; $\frac{8}{9}$; $\frac{17}{24}$.

11. $\frac{19}{20}$; $\frac{3}{10}$; $\frac{113}{180}$; $\frac{5}{9}$; $\frac{2}{3}$. 12. $\frac{13}{16}$; $\frac{11}{17}$; $\frac{17}{18}$; $\frac{25}{36}$; $\frac{19}{51}$.

13. $\frac{3}{8}$; $\frac{7}{9}$; $\frac{9}{11}$; $\frac{17}{18}$; $\frac{29}{33}$. 14. $\frac{15}{4}$ of $\frac{8}{15}$; $\frac{21}{4}$ of $\frac{5}{7}$; $\frac{5}{16}$ of $\frac{7}{10}$.

15. $\frac{31}{60}$; $\frac{47}{90}$; $\frac{13}{25}$ of $\frac{5}{26}$; $\frac{5}{9}$ of $\frac{3}{10}$. 16. $\frac{13}{44}$; $\frac{110}{121}$; $\frac{19}{132}$; $\frac{22}{110}$; $\frac{27}{33}$.

166. *To compare the magnitudes of different fractions.*

(i) If the denominators of the fractions are the same it is obvious that the fraction with the greatest numerator is the greatest, and that with the least numerator the least. Hence,

Rule. Reduce them to fractions having the least common denominator, and compare the numerators so obtained.

Ex. 1. Find the *greatest* and *least* of the fractions $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$.

The L. C. M. of the denominators is 60.

$60 \div 4 = 15$, $60 \div 5 = 12$, $60 \div 6 = 10$.

$$\therefore \frac{3}{4} = \frac{3 \times 15}{4 \times 15} = \frac{45}{60}; \quad \frac{4}{5} = \frac{4 \times 12}{5 \times 12} = \frac{48}{60}; \quad \frac{5}{6} = \frac{5 \times 10}{6 \times 10} = \frac{50}{60}.$$

$\therefore$ $\frac{5}{6}$ is the greatest and $\frac{3}{4}$ is the *least*. *Ans.*

Ex. 2. Arrange $\frac{5}{8}$, $\frac{7}{12}$, $\frac{9}{16}$, and $\frac{13}{18}$ in order of magnitude.

The L.C.M. of the denominators $= 144$.

$$\frac{5}{8}=\frac{5\times18}{8\times18}=\frac{90}{144};\quad \frac{7}{12}=\frac{7\times12}{12\times12}=\frac{84}{144};$$

$$\frac{9}{16}=\frac{9\times9}{16\times9}=\frac{81}{144};\quad \frac{13}{18}=\frac{13\times8}{18\times8}=\frac{104}{144}.$$

They should therefore be arranged thus: $\frac{13}{18}$, $\frac{5}{8}$, $\frac{7}{12}$, $\frac{9}{16}$.

(ii) It is also obvious that of fractions having the same numerator, the *greatest* is that whose denominator is the least and the *least* is that whose denominator is the greatest.

Ex. Find the greatest and least of the fractions $\frac{4}{7}$, $\frac{5}{24}$, $\frac{15}{28}$, and $\frac{8}{15}$.

The L.C.M. of the numerators $= 120$.

$$\therefore\ \frac{4}{7}=\frac{4\times30}{7\times30}=\frac{120}{210};\quad \frac{5}{24}=\frac{5\times24}{24\times24}=\frac{120}{576};$$

$$\frac{15}{28}=\frac{15\times8}{28\times8}=\frac{120}{224};\quad \frac{8}{15}=\frac{8\times15}{15\times15}=\frac{120}{225}.$$

Hence $\frac{4}{7}$ is the *greatest* and $\frac{5}{24}$ is the *least*. *Ans.*

Examples 63.

1. Which is the greater? (by the first method.)

$\frac{3}{4}$ or $\frac{4}{5}$; $\frac{5}{6}$ or $\frac{6}{7}$; $\frac{22}{25}$ or $\frac{27}{30}$; $\frac{3}{4}$ or $\frac{37}{42}$; $\frac{17}{30}$ or $\frac{22}{25}$; $\frac{8}{9}$ or $\frac{27}{30}$.

2. Which is less? (by the second method.)

$\frac{8}{25}$ or $\frac{9}{28}$; $\frac{15}{23}$ or $\frac{20}{33}$; $\frac{1}{3}$ or $\frac{5}{18}$; $\frac{10}{19}$ of $\frac{4}{5}$ of $\frac{2}{3}$ or $\frac{7}{5}$ of $\frac{2}{7}$.

3. Which is the greatest, and which the least, of the following?

(1) $\frac{3}{40}$, $\frac{7}{72}$, $\frac{1}{7}$; $\frac{9}{13}$, $\frac{7}{11}$, $\frac{9}{10}$; $\frac{11}{15}$, $\frac{17}{20}$, $\frac{19}{25}$.

(2) $\frac{17}{24}$, $\frac{19}{30}$, $\frac{23}{36}$, $\frac{28}{48}$; $\frac{73}{60}$, $\frac{11}{15}$, $\frac{13}{24}$, $\frac{31}{40}$; $\frac{16}{90}$, $\frac{7}{5}$, $\frac{37}{35}$, $\frac{13}{20}$

(3) $\frac{44}{51}$, $\frac{31}{34}$, $\frac{67}{68}$, $\frac{15}{17}$; $\frac{8}{9}$, $\frac{17}{18}$, $\frac{53}{54}$, $\frac{67}{72}$; $\frac{14}{33}$, $\frac{23}{44}$, $\frac{47}{55}$, $\frac{61}{66}$.

(4) $\frac{2}{3}$ of $\frac{4}{5}$, $\frac{6}{7}$ of $\frac{9}{10}$, $\frac{2}{5}$ of $\frac{3}{4}$; $\frac{5}{6}$ of $\frac{11}{12}$, $\frac{7}{8}$ of $\frac{5}{6}$, $\frac{5}{8}$ of $\frac{2}{3}$.

(5) $\frac{3}{7}$ of $\frac{2}{3}$, $\frac{5}{6}$ of $\frac{6}{7}$, $\frac{11}{14}$ of $\frac{37}{44}$; $\frac{2}{3}$ of $\frac{5}{6}$, $\frac{7}{8}$ of $\frac{3}{4}$, $\frac{8}{91}$ of $\frac{3}{4}$, $\frac{12}{25}$ of $\frac{10}{-}$.

4. Arrange in order of magnitude:

(1) $\frac{3}{5}$, $\frac{7}{12}$, $\frac{8}{15}$, $\frac{9}{20}$. (2) $\frac{53}{75}$, $\frac{19}{28}$, $\frac{23}{40}$, $\frac{53}{60}$. (3) $\frac{7}{27}$, $\frac{13}{45}$, $\frac{17}{64}$, $\frac{4}{15}$.

(4) $\frac{7}{20}$, $\frac{5}{16}$, $\frac{13}{40}$, $\frac{9}{20}$. (5) $\frac{7}{24}$, $\frac{18}{65}$, $\frac{27}{77}$, $\frac{35}{88}$. (6) $\frac{5}{24}$, $\frac{7}{36}$, $\frac{23}{84}$, $\frac{25}{88}$.

(7) $\frac{5}{6}$ of $\frac{3}{37}$, $\frac{18}{25}$ of $\frac{29}{36}$, $\frac{7}{9}$ of $\frac{18}{28}$, $5\frac{1}{6}$ of $\frac{9}{56}$.

(8) $\frac{4}{35}$ of $\frac{14}{75}$, $\frac{28}{25}$ of $\frac{5}{44}$, $\frac{5}{18}$ of $\frac{3}{7}$ of $\frac{38}{60}$, $\frac{21}{40}$ of $1\frac{7}{35}$ of $\frac{32}{35}$.

ADDITION OF FRACTIONS.

167. When two or more fractions have the same denominator, their **sum** is found by adding the numerators together to form the *new* numerator, the denominator remaining *the same*.

Thus, 2 *ninths* + 5 *ninths* = 7 *ninths*, or $\frac{2}{9}+\frac{5}{9}=\frac{2+5}{9}=\frac{7}{9}$.

168. If the fractions have not the same denominator, their sum is obtained by reducing the fractions to equal ones with their least common denominator, and then adding the numerator to form the *new* numerator, while the common denominator is the *new* denominator. The sum should then be reduced to its lowest terms, and, if an improper fraction, to a mixed number.

Ex. Add together $\frac{4}{7}$, $\frac{8}{21}$, $\frac{3}{14}$ and $\frac{1}{2}$.

The L. C. M. of the denominators is 42.

$$\frac{4}{7}=\frac{4\times6}{7\times6}=\frac{24}{42};\quad \frac{8}{21}=\frac{8\times2}{21\times2}=\frac{16}{42};$$

$$\frac{3}{14}=\frac{3\times3}{14\times3}=\frac{9}{42};\quad \frac{1}{2}=\frac{1\times21}{2\times21}=\frac{21}{42}.$$

$$\therefore \text{ the sum}=\frac{24}{42}+\frac{16}{42}+\frac{9}{42}+\frac{21}{42}=\frac{24+16+9+21}{42}$$

$$=\frac{70}{42}=\frac{5\times\cancel{14}}{3\times\cancel{14}}=\frac{5}{3}=1\tfrac{2}{3}.\quad \textit{Ans.}$$

169. When the given fractions are not in their lowest terms, they should be first reduced to their lowest terms.

Ex. Add together $\frac{15}{18}$, $\frac{25}{30}$, $\frac{44}{66}$, and $\frac{32}{36}$.

$$\frac{15}{18}+\frac{25}{30}+\frac{44}{66}+\frac{32}{36}=\frac{5}{6}+\frac{5}{6}+\frac{2}{3}+\frac{8}{9}=\frac{15+15+12+16}{18}$$

$$=\tfrac{58}{18}=\tfrac{29}{9}=3\tfrac{2}{9}.\quad \textit{Ans.}$$

170. When some of the fractions are mixed and some improper, it will be convenient first to reduce the improper fractions to mixed numbers, and then to add together the integral and the fractional parts separately.

Ex. Add together $2\frac{7}{8}+\frac{105}{16}+3\frac{128}{144}+\frac{5}{27}$ of $\frac{3}{40}$.

$$2\tfrac{7}{8}+\tfrac{105}{16}+3\tfrac{128}{144}+\tfrac{5}{27}\text{ of }\tfrac{3}{40}=2\tfrac{7}{8}+6\tfrac{9}{16}+3\tfrac{8}{9}+\tfrac{1}{72}$$

$$=(2+6+3)+\tfrac{7}{8}+\tfrac{9}{16}+\tfrac{8}{9}+\tfrac{1}{72}$$

$$=11+\frac{126+81+128+2}{144}$$

$$=11+\tfrac{337}{144}=11+2\tfrac{49}{144}=13\tfrac{49}{144}.\quad \textit{Ans.}$$

Examples 64.

1. Add *mentally* :

(1) $\frac{1}{3}+\frac{2}{3}$; $\frac{1}{4}+\frac{3}{4}$; $\frac{2}{5}+\frac{3}{5}$ $\frac{5}{6}+\frac{1}{6}$; $\frac{1}{5}+\frac{3}{5}+\frac{4}{5}$; $\frac{3}{7}+\frac{2}{7}+\frac{3}{7}$.

(2) $\frac{2}{5}+\frac{3}{5}+\frac{4}{5}$; $\frac{5}{9}+\frac{7}{9}+\frac{8}{9}$; $\frac{7}{10}+\frac{9}{10}+\frac{3}{10}$; $\frac{17}{25}+\frac{6}{25}+\frac{2}{25}$.

(3) $\frac{1}{2}+\frac{1}{4}$; $\frac{1}{2}+\frac{1}{3}$; $\frac{1}{2}+\frac{1}{6}$; $\frac{1}{3}+\frac{1}{4}$; $\frac{1}{3}+\frac{1}{5}$; $\frac{1}{3}+\frac{1}{6}$.

(4) $\frac{3}{4}+\frac{1}{2}+\frac{1}{6}$; $\frac{3}{4}+\frac{2}{3}+\frac{5}{6}$; $\frac{3}{4}+\frac{1}{4}+\frac{5}{12}$; $\frac{2}{5}+\frac{3}{10}+\frac{9}{20}$.

(5) $6\frac{2}{3}+7\frac{1}{3}$; $2\frac{1}{4}+4\frac{3}{4}+2\frac{1}{2}$; $4\frac{5}{6}+3\frac{1}{3}+2\frac{1}{2}$.

2. Perform the following additions :

(1) $\frac{5}{6}+\frac{3}{8}+\frac{7}{12}$; $\frac{6}{17}+\frac{7}{51}+\frac{11}{68}$; $\frac{4}{7}+\frac{7}{8}+\frac{7}{9}$; $\frac{3}{4}+\frac{61}{70}+\frac{9}{81}$.

(2) $\frac{6}{17}+\frac{3}{7}+\frac{23}{35}$; $\frac{3}{8}+\frac{31}{36}+\frac{11}{16}$; $\frac{9}{8}+\frac{17}{32}+\frac{3}{4}$; $\frac{5}{6}+\frac{5}{9}+\frac{10}{57}$.

(3) $\frac{1}{2}+\frac{2}{3}+\frac{5}{6}+\frac{5}{9}+\frac{2}{3}$.

(4) $\frac{30}{5}+\frac{7}{6}+\frac{25}{3}+\frac{5}{2}+\frac{7}{15}$.

(5) $\frac{29}{18}+\frac{29}{30}+\frac{23}{15}+\frac{7}{2}+\frac{5}{3}$.

(6) $\frac{11}{20}+\frac{33}{24}+\frac{19}{27}+\frac{2}{9}+\frac{10}{24}$.

(7) $\frac{5}{6}+\frac{46}{36}+\frac{5}{8}+\frac{17}{33}+\frac{7}{24}$.

(8) $\frac{9}{77}+\frac{39}{7}+\frac{7}{44}+\frac{5}{9}+\frac{19}{28}$.

(9) $\frac{19}{7}+\frac{15}{18}+\frac{37}{14}+\frac{81}{84}$.

(10) $\frac{7}{30}+\frac{23}{15}+\frac{48}{45}+\frac{37}{60}$.

(11) $\frac{9}{4}+\frac{8}{5}+\frac{8}{3}+\frac{21}{4}$.

(12) $\frac{42}{5}+\frac{28}{40}+\frac{78}{70}+\frac{97}{80}$.

(13) $\frac{402}{9}+\frac{473}{44}+\frac{555}{88}+\frac{17}{2}$

(14) $\frac{403}{17}+\frac{387}{51}+\frac{4002}{355}+\frac{34}{45}$.

(15) $\frac{59}{60}+\frac{13}{30}+\frac{37}{81}+\frac{44}{60}+\frac{68}{85}$.

(16) $10\frac{7}{12}+5\frac{9}{20}+7\frac{8}{15}+9\frac{5}{20}$.

(17) $9\frac{9}{16}+\frac{18}{57}+14\frac{71}{96}+27\frac{10}{19}$.

(18) $7\frac{3}{4}+2\frac{1}{3}+5\frac{1}{4}+\frac{10}{6}+\frac{4}{3}$.

(19) $19\frac{2}{3}+9+10\frac{2}{3}+12\frac{5}{6}+9\frac{1}{4}$.

(20) $\frac{1}{4}+21\frac{1}{2}+\frac{7}{8}+13\frac{4}{5}$.

(21) $3\frac{7}{8}+17+4\frac{1}{2}+7\frac{1}{3}+12\frac{7}{15}$.

(22) $27\frac{5}{6}+37\frac{2}{3}+263\frac{1}{6}+101\frac{2}{5}$.

(23) $6\frac{3}{4}$ of $\frac{4}{9}$ of $5+\frac{1}{4}$ of [illegible] $+\frac{7}{8}$ of $5\frac{3}{8}$ of $7+1\frac{3}{8}$ of $\frac{9}{10}+9\frac{4}{25}$.

(24) $6\frac{3}{4}$ of $\frac{1}{3}+\frac{8}{17}$ of $\frac{6}{7}$ of $8\frac{1}{2}+1$ of $\frac{5}{2}+\frac{7}{34}$ of $\frac{1}{5}+\frac{2}{5}$ of $\frac{5}{6}$.

(25) $\frac{2}{3}+\frac{1}{3}$ of $(\frac{4}{7}+\frac{3}{7})$ of $\frac{7}{40}$ [illegible] $\frac{93}{27}$ of $(2\frac{1}{3}+7\frac{1}{3})+\frac{5}{6}$ of $\frac{12}{35}$.

SUBTRACTION OF FRACTIONS.

171. The **Subtraction of Fractions** is effected in the same way as their Addition.

Ex. Subtract $\frac{3}{8}$ from $\frac{11}{12}$.

The L. C. M. of 8 and 12 is 24.

$$\therefore \frac{11}{12}-\frac{3}{8}=\frac{22-9}{24}=\frac{13}{24}. \quad \textit{Ans.}$$

172. The operation of subtraction can be sometimes considerably simplified, as in the following Examples.

Ex. 1. $\frac{7}{8}-\frac{7}{15}=\frac{(15-8)7}{8\times 15}=\frac{49}{120}$. *Ans.*

If both the fractions have a common numerator, multiply the difference of the denominators by the common numerator for its *numerator*, and multiply the denominators together for its *denominator*.

Ex. 2. $$1-\frac{11}{17}=\frac{17-11}{17}=\frac{6}{17}.$$

In subtracting a proper fraction from unity, subtract the numerator from the denominator for the *numerator*.

Ex. 3. $5-2\frac{4}{7}=(4+1)-(2+\frac{4}{7})=(4-2)+(1-\frac{4}{7})=2\frac{3}{7}$.

In subtracting a mixed number from a whole number, subtract the fractional part from unity as in *Ex.* 2, and the integral from the whole number diminished by unity.

Ex. 4. (*a*) $7\frac{2}{3}-3\frac{1}{6}=(7-3)+(\frac{2}{3}-\frac{1}{6})=4\frac{1}{2}$;

(*b*) $7\frac{3}{8}-4\frac{5}{8}=7-4\frac{5}{8}+\frac{3}{8}=2\frac{3}{8}+\frac{3}{8}=2\frac{6}{8}=2\frac{3}{4}$.

The first method should be followed when the fractional part of the subtrahend is less than that of the minuend; the second method, when it is greater.

Note. When an operation of subtraction is to be performed, the fractions must be first reduced to their lowest terms.

Examples 65.

1. Subtract *mentally* :

(1) $\frac{8}{9}-\frac{5}{9}$; $\frac{5}{6}-\frac{5}{7}$; $\frac{1}{2}-\frac{1}{3}$; $\frac{1}{2}-\frac{1}{4}$; $\frac{3}{4}-\frac{1}{2}$; $\frac{3}{4}-\frac{2}{3}$; $\frac{1}{2}-\frac{2}{5}$; $\frac{1}{2}-\frac{3}{8}$.

(2) $\frac{7}{8}-\frac{3}{4}$; $\frac{7}{9}-\frac{2}{3}$; $\frac{7}{12}-\frac{1}{2}$; $1-\frac{1}{2}$; $1-\frac{3}{4}$; $1-\frac{1}{3}$; $1-\frac{4}{5}$; $1-\frac{5}{8}$.

(3) $1-\frac{5}{6}$; $2-1\frac{1}{2}$; $3-2\frac{1}{4}$; $4-3\frac{2}{7}$; $7-5\frac{2}{3}$; $11\frac{2}{7}-9\frac{5}{7}$; $19\frac{1}{3}-15\frac{2}{3}$.

2. Perform the following subtractions :

(1) $\frac{17}{18}-\frac{20}{27}$; $\frac{3}{5}-\frac{5}{13}$; $\frac{35}{36}-\frac{6}{27}$; $\frac{6}{7}-\frac{8}{15}$; $\frac{49}{60}-\frac{4}{9}$.

(2) $\frac{47}{70}-\frac{67}{91}$; $2\frac{2}{3}-1\frac{4}{5}$; $6\frac{3}{5}-5\frac{7}{8}$; $3\frac{5}{8}-2\frac{3}{4}$; $16\frac{3}{32}-4\frac{3}{40}$.

(3) $161\frac{7}{8}-128\frac{23}{36}$; $12\frac{3}{4}-11\frac{7}{8}$; $212\frac{3}{40}-106\frac{5}{8}$; $256\frac{24}{49}-252\frac{45}{56}$.

3. Simplify :

(1) $3\frac{1}{3}+4\frac{5}{6}-5\frac{2}{3}$. (2) $17\frac{5}{12}+12\frac{8}{15}-21\frac{29}{30}$.

(3) $4\frac{3}{7}+5\frac{5}{8}-9\frac{3}{8}$. (4) $47\frac{5}{13}-54\frac{23}{26}+15\frac{23}{26}$.

(5) $15\frac{5}{16}-(2\frac{1}{4}+7\frac{2}{5})$. (6) $19\frac{4}{25}-(24\frac{16}{25}-15\frac{9}{50})$.

(7) $17-(4\frac{5}{6}+7\frac{2}{3})+\frac{19}{32}$. (8) $12\frac{4}{5}-(17\frac{3}{10}-18\frac{4}{15}+9\frac{17}{30})$.

(9) $(5\frac{1}{6}+7\frac{2}{5})-(6\frac{5}{9}+3\frac{11}{15}+1)$. (10) $47\frac{8}{35}-\{32\frac{9}{42}-(59\frac{13}{28}-57)\}$.

(11) $7\frac{2}{5}-\{3\frac{4}{15}-(4\frac{3}{5}-2\frac{17}{25})-\frac{9}{20}\}+4\frac{4}{35}$ of $7\frac{9}{16}+1\frac{1}{3}-2\frac{3}{4}+2\frac{1}{2}$.

(12) $4\frac{1}{9}+\{12\frac{9}{16}-(4\frac{5}{16}-\overline{3\frac{5}{8}-\frac{7}{27}-\frac{1}{4}})\}+7\frac{1}{3}-6\frac{7}{8}$.

(13) $1\frac{3}{4}-[1\frac{3}{4}-\{1\frac{3}{4}-(1\frac{3}{4}-\overline{1\frac{3}{4}-1\frac{3}{4}})\}]$.

MULTIPLICATION OF FRACTIONS.

173. $10\times\frac{5}{7}=10\times 5$ *sevenths* $=50$ *sevenths* $=\frac{50}{7}=\frac{10\times 5}{7}$.

In multiplying a fraction by an integer, we should, therefore multiply the numerator of the given fraction by the integer for the new numerator, the denominator, however, remaining unaltered. The result should always be expressed in its lowest terms : and it will often be found convenient to cancel, before multiplication, all the factors common to the multiplier and to the denominator of the fraction. Thus,

$$\frac{5}{12}\times 7=\frac{5\times 7}{12}=\frac{35}{12}=2\frac{11}{12};\ \frac{5}{12}\times 15=\frac{5\times 3\times 5}{4\times 3}=\frac{25}{4}=6\frac{1}{4}.$$

174. In multiplying a mixed number by an integer, we can either reduce the mixed number to an improper fraction and multiply as above, or multiply the integral and fractional parts separately and add the products.

Thus, (1) $2\frac{5}{7}\times 10=\frac{19}{7}\times 10=\frac{190}{7}=27\frac{1}{7}$.

(2) $2\frac{5}{7}\times 10=2\times 10+\frac{5}{7}\times 10=20+7\frac{1}{7}=27\frac{1}{7}$.

175. Where the fractional part of a mixed number differs very little from 1, or the mixed number differs very little from 10, 100, 1000, &c., the artifices explained in ARTS. 64 and 172 may be employed to obtain the product.

Thus, (1) $\frac{99}{100}\times 24=(1-\frac{1}{100})\times 24=24-\frac{24}{100}=24-\frac{6}{25}=23\frac{19}{25}$.

(2) $22\frac{83}{85}\times 7=22\times 7+(1-\frac{2}{85})7=154+7-\frac{14}{85}=160\frac{71}{85}$.

(3) $99\frac{5}{8}\times 28=100\times 28-\frac{3}{8}\times 28=2800-10\frac{1}{2}=2789\frac{1}{2}$.

Examples 66.

1. Multiply *mentally* :

(1) $\frac{1}{3}\times 2$; $\frac{3}{7}\times 3$; $\frac{2}{9}\times 4$; $\frac{7}{16}\times 5$; $\frac{5}{32}\times 8$; $\frac{7}{45}\times 5$; $\frac{7}{88}\times 11$; $\frac{4}{7}\times 5$.

(2) $\frac{8}{9}\times 5$; $\frac{3}{16}\times 12$; $\frac{7}{12}\times 3$; $\frac{8}{27}\times 9$; $\frac{55}{91}\times 13$; $\frac{17}{22}\times 11$; $\frac{7}{18}\times 6$.

(3) $\frac{32}{49}\times 14$; $\frac{2}{3}\times 6$; $\frac{8}{33}\times 11$; $\frac{27}{49}\times 7$; $\frac{15}{28}\times 42$; $\frac{13}{48}\times 24$; $\frac{7}{75}\times 30$.

2. Multiply :

(1) $\frac{25}{64}$ separately by 8, 16. 32, 128. (2) $\frac{95}{16}$ separately by 30, 40, 70.

(3) $\frac{17}{33}$ " " 3, 11, 44, 88. (4) $\frac{29}{34}$ " " 17, 51, 85.

(5) $\frac{37}{45}$ " " 108, 153. (6) $14\frac{5}{44}$ " " 4, 11, 55.

(7) $5\frac{4}{7}$ by 2, by 4, by 10, by 14. (8) $7\frac{3}{8}$ by 5, by 9, by 12, by 20.

3. Find the product of :

(1) $9\frac{9}{10}$ separately by 7, 8, 9, 10, 15. (2) $999\frac{43}{44}$ by 99, by 440.

(3) $109\frac{24}{25}$ " " 7, 10, 15, 50, 200.

(4) $499\frac{124}{125}$ " " 25, 50, 75, 100, 150, 200.

176. Since $1 \times \frac{2}{3}$ or $\frac{2}{3}$ means that *one* is to be divided into 3 equal parts and 2 of these taken, therefore $\frac{4}{5} \times \frac{2}{3}$ should mean that $\frac{4}{5}$ is to be divided into 3 equal parts and 2 of such parts are to be taken. Now $\frac{4}{5} = \frac{12}{15} = 12$ *fifteenths*, so that the three parts into which it is to be divided are each equal to 4 *fifteenths*, and 2 such parts are equal to 8 *fifteenths*, or $\frac{8}{15}$. Hence,

$$\frac{4}{5} \times \frac{2}{3} = \frac{8}{15} = \frac{4 \times 2}{5 \times 3}.$$

In multiplying one fraction by another we should therefore multiply the numerators together for the new numerator and the denominators together for the new denominator, and reduce the fraction so obtained to its lowest terms.

Thus, (1) $\frac{5}{9} \times \frac{3}{14} = \frac{5 \times 3}{9 \times 14} = \frac{15}{126} = \frac{3 \times 5}{3 \times 42} = \frac{5}{42}.$

(2) $\frac{25}{64} \times \frac{16}{27}$ of $\frac{3}{10} = \frac{25}{64} \times \frac{16}{27} \times \frac{3}{10} = \frac{25 \times 16 \times 3}{64 \times 27 \times 10}$

$$= \frac{5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 3}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 2 \times 5} = \frac{5}{72}.$$

(3) $\frac{4}{21} \times \frac{7}{15} \times 2\frac{1}{4} \times \frac{15}{35} \times 23\frac{1}{3} = \frac{4}{21} \times \frac{7}{15} \times \frac{9}{4} \times \frac{15}{35} \times \frac{70}{3}.$

$$= \frac{5 \times 7 \times 3 \times 3 \times 3 \times 4 \times 5 \times 2 \times 7}{7 \times 3 \times 3 \times 5 \times 4 \times 5 \times 7 \times 3} = \frac{2}{1} = 2.$$

(4) $\frac{3}{8} \times 3\frac{1}{3} \times \frac{4}{5} = \frac{3}{8} \times \frac{10}{3} \times \frac{4}{5} = \frac{3 \times 2 \times 5 \times 4}{2 \times 4 \times 3 \times 5} = \frac{1}{1} = 1.$

Examples 67.

1. Multiply :

(1) $\frac{1}{2}$ by $\frac{1}{3}$; $\frac{1}{3}$ by $\frac{1}{3}$; $\frac{1}{4}$ by $\frac{1}{4}$; $\frac{2}{3}$ by $\frac{4}{5}$; $\frac{3}{8}$ by $\frac{9}{10}$; $\frac{9}{11}$ by $\frac{6}{13}$.

(2) $\frac{1}{2}$ by $\frac{2}{3}$; $\frac{9}{10}$ by $\frac{15}{17}$; $\frac{9}{11}$ by $\frac{143}{162}$; $\frac{4}{13}$ by $\frac{9}{13}$; $\frac{13}{60}$ by $\frac{1}{17}$.

(3) $\frac{11}{13}$ by $\frac{9}{10}$; $\frac{12}{25}$ by $\frac{35}{36}$; $\frac{13}{60}$ by $\frac{42}{91}$; $6\frac{2}{3}$ by $6\frac{3}{11}$; $8\frac{13}{16}$ by $3\frac{2}{3}$.

(4) $\frac{1}{2}$ of $\frac{2}{3}$ by $1\frac{1}{2}$ of 2; $3\frac{1}{3}$ of $\frac{3}{8}$ by $\frac{1}{51}$ of 51; $\frac{1}{2}$ of $\frac{1}{3}$ by $5\frac{1}{2}$ of 3.

(5) $\frac{11}{13}$ of $2\frac{3}{11}$ of $\frac{1}{250}$ by $\frac{7}{12}$ of $37\frac{1}{2}$ of $6\frac{1}{7}$ of $\frac{4}{25}$.

2. Find the values of :

(1) $\frac{1}{2} \times \frac{11}{12} \times 1\frac{1}{11}$. (2) $1\frac{1}{3} \times 5\frac{1}{3} \times 3\frac{3}{4}$. (3) $2\frac{1}{4} \times \frac{22}{45} \times \frac{1}{9}$.

(4) $\frac{1}{3} \times \frac{3}{4} \times \frac{5}{6} \times \frac{4}{15} \times \frac{3}{7}$. (5) $2\frac{1}{4} \times 3\frac{1}{3} \times 3\frac{3}{7} \times \frac{21}{25} \times \frac{2}{3}$.

(6) $10\frac{1}{2} \times 5\frac{1}{3} \times \frac{3}{56} \times \frac{4}{27} \times 2\frac{1}{4}$. (7) $\frac{33}{35} \times \frac{14}{15} \times \frac{625}{506} \times \frac{168}{169}$.

(8) $\frac{11}{13}$ of $\frac{10}{9}$ of $\frac{39}{46}$ of $\frac{69}{77} \times \frac{21}{68}$ of $\frac{17}{30}$ of $\frac{25}{21}$ of $\frac{24}{25}$.

(9) $\frac{49}{50}$ of $3\frac{1}{3}$ of $4\frac{2}{10} \times 7\frac{1}{2}$ of $7\frac{1}{2}$ of $3\frac{1}{3} \times 2\frac{1}{2}$ of $1\frac{2}{13}$.

(10) $4\frac{2}{3}$ of $3\frac{3}{16} \times 2\frac{1}{2}$ of $3\frac{1}{7} \times 3\frac{7}{11}$ of $2\frac{1}{2} \times 5\frac{5}{18}$ of $5\frac{1}{7}$.

3. Simplify :

(1) $3\frac{1}{5}\times 5\frac{3}{5}-(1\frac{3}{5}-\frac{1}{3})\times 1\frac{9}{11}-2\frac{3}{5}\times(4\frac{1}{5}-3)$.

(2) $(1\frac{1}{2}-1)\times 1\frac{1}{4}\times(1\frac{2}{3}-1\frac{1}{4})\times\frac{21}{25}\times 16(2\frac{1}{4}-1\frac{4}{5})\times\frac{35}{54}\times 13\frac{6}{7}$.

(3) $(\frac{1}{6}+\frac{1}{9})\times 2\frac{1}{4}\times 10\,(\frac{2}{75}+\frac{1}{150})\times(\frac{3}{4}+\frac{3}{8})\times\frac{5}{13}\times 1\frac{4}{35}$.

(4) $(\frac{1}{28}+\frac{1}{35})$ of $\frac{5}{9}\times(2\frac{1}{6}+1\frac{5}{6})$ of $\frac{13}{67}\times(1\frac{1}{6}+1\frac{1}{7})$ of $\frac{7}{64}\times 1\frac{3}{35}$ of $18\frac{6}{13}$.

DIVISION OF FRACTIONS.

177. One number is called the **reciprocal** of another when its numerator is the denominator of the other, and *vice-versa*.

Thus, $\frac{1}{3}$ is the *reciprocal* of $\frac{3}{1}$ or 3 ; 5 or $\frac{5}{1}$ of $\frac{1}{5}$; and $\frac{4}{13}$ of $\frac{13}{4}$.

178. Since DIVISOR × QUOTIENT = DIVIDEND, (ART. 59), to divide $\frac{7}{8}$ by $\frac{2}{3}$ is to find a fraction such that, when multiplied by $\frac{2}{3}$, it becomes equal to $\frac{7}{8}$. Now $(\frac{7}{8}\times\frac{3}{2})\times\frac{2}{3}=\frac{7}{8}$. Therefore the required fraction is $\frac{7}{8}\times\frac{3}{2}$, or $\frac{7}{8}\div\frac{2}{3}=\frac{7}{8}\times\frac{3}{2}$. Hence, to divide one fraction by another, we should multiply the dividend by the *reciprocal* of the divisor.

Ex. $\frac{20}{21}\div 15=\frac{20}{21}\times\frac{1}{15}=\frac{4}{63}$; $\frac{2}{11}\times\frac{9}{5}=$

179. If the divisor is a mixed number, it should be first changed into a simple one.

Ex. $\frac{3}{20}\div 3\frac{1}{2}=\frac{3}{20}\div\frac{7}{2}=\frac{3}{20}\times\frac{2}{7}=\frac{3}{70}$.

$3\frac{1}{2}\div 5\frac{1}{4}=\frac{7}{2}\div\frac{21}{4}=\frac{7}{2}\times\frac{4}{21}=\frac{2}{3}$.

180. In an expression like $\frac{1}{4}\times\frac{5}{2}\div\frac{2}{7}$ of 3, each of the signs × or ÷ applies *only to the number immediately following*. Numbers connected by '*of*' are, however, considered as one.

Thus, $\frac{3}{2}\div\frac{5}{6}$ of $\frac{3}{2}\times 1\frac{1}{4}=\frac{3}{2}\times\frac{6}{5}\times\frac{2}{3}\times\frac{5}{4}=1\frac{1}{2}$.

Examples 68.

1. Divide *mentally* :

(1) $\frac{1}{2}$ separately by 2, 3, 4, 5, 6. (2) $\frac{7}{9}$ separately by 7, 14, 21.

(3) $\frac{32}{35}$ " " 2, 4, 8, 16. (4) $\frac{63}{80}$ " " 7, 9, 14, 21.

2. Divide .

(1) $\frac{40}{73}$ separately by 8, 16, 24, 32. (2) $\frac{35}{41}$ separately by 5, 7, 70.

(3) $\frac{126}{141}$ " " 6, 12, 14, 42. (4) $\frac{225}{256}$ " " 25, 45, 75.

(5) $4\frac{2}{9}$ " " 19, 57, 95. (6) $3\frac{2}{7}$ " " 23, 46, 92.

3. Divide *mentally* :

(1) $\frac{5}{8}$ separately by $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{8}$. (2) $\frac{4}{7}$ separately by $\frac{1}{4}$, $\frac{1}{7}$, $\frac{1}{5}$.

(3) $\frac{3}{5}$ " " $\frac{3}{7}$, $\frac{4}{5}$, $\frac{9}{35}$. (4) $\frac{4}{15}$ " " $\frac{7}{15}$, $\frac{8}{75}$, $\frac{7}{30}$.

4. Divide :

(1) $\frac{7}{16}$ separately by $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{24}$, $\frac{35}{48}$.

(2) $\frac{14}{15}$ separately by $\frac{21}{35}$, $\frac{42}{57}$, $\frac{77}{75}$.

(3) $\frac{84}{242}$ " " $\frac{7}{22}$, $\frac{14}{33}$, $\frac{56}{77}$.

(4) $\frac{28}{57}$ " " $\frac{14}{19}$, $\frac{42}{95}$, $\frac{81}{133}$.

(5) $13\frac{1}{3}$ " " $3\frac{3}{4}$, $6\frac{2}{3}$, $4\frac{2}{7}$.

(6) $24\frac{1}{5}$ " " $4\frac{2}{5}$, $8\frac{1}{5}$, $12\frac{5}{7}$.

(7) $13\frac{4}{7}$ " " $4\frac{3}{4}$, $7\frac{3}{5}$, $15\frac{1}{5}$.

(8) $15\frac{1}{6}$ " " $3\frac{5}{7}$, $5\frac{4}{7}$, $3\frac{1}{17}$.

(9) $\frac{3}{4}$ of $\frac{1}{2}$ by $3\frac{1}{2}$ of $\frac{1}{4}$.

(10) $\frac{1}{2}$ of $\frac{5}{7}$ by $\frac{6}{27}$ of $6\frac{3}{4}$.

(11) $\frac{7}{12}$ of $\frac{5}{6}$ by $\frac{14}{89}$ of $3\frac{3}{4}$.

(12) $1\frac{9}{25}$ of $\frac{2}{5}$ by $\frac{3}{15}$ of 14.

(13) $3\frac{3}{4}$ of $\frac{75}{18}$ by $1\frac{4}{12}$ of $\frac{50}{33}$.

(14) $11\frac{4}{25}$ of $16\frac{2}{3}$ by $\frac{3}{5}$ of $12\frac{1}{2}$.

(15) $(1\frac{1}{2} \times \frac{3}{4})$ by $\frac{7}{8}$ of 24.

(16) $(3\frac{1}{8} + 1\frac{5}{9})$ by $(3\frac{1}{3} + 2\frac{2}{4})$.

5. Simplify :

(1) $15\frac{3}{4} \div 1\frac{1}{4} + 7\frac{3}{4}$ of $\frac{4}{15} \div \frac{1}{3} + 6\frac{2}{3} + 10$.

(2) $10 + 5\frac{1}{4}[2\frac{1}{2} + (4\frac{1}{2} + 3 + 1\frac{1}{4}$ of $\frac{4}{5})] \div \frac{8}{21}$.

(3) $\frac{3}{5} \div [6\frac{6}{7} - 2 \div \frac{3}{4} - 3 \div (1 - \frac{6}{18})]$.

(4) $3\frac{3}{4} \div 6\frac{6}{7} \times 4\frac{4}{5} \div 7\frac{7}{8} \times 5\frac{5}{6} \div 8\frac{8}{9}$.

(5) $8\frac{5}{6}$ of $10\frac{4}{5} \div 5\frac{6}{9}$ of $7\frac{5}{7} \times 13\frac{3}{4} \div 10\frac{1}{8}$.

181. A **Complex Fraction** is one which has either a fraction or a mixed number in one or both of its terms.

Thus, $\frac{8}{9\frac{1}{6}}$, $\frac{5\frac{1}{3}}{7}$, $\frac{\frac{2}{7}}{\frac{4}{9}}$, $\frac{4\frac{2}{7}}{6\frac{1}{3}}$, $\frac{\frac{12}{5} \text{ of } \frac{7}{3}}{2\frac{2}{3}}$ are *Complex Fractions.*

182. Complex fractions are subject to the same rules as simple fractions, and are easily simplified if we remember that a fraction is the *quotient* of the numerator by the denominator.

(1) $\frac{\frac{7}{\ }}{25} = \qquad = 3\frac{1}{5}$.

(2) $\dfrac{5\frac{2}{3} - 1\frac{3}{4} + 4\frac{6}{7}}{8\frac{1}{3} \text{ of } 2\frac{1}{5}} = \dfrac{8 + \frac{56 - 63 + 72}{84}}{\frac{25}{3} \times \frac{11}{5}} = \dfrac{8\frac{65}{84}}{\frac{55}{3}} = \dfrac{737 \times 3}{84 \times 55} = \dfrac{67}{140}$.

Examples 69.

Reduce to their simple forms :

1. $\frac{\frac{7}{8}}{\frac{5}{11}}$; $\frac{3\frac{1}{4}}{4\frac{1}{3}}$; $\frac{1\frac{6}{13}}{8\frac{8}{9}}$; $\frac{7\frac{1}{2}}{\frac{5}{17}}$; $\frac{28\frac{1}{2}}{7\frac{1}{8}}$; $\frac{19\frac{1}{11}}{32\frac{4}{7}}$; $\frac{17}{4\frac{1}{4}}$; $\frac{100\frac{1}{10}}{30\frac{1}{8}}$.

2. $\frac{\frac{3}{4} \text{ of } 1\frac{5}{6}}{\frac{7}{8} \text{ of } \frac{11}{13}}$; $\frac{\frac{1}{5} \text{ of } 4\frac{1}{2}}{3\frac{3}{4} \text{ of } 5\frac{5}{7}}$; $\frac{11\frac{2}{9}}{\frac{3}{5} \text{ of } \frac{5}{6}}$; $\frac{\frac{3}{8} \text{ of } 5\frac{1}{3} \text{ of } \frac{9}{10}}{2\frac{1}{4} \times 6\frac{2}{3} \text{ of } 1\frac{1}{35}}$.

3. $\frac{2\frac{1}{3} \times 1\frac{1}{4}}{5\frac{1}{7} \times 5\frac{1}{6}}$; $\frac{3\frac{3}{4} \times 5\frac{1}{3}}{6\frac{2}{3} \times 1\frac{2}{3}}$; $\frac{\frac{11}{5} \text{ of } 18\frac{1}{3} \times 4\frac{2}{5} \text{ of } 6\frac{2}{11}}{6\frac{1}{6} \times 11\frac{1}{7} \text{ of } \frac{2}{15} \text{ of } 8\frac{4}{5}}$.

4. $\frac{\frac{1}{6}}{\frac{2}{33}} + \frac{1\frac{1}{4}}{1\frac{1}{5}}$ of $\frac{4\frac{1}{2}}{3\frac{1}{3}} \times \frac{2\frac{1}{4}}{1\frac{1}{5}} \times \frac{2\frac{7}{10}}{1\frac{1}{3}} \div \frac{3\frac{1}{3}}{\frac{19}{27}}$.

5. $\frac{4\frac{1}{2}+7\frac{1}{3}}{1\frac{1}{3}+1\frac{5}{6}}$; $\frac{3\frac{1}{4}+4\frac{1}{2}-2\frac{1}{3}}{7\frac{1}{2}-3\frac{1}{4}\text{ of }\frac{8}{9}}$; $\frac{17\frac{1}{3}-14\frac{1}{3}+2}{24\frac{1}{3}-15\frac{7}{11}+4\frac{4}{55}}$.

6. $\left(\frac{1\frac{1}{3}}{2}-\frac{1\frac{1}{3}}{3}\right)-\left(\frac{1\frac{1}{2}}{4}-\frac{1\frac{1}{3}}{5}\right)$; $\frac{\frac{4}{5}-\frac{2}{3}}{\frac{2}{3}-\frac{1}{2}}-\frac{\frac{3}{4}-\frac{1}{2}}{\frac{1}{2}-\frac{1}{12}}$.

183. A **Continued Fraction** like the following is simplified thus :

$$3-\cfrac{2}{7+\cfrac{3}{5-\frac{2}{7}}} \qquad 3-\cfrac{2}{7+\cfrac{3}{\frac{33}{7}}} \qquad 3-\cfrac{2}{7+\frac{7}{11}} \qquad 3-\cfrac{2}{\frac{84}{11}}$$

$$=\frac{5}{3-\frac{11}{42}}=\frac{5}{\frac{115}{42}}=\frac{42}{23}=1\frac{19}{23}.$$

Examples 70.

Simplify :

1. $\cfrac{1}{1+\cfrac{2}{1+\frac{3}{4}}}$. 2. $\cfrac{1}{3+\cfrac{5}{7+\frac{9}{11}}}$. 3. $\cfrac{2}{4+\cfrac{13\frac{1}{4}}{8+\frac{10}{12}}}$. 4. $\cfrac{3}{5+\cfrac{3}{5+\frac{3}{5}}}$.

5. $\cfrac{3}{4+\cfrac{3}{4+\cfrac{3}{4+\frac{3}{4}}}}$. 6. $\cfrac{2\frac{1}{2}}{2\frac{1}{2}-\cfrac{2\frac{1}{4}}{2\frac{1}{2}+\cfrac{2\frac{1}{2}}{2\frac{1}{2}-1\frac{1}{4}}}}$. 7. $\cfrac{6\frac{2}{3}}{6\frac{2}{3}+\cfrac{6\frac{1}{7}}{6\frac{2}{3}+\cfrac{5}{6\frac{2}{3}+3\frac{1}{3}}}}$.

8. $\cfrac{4}{4+\cfrac{3}{3+\cfrac{2}{2+\cfrac{1}{1+\frac{1}{2}}}}}+\cfrac{5}{5+\cfrac{4+\frac{1}{4}}{5+\cfrac{5}{5+\cfrac{13}{5+\frac{1}{5}}}}}+\cfrac{10}{7+\cfrac{23}{7+\cfrac{7}{7+\cfrac{25}{7+\frac{1}{7}}}}}$

Examples 71.

Simplify :

1. $5\frac{3}{4}+\frac{8}{15}$ of $3\frac{3}{4}-5\frac{1}{3}$. 2. $17\frac{5}{7}-5\frac{1}{2}$ of $1\frac{5}{33}$.

3. $(17\frac{5}{7}-5\frac{1}{2})$ of $1\frac{5}{33}$. 4. $\frac{9}{13}+\frac{7}{26}\times\frac{20}{27}+\frac{5}{78}$ of $1\frac{1}{11}$.

5. $27\frac{1}{3}+5\frac{2}{7}-6\frac{17}{30}$ of $(7\frac{1}{3}-2\frac{1}{2})$. 6. $3\frac{1}{3}-(4\frac{7}{8}-2\frac{1}{4})+\{5\frac{3}{4}-(2\frac{7}{15}-\frac{1}{3})\}$.

7. $\frac{1}{7}$ of $(1-\frac{2}{7})+\frac{1}{5}$ of $\frac{1}{10}+\frac{3}{8}$ of $(\frac{1}{3}+\frac{1}{14})+\frac{3}{70}$ of $(\frac{2}{7}+\frac{4}{5})$.

8. $(3\frac{1}{3}+7\frac{1}{4})$ of $\frac{3}{5}+9\frac{3}{4}$. 9. $3\frac{1}{3}+7\frac{1}{4}$ of $\frac{3}{5}+9\frac{3}{4}$.

10. $3\frac{1}{3}+6\frac{1}{4}$ of $(\frac{3}{5}+7\frac{3}{4})$. 11. $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+(\frac{3}{4}-\frac{1}{3})$.

12. $\frac{2\frac{2}{35}}{1+2\frac{4}{5}}$ of $\left\{58\frac{3}{4}\times\frac{1}{9\frac{1}{3}}+\frac{47}{4\frac{1}{6}}\right\}$. 13. $\frac{1+3\frac{3}{4}(1+3\frac{3}{4})}{1+3\frac{3}{8}(1+3\frac{3}{8})}$.

14. $\dfrac{3\frac13\times3\frac13\times3\frac13-1}{3\frac13\times3\frac13-1}$.

15. $\dfrac{5\frac13-3\frac27}{10\frac17-1\frac27}+\dfrac{12\frac38-7\frac{5}{24}}{17\frac12-8\frac{1}{14}}$.

16. $\dfrac{7\frac13}{6\frac14}-\dfrac{4\frac13-2\frac13}{4\frac13+2\frac13}-\dfrac{1\frac13}{2\frac{1}{10}}$.

17. $\dfrac{1\frac{1}{12}-\frac79\text{ of }\frac37-\frac17}{(1\frac{1}{12}-\frac79)\text{ of }(\frac37-\frac17)}$.

18. $\dfrac{5\frac38}{10\frac12}-\dfrac{22\frac12}{30}\text{ of }\left(\dfrac{4}{15}-\dfrac{2-1\frac14}{14\frac14-9\frac14}\right)$.

19 $\dfrac{1\frac35}{3\frac12}+\dfrac{1\frac14}{3}+9+\dfrac34\text{ of }\dfrac89\text{ of }\dfrac{5\frac58}{7}$.

20. $\dfrac{1\frac{17}{18}-1\frac{14}{54}}{\frac{5}{36}\text{ of }(\frac19+\frac{5}{63})}\div5\frac35$.

21. $\dfrac{1\frac78}{1\frac{1}{10}}+\dfrac{2\frac78-\frac35}{11\frac13+2\frac25}\times\dfrac{139}{2\frac25}-\dfrac{1\frac{14}{27}}{\frac15}$.

22. $\dfrac{5+4\frac45(5+4\frac45)}{5+4\frac15(5+4\frac15)}$.

23. $\dfrac{27\frac13+3\frac34-1\frac14}{5\frac56+2\frac76}+$ [illegible]

24. $\dfrac{27\frac12-9\frac14}{17\frac12+5\frac58}+\dfrac{7}{12}\text{ of }\dfrac{8\frac14\times13\frac34}{3\frac{1}{10}}\times\dfrac{1}{27\frac12}-\dfrac{48\frac13}{42}+\dfrac{20\frac14}{7\frac15}-\dfrac{22\frac12+4\frac65}{6\frac{5}{107}\times5\frac15}$.

25. $\left(\frac47+\dfrac{5\frac17}{11\frac15}+1\frac{17}{49}+\frac{20}{21}-1\frac57\right)\div6\frac23\text{ of }\frac{5}{36}\times\dfrac{1\frac37}{4\frac34}-\dfrac{7-4\frac27}{11\frac13-8\frac{13}{21}}$.

26. $\dfrac{5\frac17-2\frac47}{5\frac17+2\frac35}\text{ of }\dfrac{5\frac{8}{11}}{2\frac{4}{11}}\div\dfrac{\frac47}{1\frac97-\frac59}+\dfrac{7\frac{7}{12}}{2\frac16}-\dfrac{\frac13}{\frac13-\frac{3}{117}}$.

27. $\left\{\dfrac{1\frac14}{1\frac{1}{12}}\text{ of }\frac34+\dfrac{2\frac13-1\frac56}{\frac14+1\frac56}-\dfrac{8\frac14}{7\frac47}\right\}\div\dfrac23$.

28. $\dfrac{1\frac67}{2\frac23\times5\frac13}+\dfrac{1\frac{13}{20}\div\frac{7}{10}}{2\frac13}-\dfrac{29\frac13\times10\frac38\text{ of }\frac14}{13\frac12+1\frac18}+\dfrac{\frac25-\frac{4}{15}}{\frac15-\frac{4}{15}}$.

29 $\dfrac{\frac35-\frac37}{\frac14-\frac15}\text{ of }\dfrac{1-\frac17}{2+\frac13}+\dfrac{\frac{6}{35}\div\frac{3}{25}\times6+9\frac13-4\frac{1}{15}\text{ of }1\frac18\times2}{4\frac13\times\frac37-11\frac15+1\frac{13}{30}}\times\dfrac{94\frac15}{7}-\dfrac{22\frac13}{30}$.

30. $2\frac16+1\frac97+(6\frac13+3\frac14+2\frac{1}{10})\div\dfrac{3\frac14}{10\frac12}-\dfrac{3\frac14}{9}\div\dfrac{21\frac38-13}{2\frac12-1\frac38}$.

31. $\dfrac{27\frac16-19\frac14}{44\text{ of }14\frac34+60\times5\frac17}\times\dfrac{93-7\frac47}{7+5\frac14}-\dfrac{\frac12-\frac18}{\frac12+\frac16}+\dfrac{1\frac35}{2\frac13}\text{ of }\left\{\dfrac{\frac{9}{17}\times2\frac12}{4\frac35\times\frac{1}{13}}-\dfrac{1\frac13}{2\frac38}\right\}$.

32. $\dfrac{3\frac14+2\frac56}{113-7\frac29\text{ of }\dfrac{12}{2\frac16}}\div\dfrac{6\frac45+4\frac14}{11\frac{9}{10}\text{ of }\dfrac{1\frac37}{14}-\frac{17}{420}}+\dfrac{3\frac13}{6\frac35+\dfrac{1}{2+\frac12}}+\dfrac{26\frac47\text{ of }\dfrac{3\frac12}{117}}{5-\frac35\text{ of }\dfrac{2\frac35}{1-\frac35}}$.

33. $\dfrac{11+27\frac12+36\frac23}{\dfrac{11}{\frac34}+\dfrac{22}{1\frac16}+\dfrac{33}{1\frac58}}\times\dfrac{222\frac23+2\frac13}{\dfrac{4\frac{8}{11}\text{ of }54\frac23}{8}}+\dfrac{2^1-\frac56}{2\frac12+\frac56}+\dfrac{6\frac23}{15}$.

34. $\dfrac{3\frac38}{3\frac12+\dfrac{1\frac13}{4-2\frac14}}\times\dfrac{673\frac34}{548\frac14}\div(7\frac{1}{12}-3\frac34)+\dfrac{2}{[\text{illegible}]+\dfrac{1}{3+\frac14}}-\dfrac{4}{9-\dfrac{3}{1-\frac25}}$.

35. $\dfrac{3\frac12}{2\frac13-1\frac13}+\dfrac{\frac38-\dfrac{2}{15-7\frac12}}{\frac45-\frac{3}{10}}-\dfrac{1\frac27}{2\frac14}\text{ of }\left\{\dfrac{5}{7\frac14}+\dfrac{1\frac17}{14}\text{ of }\dfrac{16\frac23-12\frac13}{11\frac{4}{21}-10}\right\}$.

36. $\dfrac{\frac{2}{7}}{\frac{3}{7}-\frac{1}{7-3\frac{1}{2}}}-2\frac{1}{3}\text{ of }\left(\frac{5}{7}-\frac{2}{10\frac{1}{2}-1\frac{1}{6}}\right)\div\dfrac{6\frac{1}{3}+16\frac{1}{4}}{28\frac{1}{2}}.$

37. $\dfrac{1\frac{3}{5}-\frac{1}{3-1\frac{1}{4}}-\frac{15}{16}\text{ of }1\frac{7}{9}+\frac{7\frac{1}{3}}{1\frac{1}{2}}+\frac{1\frac{2}{3}}{\frac{2}{3}}}{\frac{1}{5}\left(15+\frac{8\frac{4}{7}}{\frac{1}{2}}\right)-6\frac{2}{3}}.$ 38. $\dfrac{\frac{\frac{2}{5}}{1-\frac{1}{25}}+\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{7}\text{ of }\left\{\frac{\frac{2}{5}}{1-\frac{2}{5}}+\frac{1}{3}\right\}}$

39. $\dfrac{2\frac{2}{7}}{1\frac{6}{7}}-\dfrac{4\frac{1}{2}-1\frac{31}{36}}{[illegible]-1\frac{5}{12}}+\dfrac{4\frac{3}{5}}{11\frac{1}{2}}-\dfrac{13\frac{7}{8}}{15\frac{5}{12}}\text{ of }\left\{\frac{5\frac{3}{5}}{26\frac{1}{9}}-\frac{8\frac{1}{4}-5\frac{1}{2}}{78\frac{3}{8}-53\frac{1}{6}}\right\}.$

40. $\dfrac{14\frac{7}{16}}{10\frac{5}{12}-5\frac{5}{9}}+\dfrac{3-\frac{6\frac{4}{5}}{10\frac{1}{5}-5\frac{1}{10}}}{(9-3\frac{3}{8})\div2\frac{1}{4}}-\dfrac{6\frac{3}{7}}{9}\text{ of }\left\{\frac{1\frac{2}{5}}{2}+\frac{25\frac{1}{2}}{21\frac{1}{4}}\text{ of }\frac{7\frac{5}{11}-6\frac{4}{55}}{5\frac{1}{4}-3\frac{7}{44}}\right\}.$

41. $\dfrac{(\frac{3}{5}+\frac{1}{4})\times(\frac{3}{5}-\frac{1}{4})}{\frac{9}{10}\left\{\frac{1}{2}-2\left(\frac{1}{3}-\frac{1}{6}\right)\div\frac{\frac{3}{4}+\frac{7}{28}}{\frac{1}{2}+\frac{1}{10}+\frac{14}{35}}\right\}}+\dfrac{(5\frac{1}{2}\text{ of }\frac{2}{9}\text{ of }\frac{4}{7})\div(1\div\overline{\frac{1}{5}+\frac{1}{2}})}{1-\frac{3}{14}\text{ of }\left(\frac{1}{2}+\frac{1}{2}\text{ of }\frac{\frac{1}{30}}{\frac{1}{7}\times1\frac{1}{20}}\right)}$

42. $11-\dfrac{1}{1\frac{1}{13}-\frac{1}{1-\frac{1}{8\frac{1}{11}}}}.$ 43. $\dfrac{1}{7-\frac{1}{7+\frac{1}{7-\frac{1}{7}}}}.$ 44. $\dfrac{1}{2+\frac{7}{1+\frac{5}{5+\frac{1}{3}}}}.$

45. $\dfrac{2\frac{2}{3}}{1+\frac{3}{1+\frac{3}{2+\frac{9}{5+\frac{1}{7}}}}}.$ 46. $\dfrac{2}{1+\frac{5}{3+\frac{11}{4+\frac{5}{3+\frac{1}{5}}}}}.$ 47. $\dfrac{1}{3+\frac{2}{3+\frac{2}{3+\frac{2}{3+\frac{2}{3}}}}}.$

48. $9\dfrac{5}{6\frac{1}{4}}\text{ of }\dfrac{11\frac{1}{14}}{13\frac{1}{13}-12\frac{1}{2}}+\dfrac{13}{(\frac{1}{2}-\frac{1}{3})(\frac{1}{5}-\frac{4}{25})}-7\div\dfrac{2\frac{2}{7}-2\frac{1}{4}}{1\frac{9}{11}\left(13\frac{1}{2}+\frac{7\frac{1}{3}}{7}\right)}\text{ of }\dfrac{2}{2+\frac{2}{4-\frac{3}{4}}}.$

CHAP. XVIII. G. C. M. & L. C. M. OF FRACTIONS.

184. To find the G. C. M. of fractions.

RULE. *Find the G.C.M. of the numerators for the numerator, and the L. C. M. of the denominators for the denominator.*

Ex. Find the G. C. M. of $\frac{5}{6}$, $\frac{25}{24}$, $\frac{35}{36}$.

The G. C. M. of the numerators 5, 25, 35, is 5; and the L. C. M. of the denominators 6, 24, 36, is 72. Thus the reqd. G. C. M. is $\frac{5}{72}$. *Ans.*

185. To find the L. C. M. of fractions.

RULE. *Find the L. C. M. of the numerators for the numerator and the G. C. M. of the denominators for the denominator.*

Ex. Find the L. C. M. of $\frac{5}{6}, \frac{25}{24}, \frac{35}{36}$.

The L C. M. of the numerators 5, 25, 35, is 175, and the G. C M. of the denominators 6, 24, 36, is 6. Thus the reqd. L. C. M. is $\frac{175}{6}$. *Ans.*

Note. Mixed numbers must be reduced to improper fractions and compound fractions to simple ones, in their lowest terms.

Examples 72.

1. Find the G. C. M. and L. C. M. of :

(1) $\frac{3}{4}, \frac{7}{9}$. (2) $\frac{5}{12}, \frac{15}{28}$. (3) $\frac{8}{35}, \frac{24}{77}$. (4) $\frac{9}{10}, 1\frac{17}{40}$.
(5) $\frac{5}{9}, \frac{20}{27}, \frac{25}{33}$. (6) $7\frac{1}{2}, 5\frac{1}{4}, 6\frac{3}{4}$. (7) $\frac{26}{27}, \frac{39}{45}, \frac{52}{75}$. (8) $5\frac{1}{3}, 6\frac{6}{7}, 9\frac{7}{9}$.

2. R233¾ and R272 were respectively distributed among two companies of beggars. Supposing the beggars were paid equally, find the greatest possible sum that each could get, and the number of beggars in each company.

3. There is a path all round a park whose circuit is 1650 yards. 5 boys begin to run the same way round it, at the rate of 5, $6\frac{1}{4}$, $7\frac{1}{3}$, $8\frac{1}{4}$ and $6\frac{3}{8}$ yards a second respectively. How soon will they be together again at the starting point ?

4. Three persons start together from the same place to sail round an island 314 mi. round, the first two sailing one way and the third the other way. If they sail $3\frac{5}{9}$, $4\frac{4}{5}$, and $5\frac{1}{3}$ miles per hour respectively, how soon will they all be together at the starting point ?

5. Three wheels are respectively $7\frac{1}{3}$ ft., $10\frac{1}{4}$ ft., and $85\frac{3}{8}$ ft. round. Find the nearest distance in which, they will all make complete revolutions.

***6.** A viaduct is made of 3 series of arches built over one another, the spans of the arches being $12\frac{2}{3}$, $8\frac{1}{2}$ and $4\frac{1}{4}$, yards, respectively. The piers on which they rest are all 4 yds. wide : find the least length of the viaduct.

Problems worked out.

Ex. 1. The sum of two fractions is $\frac{1}{3}$ and their difference is $\frac{1}{5}$; what are the fractions.

The greater of the two fractions $= (\frac{1}{3} + \frac{1}{5}) \times \frac{1}{2} = \frac{4}{15}$;
∴ „ smaller „ „ $= \frac{1}{3} - \frac{4}{15} = \frac{1}{15}$. } *Ans.*

Ex. 2. What least fraction must be added to $\frac{17}{25}$ to make it an integer ?

The fractional part subtracted from unity is the required fraction ;

∴ the required fraction $= 1 - \frac{17}{25} = \frac{8}{25}$. *Ans.*

Ex. 3. What fraction added to the sum of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ will make $2\frac{2}{3}$?

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = 1\frac{17}{60}$.

∴ the required fraction $= 2\frac{2}{3} - 1\frac{17}{60} = 1\frac{23}{60}$. *Ans.*

Ex. 4. If a person gets a bequest of $\frac{5}{7}$ of an estate, and sells $\frac{2}{3}$ of his portion, what part of the whole does he still retain ?

$\frac{2}{3}$ of his share being sold, there will remain $(1 - \frac{2}{3})$. But his share $\frac{5}{7}$ of the estate. Therefore he retains $\frac{5}{7}(1 - \frac{2}{3})$ or $\frac{5}{21}$. *Ans.*

Ex. 5. If I sell up $\frac{1}{3}$ of my property to one person, then $\frac{1}{4}$ of the remainder to another, and then $\frac{1}{5}$ of what still remains to a third; what part of the whole should I still possess?

Reqd. part $=(1-\frac{1}{3})(1-\frac{1}{4})(1-\frac{1}{5})=\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}=\frac{2}{5}$. *Ans.*

Examples 73.

1. Divide the sum of $2\frac{2}{3}$ and $1\frac{5}{6}$ by the product of their sum and difference.

2. Divide the product of $3\frac{3}{4}$ and $(1+16\frac{4}{5})$ by half the sum of $\frac{3}{4}$ and $\frac{5}{6}$.

3. What fraction added to $13\frac{5}{8}-4\frac{7}{12}+2\frac{1}{6}-5\frac{4}{15}$ will give 20?

4. Divide the sum of 12 and $3\frac{3}{4}$ by their difference, and the difference by their sum; and find the sum and difference of the two quotients.

5. Multiply the sum of $1\frac{2}{3}$, $5\frac{1}{8}$ and $6\frac{1}{4}$ by the difference of $9\frac{6}{7}$ and $7\frac{5}{6}$; and divide the product by the sum of $84\frac{3}{8}$ and $103\frac{4}{9}$.

6. Two persons *A* and *B* jointly hold $\frac{7}{9}$ of an estate; if *A*'s share exceeds that of *B* by $\frac{3}{4}$ of $\frac{2}{3}$, what are their respective shares?

7. The sum of two numbers is $\frac{5}{9}$ of $16\frac{4}{7}$, and their difference is $4\frac{3}{7}$; find the product of the numbers.

8. What least fraction must be added to $2\frac{1}{3}$ of $9\frac{3}{5}$ to make it an integer?

9. What is that number, $\frac{3}{5}$ of which is $2\frac{7}{40}$?

10. What is that sum of money, $\frac{17}{75}$ of which is R51?

11. What fraction diminished by $5\frac{1}{6}+6\frac{2}{3}+7\frac{1}{4}+15\frac{7}{8}$ is 50?

12. What number is that which, if multiplied by $2\frac{1}{4}$ of $\frac{3}{32}$ of $21\frac{1}{3}$, will give $\frac{3}{8}$ of a unit?

13. Of what number does the half exceed the fifth by 216?

14. How often is $(\frac{2}{5}+\frac{2}{3}-\frac{4}{15})$ contained in $(\frac{4}{7}+\frac{1}{3}-\frac{1}{7})$?

15. What number is that, from which if you deduct $\frac{3}{7}$ of $1\frac{5}{6}$ and to the remainder add $\frac{4}{63}$ of $\frac{7}{8}$, the sum will be 9?

16. Add together the sum, difference, product and quotient of the sum and difference of $\frac{4}{7}$ and $\frac{2}{3}$.

17. By what number must the product of the sum and difference of $5\frac{3}{4}$ and $4\frac{3}{4}$ be divided, to give the quotient $\frac{1}{8}$?

18. Find the number to which if $7\frac{7}{8}$ be added, the sum multiplied by $27\frac{1}{3}$, the product increased by 15, the result divided by $7\frac{1}{2}$, the quotient is 45.

19. Find the number, so that the difference of its third and fifth parts may exceed $\frac{1}{3}$ of the sum of its fourth and sixth parts by 27.

20. What are the nearest integers to $18\frac{4}{7}$ and to $21\frac{5}{8}$? Give reasons for your answers.

21. In 1890 *A* was $15\frac{1}{8}$ years younger than *B* and $17\frac{1}{4}$ years older than *C*, who was $21\frac{1}{2}$ years old in 1850; how old were *A*, *B*, and *C* respectively in 1892?

22. A man bequeathed $\frac{5}{12}$ of his estate to one son, $\frac{4}{7}$ of the remainder to another, and the rest to his wife. What part of the whole does the widow get ?

23. A general after sending $\frac{1}{3}$ of his men to forage in one direction and $\frac{2}{5}$ of them in another, had 960 men left in camp. How many did he command ?

24. In an examination, 600 marks are allotted to four branches: to English $\frac{1}{3}$ of the whole number, to Mathematics $\frac{4}{5}$ of that of English, to Sanskrit $\frac{1}{3}$ of the sum of both, and to History and Geography the rest. Find the number of marks allotted to each branch.

25. A post has $\frac{1}{5}$ stuck in mud, $\frac{2}{5}$ under water, and 10 ft., above water: what is the length of the post ?

26. *A* and *B* were joint owners of an estate, *A*'s share being $\frac{2}{3}$ and *B*'s $\frac{1}{3}$ of the whole. *A* sold $\frac{1}{5}$ of his share to *B* and $\frac{4}{9}$ of the remainder to *C*; what portions of the estate do *A*, *B*, *C*, now respectively possess ?

27. If I pay away $\frac{1}{4}$ of my money, then $\frac{2}{3}$ of the remainder, then $\frac{1}{3}$ of what still remains, and then $\frac{1}{16}$ of the whole; what part of my original money have I still left to myself ?

28. A man buys a piece of land and intends to build on $\frac{29}{64}$ of it, to dig a tank on $\frac{26}{64}$ of it, and to use the remainder which is 120 bighas as a garden. How many bighas were built upon ?

CHAP. XIX. CONCRETE FRACTIONS.

186. The following will illustrate the application of fractional measures to compound quantities.

Examples 74.

1. Add together:

(1) R	a.	p.	(2) R	a.	p.	(3) £	s.	d.	(4) £	s.	d.
13	7	$11\frac{5}{8}$	47	12	$7\frac{1}{3}$	13	11	$4\frac{5}{7}$	15	12	$6\frac{4}{5}$
14	12	$10\frac{7}{8}$	94	13	$1\frac{1}{3}$	18	12	$5\frac{1}{14}$	13	14	$11\frac{2}{5}$
9	14	$9\frac{4}{7}$	44	15	7	19	13	$4\frac{1}{4}$	14	11	$4\frac{3}{5}$
15	12	$11\frac{4}{5}$	11	7	$9\frac{5}{6}$	12	6	$5\frac{5}{6}$	9	12	$6\frac{3}{4}$

2. Perform the following subtractions:

(1) R	a.	p.	(2) R	a.	p.	(3) £	s.	d.	(4) £	s.	d.
15	4	$7\frac{1}{3}$	73	12	$9\frac{1}{3}$	15	12	$9\frac{5}{6}$	45	17	$7\frac{2}{7}$
12	5	$11\frac{1}{3}$	63	13	$10\frac{2}{7}$	14	13	$7\frac{9}{13}$	42	17	$9\frac{3}{5}$

3. Multiply:

(1) R4. 7*a*. $8\frac{2}{3}$*p*. separately by 8, 14, 22, 33.

(2) £15. 9*s*. $7\frac{2}{17}$*d*. " " 9, 16, 34, 51.

(3) 15 mds. 4 sr. $7\frac{1}{18}$ ch. separately by 9, 12, 36.

(4) 5 tons 7 cwt. 1 qr. $12\frac{8}{21}$ lbs. separately by 21, 42, 63.

4 Divide :

(1) R 5 7a. $8\frac{4}{7}p$ separately by 12, 14, 21.

(2) £19 11s. $4\frac{8}{10}d$. „ „ 13, 36, 57.

(3) R17. 14a. $9\frac{1}{2}p$. „ „ 15a. $8\frac{1}{8}p$., and R3. 4a. 6p.

(4) £25. 17s. $7\frac{7}{13}d$. „ „ 19s. $7\frac{7}{9}d$, „ £5. 14s. 9d.

187. To find the value of any fraction of a simple quantity, multiply by the numerator and divide the product by the denominator of the fraction.

Ex. 1. Find the value of $\frac{5}{12}$ of *Rs.* 2.

$\frac{5}{12}$ of *Rs.* 2 = *R.* $\frac{2\times 5}{12}$ = *Re.* $\frac{5}{6}$

∴ the required value = <u>13a 4p.</u>

$$\begin{array}{l} 6)Rs.\ 5(\\ \quad 16 \\ \quad \overline{80}(13a. \\ \quad \underline{78} \\ \quad\ \ 2 \\ \quad \underline{12} \\ \quad 24(4p. \end{array}$$

Ex. 2. Find the value of £$\frac{1}{3}$

£$\frac{1}{3}$ = $\frac{1}{3}$ of £1 = $\frac{20}{3}s.$ = 6s 8d.

The following Table will be of considerable use to the student, and should be learnt by heart.

R $\frac{1}{2}$ = 8a.	R $\frac{1}{4}$ = 4a.;	£$\frac{1}{2}$ = 10s.;	£$\frac{1}{4}$ = 5s.;
„ $\frac{3}{4}$ = 12a	„ $\frac{1}{8}$ = 2a.;	£$\frac{3}{4}$ = 15s.;	£$\frac{1}{8}$ = 2s. 6d.
„ $\frac{1}{3}$ = 5a. 4p.;	„ $\frac{2}{3}$ = 10a. 8p.	£$\frac{1}{3}$ = 6s. 8d.;	£$\frac{2}{3}$ = 13s. 4d.
„ $\frac{1}{12}$ = 1a. 4p.;	„ $\frac{1}{16}$ = 1a.	£$\frac{1}{12}$ = 1s. 8d.;	£$\frac{1}{20}$ = 1s.

£$\frac{1}{5}$ = 4s.; £$\frac{1}{10}$ = 2s.

188. When the given fraction is a mixed number, multiply the simple quantity separately by the integral and fractional parts, and add the products so obtained.

Ex. Find the value of $2\frac{7}{24}$ of 3 maunds.

The required value = 3 mds. × 2 + 3 mds. × $\frac{7}{24}$ = 6 mds. + $\frac{7}{8}$ md.

= $6\frac{7}{8}$ mds = 6 mds. 35 sr. *Ans.*

189. Compound and complex fractions should first be reduced to simple ones, and the above methods then followed.

Ex. 1. Find the values of $\frac{80}{9}$ of $\frac{9}{140}$ of 5 cwt., and of $3\frac{7\frac{1}{2}}{45}$ of 27 ft.

$\frac{80}{9}$ of $\frac{9}{140}$ = $\frac{4}{7}$.

$\frac{4}{7}$ of 5 cwt. = $\frac{20}{7}$ cwt. = $2\frac{6}{7}$ cwt.

= <u>2 cwt. 3 qr. 12 lbs.</u>

$3\frac{7\frac{1}{2}}{45} = 3 \times \frac{15}{2} \times \frac{1}{45} = 3\frac{1}{6}$.

∴ required value = $3\frac{1}{6}$ of 27 ft.

= 3 × 27 ft. + $\frac{1}{6}$ × 27 ft.

= (81 + $4\frac{1}{2}$) ft. = <u>85 ft. 6 in.</u>

Ex. 2. Find the value of $\frac{7}{8}$ of *Re.* 1 + $\frac{2}{3}$ of *Rs.* 2 + $\frac{2}{5}$ of *Rs.* 3.

$\frac{7}{8}$ of *Re.* 1 = $\frac{7}{8}$ × 16a. = 7 × 2a. = 14a.

$\frac{2}{3}$ of *Rs.* 2 = *Rs.* $\frac{4}{3}$ = *Rs.* $1\frac{1}{3}$ = *Re.* 1. 5a. 4p.

$\frac{2}{5}$ of „ 3 = „ $\frac{6}{5}$ = „ $1\frac{1}{5}$ = <u>„ 1. 3a. $2\frac{2}{5}p$.</u>

∴ the required value = *Rs.* 3. 6a. $6\frac{2}{5}p$. *Ans.*

Ex. 3. Find the value of $\frac{3}{8}$ of £1 − $\frac{2}{3}$ of £1 + $\frac{7}{8}$ of £2 − $\frac{1}{3}$ of £2.

$\left.\begin{array}{ll}\frac{3}{8} \text{ of } £1 = & 7s.\ 6d.\\ \frac{7}{8} \text{ of } ,,2 = & £1.\ 15s.\ 0d.\end{array}\right\} = £2.\ 2s.\ 6d.$

$\left.\begin{array}{ll}\frac{2}{3} \text{ of } ,,1 = & 13s.\ 4d.\\ \frac{1}{3} \text{ of } ,,2 = & 13s.\ 4d.\end{array}\right\} = \underline{£1\ \ 6s.\ 8d.}$

∴ the required value = 15*s*. 10*d*. *Ans.*

Examples 75.

1. Find the value of :

(1) $\frac{3}{4}$ of R1 ; $\frac{3}{16}$ of R1 ; $\frac{7}{16}$ of R4 ; $12\frac{2}{3}$ of $\frac{34}{35}$ of R5.

(2) $\frac{3\frac{1}{4}}{39}$ of R9 ; $\frac{7\frac{1}{2}}{4\frac{2}{7}}$ of R15 ; $8\frac{8\frac{1}{4}}{11\frac{1}{2}}$ of R7 ; $4\frac{4\frac{1}{4}}{10\frac{5}{8}}$ of R11.

(3) £$\frac{3}{4}$; $\frac{7}{8}$ of £1 ; $\frac{9}{16}$ of £3 ; $\frac{3}{7}$ of a guinea ; $\frac{7}{14}$ of a moidore ; $2\frac{3}{7}$ of 2 crowns ; $9\frac{3}{4}$ of £3 ; $5\frac{1}{3}$ of £4 ; $\frac{2}{3}$ of $\frac{4}{9}$ of $\frac{3}{26}$ of £13.

(4) $\frac{6\frac{1}{4}}{8\frac{3}{4}}$ of £2 ; $10\frac{2\frac{1}{2}}{8\frac{1}{3}}$ of £7 ; $(4+\frac{5\frac{1}{3}}{1\frac{7}{9}})$ of £4 ; $\frac{2}{3\frac{3}{11}}$ of £13.

(5) $\frac{3}{4}$ of 1 md. ; $\frac{7}{8}$ of 1 md. ; $\frac{9}{14}$ of $3\frac{1}{2}$ mds ; $5\frac{1}{14}$ of 5 mds.

(6) $\frac{3}{13}$ of 1 ℔. Troy ; $\frac{27}{40}$ of 15 dwt. ; $\frac{33}{80}$ of 5 ℔s., Avoir. ; $\frac{13}{30}$ of 7 cwt.

(7) $7\frac{5}{12}$ of 5 yds ; $4\frac{37}{55}$ of 1 mi. ; $7\frac{3}{4}$ of 4 ro. ; $\frac{17}{24}$ of 3 ac.

(8) $\frac{7}{30}$ of 1 bus. ; $7\frac{1}{2}$ of $\frac{1}{14}$ of 3 hhd. ; $3\frac{1}{3}$ of $\frac{2}{5}$ of 1 pipe ; $3\frac{1}{4}$ of 20 gal.

2. Find the value of :

(1) $\frac{2\frac{1}{3}}{1\frac{1}{6}}$ of R2 + $\frac{7\frac{1}{2}}{3\frac{3}{4}}$ of $\frac{1}{8}$ of R2 − $\frac{7\frac{3}{4}}{12\frac{2}{5}}$ of $\frac{7}{8}$ of R1 − $\frac{2\frac{3}{4}}{10}$ of R1.

(2) £$\frac{3}{8}$ + £$\frac{7}{8}$ − $\frac{3}{7}$ of a guinea − $\frac{3}{8}$ of a crown + $2\frac{3}{4}$ of £1 − $\frac{3}{8}$ of 12*s*.

(3) $\frac{3}{50}$ of 1 md + $\frac{9}{16}$ of 1 md − $\frac{7}{28}$ of 3 mds. + $\frac{37}{48}$ of 3 mds.

(4) $\frac{5}{22}$ of 1 mi. − $\frac{3}{33}$ of 1 mi. − $\frac{3}{4}$ of $\frac{3}{4}$ of 7 fur. + $2\frac{9}{35}$ of 1 mi.

(5) $\frac{5}{33}$ of 1 ac. − $\frac{3}{32}$ of 1 ac. + $\frac{3}{4}$ of 3 ro. − $\frac{3}{35}$ of 2 ro.

(6) $\frac{3}{4}$ of 1 bus. − $\frac{5}{6}$ of 1 bus. + $\frac{3}{8}$ of 2 qr. − $\frac{7}{8}$ of 1 bus. + $\frac{1}{4}$ of 3 qr.

(7) $\frac{1}{60}$ of 1 da. − $\frac{3}{35}$ of 7 hrs + $\frac{1}{12}$ of $3\frac{1}{5}$ min. + $\frac{5}{8}$ of $2\frac{3}{5} \times 11\frac{1}{4}$ of 6 sec.

190. To find the value of any fraction of a compound quantity, follow the method shewn in Art. 187.

Ex. 1. Find the value of $\frac{5}{7}$ of £4. 17*s*. 5*d*.

The required value = (£4. 17*s*. 5*d* ÷ 7) × 5

= 13*s*. 11*d*. × 5 = £3. 9*s*. 7*d*. *Ans.*

Ex. 2. Find the value of $5\frac{3}{4}$ of *Rs*. 2. 3*a*. 4*p*.

The required value = *Rs*. 2. 3*a*. 4*p*. × 5 + *Rs*. 2. 3*a*. 4*p*. × $\frac{3}{4}$

= *Rs*. 11. 0*a*. 8*p*. + *Rs*. 1. 5*a*. $2\frac{1}{2}$*p*. = <u>*Rs*. 12. 5*a*. $10\frac{1}{2}$*p*.</u>

191. To divide a compound quantity by a fraction, multiply by the *reciprocal* of the divisor.

Ex. Divide £4. 15s. 4d. by $2\frac{3}{5}$.

£4. 15s. 4d. $\div 2\frac{3}{5}$ = £4. 15s. 4d. $\times \frac{5}{13}$ = £1. 16s. 8d. *Ans.*

Note.—Both in multiplication and in division compound and complex fractions must first be reduced to simple ones.

Examples 76.

1. Multiply :

(1) R7. 14*a.* $7\frac{1}{8}$*p.* separately by $\frac{5}{6}$, $\frac{3}{7}$, $\frac{7}{8}$, $\frac{33}{40}$.

(2) „ 4. 11*a.* $9\frac{5}{8}$*p.* „ „ $2\frac{4}{9}$, $5\frac{1}{6}$, $13\frac{1}{4}$.

(3) £7. 4*s.* $10\frac{3}{5}$*d.* „ „ $\frac{6}{7}$, $\frac{28}{33}$, $3\frac{7}{12}$, $24\frac{1}{8}$.

2. Divide :

(1) R34. 7*a.* $5\frac{1}{8}$*p.* separately by $\frac{5}{6}$, $\frac{3}{3\frac{4}{5}}$, $\frac{37}{45}$, $\frac{37}{60}$.

(2) „ 17. 11*a.* $4\frac{2}{3}$*p.* „ „ $5\frac{5}{9}$, $21\frac{3}{8}$, $41\frac{4}{7}$.

(3) £19. 17*s.* $7\frac{1}{8}$*d.* „ „ $\frac{9}{80}$, $3\frac{3}{8}$, $5\frac{1}{7}$, $19\frac{2}{7}$.

3. Simplify :

(1) $2\frac{4}{5}$ of R7. 14*a.* 7*p.* ; $\frac{1}{8}$ of $\frac{3}{8}$ of R19. 10*a.* 8*p.*

(2) $\frac{3}{4\frac{2}{3}}$ of £14. 6*s.* 8*d.* ; $6\frac{1}{4}$ of £11. 12*s.* 6*d.* ; $9\frac{1}{5}$ of £15. 8*s.* 8*d.*

(3) $3\frac{7}{8}$ of 15 mds. 20 sr. ; $\frac{7}{9}$ of $2\frac{1}{4}$ of 16 mds. 14 sr. 8 ch.

(4) $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of 3 tons ; $\frac{7}{8}$ of $\frac{3}{76}$ of $\frac{22}{35}$ of 4 tons 15 cwt.

(5) $\frac{23}{40}$ of 1 mi. 3 fur. ; $5\frac{2}{3}$ of $2\frac{1}{3}$ of 7 mi. 5 fur. 35 po. 4 yds. 2 ft. 7 in.

(6) $7\frac{3}{34}$ of 15 da. 11 hrs. 30 min. ; $4\frac{8}{25}$ of 16 wks. 10 da. 12 hrs.

4. Find the value of :

(1) $\frac{3}{5}$ of R7. 14*a.* + $\frac{4}{9}$ of R19. 3*a.* 6*p.* − $\frac{3}{8}$ of R3. 4*a.* − $\frac{3}{8}$ of R2.

(2) $5\frac{3}{5}$ of R25. 3*a.* 4*p.* + $\frac{3}{5}$ of $\frac{5}{9}$ of R10. 8*a.* + $\frac{4}{9}$ of $2\frac{3}{4}$ of R5 − $7\frac{1}{4}$ of R12. 10*a.* 8*p.*

(3) $12\frac{4}{7}$ of £21. 14*s.* $7\frac{7}{25}$*d.* − $4\frac{3}{8}$ of £7. 14*s.* $7\frac{1}{4}$*d.* + $4\frac{7}{25}$ of 9*s.* $7\frac{1}{2}$*d.* − $8\frac{1}{3}$ of £8. 9*s.* $10\frac{1}{3}$*d.*

(4) $\frac{9}{64}$ of £1. 12*s.* − $\frac{3}{8}$ of 10*s.* 6*d.* + $\frac{7}{8}$ of a moidore − $\frac{1}{3}$ of a guinea.

(5) $\frac{7}{30}$ of 3 mds. 2 sr. 8 ch. − $\frac{24}{35}$ of 5 mds. 25 sr. + $\frac{23}{25}$ of 17 mds. 8 sr.

(6) $\frac{7}{8}$ of 1 ton 6 cwt. + $\frac{7}{8}$ of 14 tons − $\frac{3}{7}$ of 10 cwt. − $\frac{3}{4}$ of 3 qr.

192. To reduce quantities of lower denominations to fractions of a higher denomination of the same kind, the operation should be performed as follows.

Ex. 1. Reduce 6*p.*, 2*a.* 6*p.*, 5*a.* 4*p.*, to fractions of a rupee.

$$\left.\begin{aligned} 6p. &= \tfrac{6}{12}a. = \tfrac{1}{2}a. = \tfrac{1}{2} \times Re.\tfrac{1}{16} = Re.\tfrac{1}{32};\\ 2a.\ 6p. &= 2a. + \tfrac{6}{12}a. = 2\tfrac{1}{2}a. = \tfrac{5}{2} \times \text{,,}\ \tfrac{1}{16} = \text{,,}\ \tfrac{5}{32};\\ 5a.\ 4p. &= 5a. + \tfrac{4}{12}a. = 5\tfrac{1}{3}a. = \tfrac{16}{3} \times \text{,,}\ \tfrac{1}{16} = \text{,,}\ \tfrac{1}{3}. \end{aligned}\right\} Ans.$$

Ex. 2. Reduce 6*d*., 2*s*. 6*d*., 6*s* 8*d*., to pounds.

$$\left.\begin{array}{l} 6d. = \frac{1}{2}s. = \frac{1}{2} \times £\frac{1}{20} = £\frac{1}{40}\,; \\ 2s.\ 6d. = 2\frac{1}{2}s. = \frac{5}{2} \times\ ,,\ \frac{1}{20} =\ ,,\ \frac{1}{8}\,; \\ 6s.\ 8d. = 6\frac{2}{3}s. = \frac{20}{3} \times\ ,,\ \frac{1}{20} =\ ,,\ \frac{1}{3}. \end{array}\right\} \textit{Ans.}$$

Ex. 3. Express *Rs*.7. 13*a*. 6*p*. as a fraction of a rupee.

$13a.\ 6p. = 13\frac{1}{2}a. = \frac{27}{2} \times \frac{1}{16}Re. = \frac{27}{32}\ Re.$:

$\therefore\ Rs.7.\ 13a.\ 6p. = Rs.7\frac{27}{32}.$ *Ans.*

Ex. 4. Express £10. 17*s*. 6*d*. in pounds.

$17s.\ 6d = 17\frac{1}{2}s. = \frac{35}{2} \times £\frac{1}{20} = £\frac{7}{8}$;

$\therefore\ £10.\ 17s.\ 6d. = £10\frac{7}{8}$ *Ans.*

Ex. 5. Reduce 7 mi. 330 yds. to miles.

7 mi. 330 yds. $= 7\text{m.} + \frac{330}{1760}$ mi. $= 7\frac{3}{16}$ miles. *Ans.*

Examples 77.

1. Express 1*a*. 6*p*. ; 2*a*. 9*p*. ; 3*a* 7*p*. ; 8*a*. $4\frac{1}{2}p.$; 13*a*. 11*p*. ; 9*a*. $5\frac{1}{3}p.$; 12*a*. $1\frac{1}{3}p.$; each as a fraction of a rupee.

2. Reduce 1*s*. $7\frac{1}{2}d$; 5*s*. 4*d*. ; 7*s* 8*d* ; 15*s*. $4\frac{1}{2}d.$; 13*s*. 9*d*. ; 12*s*. 6*d*. ; 11*s*. 11*d*. ; each to a fraction of a pound.

3. Express R7. 11*a*. 4*p*., R13. 14*a* 8*p*., R6. 7*a*. in rupees.

4. Express £13. 13*s*. 5*d*., £4. 11*s*. 9*d*., £12. 4*s*. $6\frac{1}{3}d.$ in pounds.

5. Reduce 5 mds. 13 sr. $5\frac{1}{8}$ ch. to maunds ; and 5 tons 15 qr. to tons.

6. Express 5 mi. 220 yds. as miles ; and 13 hhd. 27 gal. as pipes.

193. *To express a simple or a compound quantity as the fraction of another of the same kind.*

RULE. Reduce both the quantities to the same denomination, and divide the first by the second.

Ex. 1. Express *Re*.1. 3*a*. 4*p*. as the fraction of *Rs*.7. 4*a*.

Re.1 3*a*. 4*p*. = 232*p*. : and *Rs*.7 4*a*. = 1392*p*.

$\therefore$ the reqd. fraction $= \frac{232}{1392} = \frac{1}{6}.$ *Ans.*

Ex. 2. What fraction of 2 guineas is $\frac{7}{8}$ of £1 ?

$\frac{7}{8}$ of $£1 = £\frac{7}{8}$; and 2 guineas $= £2.\ 2s. = £2\frac{1}{10} = £\frac{21}{10}.$

$\therefore$ the required fraction $= \frac{7}{8} \div \frac{21}{10} = \frac{5}{12}.$ *Ans.*

Ex. 3. What part is 7*s*. 6*d*. of 13*s*. 4*d*. ?

7*s*. 6*d*. = 90*d*. ; and 13*s*. 4*d*. = 160*d*.

$\therefore$ the required fraction $= 90 \div 160 = \frac{9}{16}.$ *Ans.*

Ex. 4. How often is 2 mi. 5 fur. contained in 1 mi. 7 fur. ?

2 mi. 5 fur = 21 fur. and 1 mi. 7 fur. = 15 fur.

$\therefore$ the result required $= 15 \div 21 = \frac{5}{7}.$ *Ans.*

Ex. 5. Reduce $\frac{5}{16}$ of £5 + $\frac{8}{9}$ of £7. 17*s*. 6*d*. + $\frac{5}{7}$ of £6. 14*s*. 2*d*. to the fraction of £26. 14*s*. 2*d*.

$\frac{5}{16}$ of £5 = £1. 11*s*. 3*d*.; $\frac{8}{9}$ of £7. 17*s*. 6*d*. = £7; and $\frac{5}{7}$ of £6. 14*s*. 2*d*. = £4. 15*s*. 10*d*.

£1. 11*s*. 3*d*. + £7 + £4. 15*s*. 10*d*. = £13. 7*s*. 1*d*. = £$13\frac{17}{48}$ = £$\frac{641}{48}$;
also £26. 14*s*. 2*d*. = $26\frac{17}{24}$ = £$\frac{641}{24}$.

∴ the required fraction = $\frac{641}{48} \div \frac{641}{24} = \frac{1}{2}$. *Ans.*

Examples 78.

1. Express 11*a*. 6*p*. as the fraction of R1. and of R3.

2. Express R2. 1*a*. 4*p*. as the fraction of R3. 2*a*. and of R6. 4*a*.

3. Express R26. 10*a*. 8*p*. as the fraction of R41. 10*a*. 8*p*. and of R20. 13*a*. 4*p*.

4. Express £4. 13*s*. 9*d*. as the fraction of £5. 17*s*. 6*d*. and of £6. 5*s*.

5. Reduce £3. 12*s*. 9*d*. to the fraction of £7. 8*s*. 8*d*. and of £16. 6*s*.

6. Reduce 3 mds. 36 sr. 4 ch to the fraction of 5 mds. 34 sr. 6 ch. and of 12 mds. 20 sr.

7. Express 15 cwt. 3 qr. 21 lb. as the fraction of 3 tons 19 cwt. 2 qr. 21 lb.

8. Express 12 mi, 3 fur. 40 yds. as the fraction of 14mi. 7fur. 110yds.

9. What part is 15 da. 18 hr. 30 min. of 27 da. 19 hr. 45 min.?

10. Reduce 21 gal. 3 qt. 1 pt to the fraction of 1 hhd. 14 gal.

11. What fraction is $\frac{1}{7}$ of £2. 2*s*. 7*d* of 6*s*. 8*d*., and of 13*s*. 4*d*.?

12 What fraction of £3. 11*s* 3*d*. is $\frac{7}{12}$ of £2. 11*s*. 6*d*.?

13. What part is $\frac{13}{17}$ of R4. 5*a*. 5*p*. of $\frac{19}{22}$ of R15. 10*a*. 1*p*.?

14. What fraction is 2 mds. 20 sr. of $\frac{7}{18}$ of 4 mds. 5 sr. 6 ch.?

15. What part is $\frac{7}{11}$ of 3 ro. 5 po. of $\frac{5}{8}$ of an acre?

16 What part of 14 gal. is $\frac{7}{9}$ of a pint?

17. What fraction of a week is $\frac{11}{25}$ of 5 days 20 hours?

18. What part of a guinea is $\frac{7}{13}$ of 5*s*. 5*d*.?

19. Reduce $2\frac{3}{4}$ of $\frac{5}{9}$ of 1 lb. Troy to the fraction of 1 lb. Avoir.

20. Reduce the difference between 3 tons 10 cwt. 1 qr. and 1 ton 11 cwt. 5 lbs. to the fraction of 2 tons.

21. How often is R7. 15*a*. 4*p*. contained in R3. 15*a*. 8*p*.?

22. How often is £15. 17*s*. 8*d*. contained in £3. 19*s*. 5*d*.?

23. How often is 17 mds. 35 sr. 11 ch. contained in 11mds. 37sr. 2ch.

24 How often is 12 cwt. 2 qr. contained in 5 cwt. and in 1 ton?

25. What part is $\frac{5}{33}$ of $4\frac{2}{5}$ of $\frac{2}{3}$ of R7. 12*a*., of $\frac{3}{11}$ of R13. 4*a*.?

CHAP. XX. MISCELLANEOUS PROPOSITIONS.

194. **The Unitary Method.** We have already given the student some idea of this method. The following solutions will, we hope, make the method still more clear to him.

Ex. 1. If a maund of rice costs *Rs.*4, what is the cost of 75 mds. 30 sr.?

75 mds. 30 sr. $= 75\frac{3}{4}$ mds.; also the cost of 1 md. $=$ *Rs.* 4.

$\therefore$ the cost of $75\frac{3}{4}$ mds. $=$ *Rs.* $4 \times 75\frac{3}{4} =$ *Rs.* $4 \times 75 +$ *Rs.*$4 \times \frac{3}{4} =$ *Rs.* $\underline{303}$.

Ex. 2. If a piece of cloth be 30 yds. 2 ft. 9 in. long, what will be the length of $15\frac{3}{4}$ such pieces?

The length of 1 piece $=$ 30 yds. 2 ft. 9 in.

$\therefore$ " " $15\frac{3}{4}$ " $=$ 30 yds. 2 ft. 9 in. $\times\, 15\frac{3}{4} =$ $\underline{\text{484 yds. 1 ft. 1 in.}}$

Ex. 3. If the cost of $6\frac{1}{4}$ yds. be *Rs.* 37. 8*a.*, find the cost of 1 yd. of the same quality.

The required cost per yd. $=$ *Rs.* 37. 8*a.* $\div 6\frac{1}{4} =$ *Rs.* 6. *Ans.*

Ex. 4. If $3\frac{1}{2}$ yds. of silk cost 4*s.* 8*d.*, how many yds. can be bought for £49. 1*s.* 4*d.*?

$3\frac{1}{2}$ yds. cost 4*s.* 8*d.*; $\therefore$ 1 yd. costs (4*s.* 8*d.* $\div 3\frac{1}{2}$) or 1*s.* 4*d.*;

$\therefore$ the required number of yds. $=$ £49. 1*s.* 4*d.* $\div$ 1*s.* 4*d.* $=$ 736. *Ans.*

Ex. 5. If 18 mds. 20 sr. of sugar cost *Rs.* 263. 10*a.*, what will 56 mds. 30 sr. cost?

The cost of $18\frac{1}{2}$ mds. $=$ *Rs.* 263. 10*a.*;

$\therefore$ " " 1 " $=$ " 263. 10*a.* $\div 18\frac{1}{2} =$ *Rs.* 14. 4*a.*

$\therefore$ " " $56\frac{3}{4}$ " $=$ " 14. 4*a.* $\times\, 56\frac{3}{4} =$ " 808. 11*a.* *Ans.*

Ex. 6. If $\frac{7}{18}$ of an estate be worth *Rs.* 2100, what is $\frac{5}{9}$ of it worth?

$\frac{7}{18}$ of the estate costs *Rs.* 2100;

$\therefore$ the whole estate costs *Rs.* $2100 \div \frac{7}{18} =$ *Rs.* 5400.

$\therefore$ $\frac{5}{9}$ of the estate costs *Rs.* $5400 \times \frac{5}{9} =$ *Rs.* 3000. *Ans.*

Ex. 7. If $15\frac{3}{4}$ mds. be carried $32\frac{4}{7}$ miles for *Rs.*8, how far will $12\frac{2}{3}$ mds. be carried for the same sum?

$15\frac{3}{4}$ mds. are carried $32\frac{4}{7}$ mi.;

$\therefore$ 1 md. should be carried $(32\frac{4}{7} \times 15\frac{3}{4})$ mi. $=$ 513 mi.;

$\therefore$ $12\frac{2}{3}$ mds. should be carried 513 mi. $\div 12\frac{2}{3} = 40\frac{1}{2}$ miles. *Ans.*

Ex. 8. If the four-penny loaf weighs 3 lbs. when wheat is £5 a quarter, what should it weigh when wheat is £6. 5*s.* a quarter?

When wheat is 100*s.* a quarter, the weight of the loaf is 3 lbs.;

$\therefore$ " " is 1*s.* " " should be 300 lbs.;

and " " " 125*s.* " " " $\frac{300}{125}$ lbs.;

$\therefore$ the required weight $= \frac{300}{125}$ lbs. $= 2\frac{2}{5}$ lbs. *Ans.*

Ex. 9. *A* borrowed of *B*, *Rs.* 1600 for $4\frac{1}{2}$ months; for how long should *A* lend *B Rs.* 2400 in return?

Lending *Rs.* 1600 for $4\frac{1}{2}$ mo = lending *Rs.* 100 for $(16 \times \frac{9}{2})$ or 72 mo.;

$\therefore$ *Rs.* 2400 should be lent for $\frac{72}{24}$ or 3 mo. *Ans.*

Ex. 10. If the monthly expenditure of a family be *Rs.* 50 when rice is 20 seers the rupee, and *Rs.* 48 when rice is 24 seers the rupee, what should the expenditure be when rice is 30 seers the rupee?

The price of 1 sr. being first reduced from $R\frac{1}{20}$ to $R\frac{1}{24}$, *i.e.* by $R\frac{1}{120}$ there is a saving of *Rs.* (50 − 48) or *Rs.* 2 in the expenditure.

The price of 1 sr. is ultimately reduced by $(\frac{1}{20} - \frac{1}{30})$ or $\frac{1}{60}$ *Re.*

Now $\frac{1}{60} = 2 \times \frac{1}{120}$; $\therefore$ the final saving = *Rs.* 2×2 = *Rs.* 4.

$\therefore$ the required expenditure will be *Rs.* (50 − 4) or *Rs.* 46. *Ans.*

Ex. 11. If a garrison of 2000 men having provisions for 120 days be reinforced after 8 days by 1200 men, how long will the remaining provisions last?

After 8 days, 2000 men have provisions for 112 days;

$\therefore$ 200 men have provisions for 112×10 days;

$\therefore$ 3200 men have provisions for $\frac{112 \times 10}{16}$ or 70 days.

Examples 79.

1. If the length of a piece of cloth be 27 yds. 1 ft. 8 in., what is the length of $25\frac{3}{8}$ such pieces?

2. If a rupee can buy 3 mds. 16 sr. 12 ch. of coal, how much can R25. 8*a*. buy?

3. If 5 tons 16 cwt. of goods be carried 30 miles for £1, what weight ought to be carried the same distance for £56. 15*s*.?

4. A man can walk 3 mi. 220 yds. in an hour: how far will he walk in 15 days, supposing that he rests 14 hours each day?

5. The wages of a servant for a week is 5*s*. $6\frac{1}{4}$*d*.; how much must he receive for $42\frac{2}{3}$ weeks' service?

6. If the cost of painting $256\frac{1}{16}$ sq. ft. be £5. 7*s*. $0\frac{7}{12}$*d*., how much is it per square foot?

7. If a maund of sugar cost R11. 5*a*. 4*p*., what quantity can be bought for R186. 4*a*. 8*p*.?

8. The length of an iron bar is 16 ft. $7\frac{1}{4}$ in.: how many bars placed in a straight line will be 443 ft. 4 in. long?

9. The value of 5 tons 6 cwt. of wheat is £57. 8*s*. 4*d*.: find that of 16 tons 15 cwt.

10. If $26\frac{1}{4}$ acres of land be rented for £114. 16*s*. 8*d*. per annum, what will 32 ac. 2 ro. be rented for?

11. If 150 yards of velvet be sold for £81. 13*s*. 4*d*, how much can I buy for £111 13*s* $5\frac{1}{4}$*d*. ?

12. If the daily wages of 11 carpenters amount to ₨7. 3*a*. $7\frac{3}{8}$*p*., what will be the daily wages of 72 carpenters ?

13. What is the value of 729 apples at $11\frac{1}{4}$*d*. per score ?

14. If $15\frac{3}{4}$ lbs. of tea cost £2. 15*s*. $1\frac{1}{2}$*d*., what will $2\frac{5}{7}$ cwt. cost ?

15. If $\frac{5}{7}$ of a md. cost ₨10. 5*a*., what will 8 sr. cost ?

16. What quantity of sugar can be bought for ₨2. 13*a*., when $11\frac{1}{4}$mds. cost ₨132. 3*a*. ?

17. If an estate of ₨25600 yield an income of ₨128 per month, what ought to be the value of another estate which yields an income of ₨2560. 8*a*. per month ?

18. If 16 men or 24 boys earn ₨28 in a certain time, how much will 10 men and 14 boys earn in the same time ?

19. A train runs six times as fast as a tram car, which goes 88 ft. in every 15 sec. ; how far can the train run in 2 hrs. 15 min. ?

20. How many rupees must be paid for a bill of £802. 17*s*. $8\frac{1}{4}$*d*., when another of 16*s*. $0\frac{1}{3}$*d*. is paid up with ₨11 ?

21. A dealer in cloth uses a yard measure which is half an inch too short : by how much will a customer be cheated who buys 51 yds. ?

22. If the penny loaf weighs 8 oz when wheat is £5. 8*s*. a quarter, what should it weigh when wheat is £5 4*s*. a quarter ?

23. If the two-anna loaf weighs 8 ch. when wheat is ₨4. a maund, what should its weight be when wheat is ₨$5\frac{1}{3}$ a maund ?

24. If the expenses of a family of 12 persons for 3 months amount to ₨240, how long will the same sum support a family of 9 persons ?

25. I borrowed of *B*, ₨4500 for 16 months : what sum must I lend him for 18 months to repay the obligation ?

26. A garrison of 800 men have provisions for 72 days, and after 18 days are reinforced by 400 : how long will the remaining provisions last ?

27. In a fort there are provisions for 60 days at $1\frac{1}{2}$ sr. daily per head : what should each man's allowance be, if they are to last 80 days ?

28. If 35 men can complete a piece of work in 6 weeks, how many men must be added to the number after a week and a half to have it completed in 24 days ? (A week contains 6 working days.)

29. If 16 men or 28 boys can do a piece of work in 10 days, in what time can 12 men and 14 boys do a piece of work thrice as great ?

30. If the cost of maintaining a family be ₨220 a month when rice is ₨3 a maund and ₨240 when rice is 12 seers the rupee, what will the cost be when rice is only 10 seers the rupee ?

195. **Cisterns.** If one or more pipes fill a cistern in 6 min., they fill $\frac{1}{6}$th of it per min. ; and *conversely*, if they fill $\frac{1}{6}$th of it per min., they fill the whole in 6 min. Similarly, if

they fill a cistern in $7\frac{1}{2}$ hrs., they fill $(1 \div 7\frac{1}{2})$ or $\frac{2}{15}$ of it in 1 hr.; and *conversely*, if they fill $\frac{2}{15}$ of it per hour, they will fill the cistern in $(1 \div \frac{2}{15})$ or $7\frac{1}{2}$ hrs.

Ex. 1. Two pipes can fill a cistern in 10 and 20 minutes, respectively; how soon will the cistern be filled, if both pipes are open?

The first pipe fills $\frac{1}{10}$ per min. and the second $\frac{1}{20}$ per min.; therefore they together fill $\frac{1}{10}+\frac{1}{20}=\frac{3}{20}$ per min. Hence they will fill the cistern in $(1 \div \frac{3}{20})$ min., or $\frac{20}{3}$ min, or 6 min. 40 sec. *Ans.*

Ex. 2. Two pipes can separately fill a cistern in $7\frac{1}{2}$ min., and 9 min., and an escape pipe can empty it in $5\frac{2}{5}$ min. If all three are opened, how long will it be before the cistern is full?

The first pipe *fills* in 1 min. $(1 \div 7\frac{1}{2})$ or $\frac{2}{15}$ of the cistern;
,, second ,, ,, ,, $(1 \div 9)$ or $\frac{1}{9}$,, ,, ;
,, third ,, *empties* ,, $(1 \div 5\frac{2}{5})$ or $\frac{5}{27}$,, ,, ;

$\therefore$ if all three be opened, there will be filled every minute $\frac{2}{15}+\frac{1}{9}-\frac{5}{27}$ or $\frac{8}{135}$; $\therefore$ the cistern will be filled in $(1 \div \frac{8}{135})$ min. or $16\frac{7}{8}$ min. *Ans.*

Ex. 3. Two pipes can separately empty a reservoir in $4\frac{2}{3}$ hours and 7 hours. They are both opened until the reservoir is half empty, when the first pipe is turned off; when will the reservoir become empty?

The first pipe empties $(1 \div 4\frac{2}{3})$ or $\frac{3}{14}$ and the second $(1 \div 7)$ or $\frac{1}{7}$ in one hour. Therefore they together empty $(\frac{3}{14}+\frac{1}{7})$ or $\frac{5}{14}$ in 1 hour, or the whole in $\frac{14}{5}$ hrs, or half in $\frac{7}{5}$ hrs. or 1 hr. 24 min

Again the second pipe empties the whole in 7 hrs. and therefore half in $3\frac{1}{2}$ hrs. Therefore the whole time taken is 1 hr. 24 min. + 3 hrs. 30 min. = 4 hrs. 54 min. *Ans.*

Examples 80.

1. A water tub can be filled by one pipe in 10 hrs. and by another in 20 hrs. In what time will the tub be filled if both be opened at once?

2. A cistern can be filled by one pipe in 15 min. and emptied by another in 25 min.: if both the pipes be opened together, in how many minutes will the cistern be filled?

3. If $\frac{1}{8}$ of a cistern can be filled by one pipe in $2\frac{2}{5}$ days, and $\frac{3}{8}$ of it by another in $4\frac{1}{2}$ days; in what time will it be filled when both are opened together?

4. Two pipes can fill a cistern in 15 min. and 20 min. respectively, and a third can empty it in 18 min., what part of it will be filled in 5 min., all three being opened together?

5. A tank has 3 pipes; by the first $\frac{3}{4}$ of it can be filled in 4 hours by the second $\frac{1}{8}$ in 3 hrs., and by the third $\frac{3}{4}$ of it can be emptied in 5 hrs. If all three be opened at once, how long will it be before it is full?

6. When $\frac{5}{8}$ths of a reservoir is full, two pipes are opened, one of which can fill it in 30 min., while the other can empty it in 20 min.; when will the cistern become empty?

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7. There are two pipes attached to a water tub: after one has filled $\frac{3}{4}$ of it in 24 min., the other is opened, and then the tub becomes empty in 24 min. In what time can the second pipe empty the full tub?

8. Two pipes can fill a cistern in 25 and 30 minutes, respectively. Both pipes being opened, find when the first pipe must be turned off, that the cistern may just be filled in 15 min.

9. Two pipes can fill a cistern in 30 and 36 hours respectively. The two pipes are opened together, and after 5 hours the first is stopped. In how many hours more will the cistern become full?

10. A cistern has three pipes *A*, *B* and *C*; *A* and *B* can separately fill it in 8 and 12 hours, and *C* can empty it in 3 hours. The pipes are opened successively at 7, 8, and 9 A.M.; when will the cistern be empty?

11. Two pipes can separately fill a cistern in 6 and 8 hours, and an escape pipe can empty it in 4 hours. The three pipes are opened at 1, 2, and 3 P.M. respectively; when will the cistern be full.

12. Two pipes can respectively fill a tank in 8 and 10 hours; but when the discharging pipe is opened, they take $13\frac{1}{3}$ hrs. to fill it together. In what time can the discharging pipe empty the tank?

13. *A* and *B* are two supplying pipes and *C* a waste pipe all attached to a tank. *A* can fill it in 25 hrs. and *B* in 30 hrs. The empty tank takes 15 hrs. to fill when all three are opened at once. In what time will *C* empty it, *A* and *B* being closed just as it becomes full?

196. **Work and Time.** The following points should be remembered in working out problems on **Work** and **Time.**

(1) If one or more men can do a piece of work in **4** days they can do $\frac{1}{4}$ of the work in 1 day; so, if a piece of work can be done in $3\frac{1}{2}$ days, $(1 \div 3\frac{1}{2})$ can be done in 1 day; *conversely*, if $\frac{2}{7}$ths of a piece of work is done in 1 day, the whole work will be done in $(\mathbf{1} \div \frac{2}{7})$ days.

(2) If one man do a piece of work in **5** days, then he will do $\frac{1}{5}$th of the work in one day; hence 2 men will do $\frac{2}{5}$ths of the work in 1 day, and therefore the whole work $(1 \div \frac{2}{5})$ or $(5 \div 2)$ days. So if **1** man can do a piece of work in **10** days, **3** men will do the work $(\mathbf{10} \div \mathbf{3})$ days.

(3) If a man can do $\frac{2}{7}$ of a piece of a work in **4** days, he can do $\frac{1}{4}$ of $\frac{2}{7}$ of it in 1 day, and therefore the whole work in $(1 \div \frac{1}{4}$ of $\frac{2}{7})$ or $(\mathbf{4} \div \frac{2}{7})$ days.

(4) If **4** men can do a piece of work in **3** days, then they will do $\frac{1}{3}$ of the work in one day; therefore 1 man will do $\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$ of the work in 1 day, or the whole work in 12 or $(\mathbf{4} \times \mathbf{3})$ days.

Ex. 1. **A can do a piece of work in 4 days, and *B* in 6 days; in what time can *A* and *B* together do it?**

A can do $\frac{1}{4}$ of the work in 1 day; *B* can do $\frac{1}{6}$ of it in 1 day;

∴ *A* and *B* together can do $(\frac{1}{4}+\frac{1}{6})$ or $\frac{5}{12}$ of the work in 1 day.

∴ they can together do the whole in $(1 \div \frac{5}{12})$ or $2\frac{2}{5}$ days. *Ans.*

Ex. 2. *M* can alone do a piece of work in 10 days, but with the help of *N* he can do it in $7\frac{1}{2}$ days; in what time can *N* alone do the work?

M can do the work in 10 days; ∴ he can do $\frac{1}{10}$ of it in 1 day or $(\frac{1}{10}\times 7\frac{1}{2})$ or $\frac{3}{4}$ of it in $7\frac{1}{2}$ days; ∴ *N* can do $(1-\frac{3}{4})$ or $\frac{1}{4}$ of it in $7\frac{1}{2}$ days, or the whole in $(7\frac{1}{2}\div\frac{1}{4})$ or 30 days. *Ans.*

Otherwise thus:

M can do the work in 10 days: ∴ he can do $\frac{1}{10}$ of it in 1 day;

M and *N* can do the work in $7\frac{1}{2}$ days, ∴ they can do $\frac{2}{15}$ of it in 1 day.

∴ *N* can do $(\frac{2}{15}-\frac{1}{10})$ or $\frac{1}{30}$ of work in 1 day;

∴ he can do the whole work in $(1\div\frac{1}{30})$ or 30 days. *Ans.*

Ex. 3. *M* and *N* can do a piece of work in 50 days and 36 days, respectively. They begin together, but after 12 days *N* goes away. After 5 days, *P* is called to help *M*, and the work is finished in 8 days more. In what time can *P* do it alone?

M works with *N* for 12 days, alone for 5 days, and with *P* for 8 days, *i.e.*, for 25 days in all; also *N* works for 12 days, and *P* for 8 days.

M can do $\frac{1}{50}$ of the work in 1 day; ∴ he does $\frac{1}{2}$ of it in 25 days;
N can do $\frac{1}{36}$ " " " ; ∴ " $\frac{1}{3}$ " 12 " ;
∴ *P* does $\{1-(\frac{1}{2}+\frac{1}{3})\}$ or $\frac{1}{6}$ of the work in 8 days;
∴ he can do the work in (8×6) or 48 days. *Ans.*

Ex 4. *M* can do a piece of work in 8 days and *N* in 12 days. They begin together, but *M* goes away 2 days before the work is finished. How long did the work last?

It is clear that *M* leaves when there remains such a portion of the work as can be done by *N* in 2 days.

Since *N* can do $\frac{1}{12}$ of the work in 1 day; ∴ he can do $\frac{1}{6}$ of it in 2 days; ∴ *M* and *N* have together done $(1-\frac{1}{6})$ or $\frac{5}{6}$ of the work.

But as *M* can do $\frac{1}{8}$ of the work in 1 day and *N*, $\frac{1}{12}$; ∴ they can together do $(\frac{1}{8}+\frac{1}{12})$ or $\frac{5}{24}$ in 1 day, and ∴ $\frac{5}{6}$ of it in $(\frac{5}{6}\div\frac{5}{24})$ or 4 days.

∴ the work lasted for $(4+2)$ or 6 days. *Ans.*

Ex. 5. *A* can do a piece of work in 6 days, *B* in 8 days, and *C* in 12 days. They all begin the work together; but *A* leaves after 2 days, and *B*, 3 days before the work is finished. How long did the work last?

A worked for 2 days only and, therefore, did $\frac{2}{6}$ or $\frac{1}{3}$ of the work. Hence the remaining $(1-\frac{1}{3})$ or $\frac{2}{3}$ must have been done by *B* and *C*. Now, *C* worked 3 days more than *B* and during this time did $\frac{3}{12}$ or $\frac{1}{4}$ of the work. Therefore, they together did only $(\frac{2}{3}-\frac{1}{4})$ or $\frac{5}{12}$ of the work. But since *B* does $\frac{1}{8}$ and *C* $\frac{1}{12}$ of the work in 1 day, they can together do $\frac{5}{24}$ of the work in 1 day. Therefore they took $(\frac{5}{12}\div\frac{5}{24})$ or 2 days in doing $\frac{5}{12}$ of it. Therefore, the whole time taken = the number of days that they worked together + the number of days that *C* worked alone = 2 days + 3 days = 5 days. *Ans.*

Otherwise: A did in 2 days $\frac{2}{6}$ or $\frac{1}{3}$ of the work. Therefore B and C did $(1-\frac{1}{3})$ or $\frac{2}{3}$ of the work. Now, if B had also worked for the last 3 days, he would have done $\frac{3}{8}$ of the work, and therefore, the two would together have done $(\frac{2}{3}+\frac{3}{8})$ or $\frac{25}{24}$ of work. But they do $\frac{5}{24}$ in 1 day; therefore they would have taken $(\frac{25}{24}\div\frac{5}{24})$ or 5 days. *Ans.*

Ex. 6. A can do a piece of work in 9 days, B in 12 days, and C in 18 days. They all begin together. A leaves the work 3 days, and B 2 days, before the work is finished. How long did the work last?

A can do in 1 day $\frac{1}{9}$ of the work, B in 1 day $\frac{1}{12}$, and C in 1 day $\frac{1}{18}$;
$\therefore$ they can together do $(\frac{1}{9}+\frac{1}{12}+\frac{1}{18})$ or $\frac{1}{4}$ of the work in 1 day.

Now if A and B had not left, they would have done $(\frac{3}{9}+\frac{2}{12})$ or $\frac{1}{2}$ of the work more: *i.e.*, they would together have done $(1+\frac{1}{2})$ or $\frac{3}{2}$ of the work;

$\therefore$ the required number of days $=\frac{3}{2}\div\frac{1}{4}=6$. *Ans.*

Ex. 7. P and Q can do a piece of work in $5\frac{1}{7}$ days, P and R in 6 days, and Q and R in $7\frac{1}{5}$ days. In what time can they do it all working together, and also each working singly?

P and Q can together do the work in $5\frac{1}{7}$ days, or $\frac{7}{36}$ of it in 1 day;
P ,, R ,, ,, ,, 6 ,, $\frac{1}{6}$,, ,, ;
Q ,, R ,, ,, ,, $7\frac{1}{5}$,, $\frac{5}{36}$,, ,, ;

$\therefore$ 2 men like P, assisted by 2 men like Q and 2 men like R, can do $(\frac{7}{36}+\frac{1}{6}+\frac{5}{36})$ or $\frac{1}{2}$ of the work in 1 day. Therefore the three men can together do $\frac{1}{4}$ of the work in 1 day, or the whole work in 4 days;
Also, P can do $(\frac{1}{4}-\frac{5}{36})$ or $\frac{1}{9}$,, ,, ,, 9 ,, ;
Q ,, ,, $(\frac{1}{4}-\frac{1}{6})$ or $\frac{1}{12}$,, ,, ,, 12 ,, ;
R ,, ,, $(\frac{1}{4}-\frac{7}{36})$ or $\frac{1}{18}$,, ,, ,, 18 ,, ; *Ans.*

Ex. 8. If 15 men can dig 10 loads of earth in 15 hrs. and 18 others can dig 12 loads in 16 hrs.; in what time can they dig 170 loads working together?

The first set of men can dig $\frac{10}{15}$ or $\frac{2}{3}$ load in 1 hour; the second set $\frac{12}{16}$ or $\frac{3}{4}$ load in 1 hour.

$\therefore$ they can together dig $(\frac{2}{3}+\frac{3}{4})$ or $\frac{17}{12}$ loads in 1 hour:

$\therefore$ they can ,, ,, 1 load in $\frac{12}{17}$ hour;

$\therefore$ they can dig 170 loads in $(\frac{12}{17}\times 170)$ or 120 hrs. *Ans.*

Ex. 9. C can do as much work in 6 hours as B can in 8 hours or as A can in 12 hours; one-half of the work is done by C working 16 hrs. and B working 24 hours: in what time can A finish the other half?

$$6 \text{ hrs. of } C = 12 \text{ hrs. of } A;$$

$\therefore$ 1 hr. of $C=2$ hrs. of A, or 16 hrs. of $C=32$ hrs. of A.

$$8 \text{ hrs. of } B = 12 \text{ hrs. of } A;$$

$\therefore$ 1 hr. of $B=\frac{3}{2}$ hrs. of A, or 24 hrs. of $B=36$ hrs. of A.

$\therefore$ 16 hrs. of $C+24$ hrs. of $B=(32+36)$ or 68 hrs. of A.

But $\frac{1}{2}$ of the work is done by C working 16 hrs. and B working 24 hrs;

$\therefore$ A can finish the remaining half in 68 hours. *Ans.*

Examples 81.

1. *A* can do a piece of work in 2 days, and *B* in 3 days ; in what time can they do it together ?

2. *A* can do a piece of work in $6\frac{2}{3}$ days and *B* in $8\frac{1}{3}$ days ; in what time can they jointly do it ?

3. *A* can do a piece of work alone in 8 hrs. and with *B*'s help in 6 hrs. ; in what time can *B* do it alone ?

4. *A* can do a piece of work in 10 days ; after *A* has been at the work for 5 days, *B* comes to help him and the work is finished in 3 days more. Find the time that *B* must take working singly.

5. *A* alone can do a piece of work in 4 days, *B* alone in 6 days, and *C* alone in 8 days. In what time can they together do it ?

6. *A* can do a piece of work in 10 hrs., *B* in 15 hrs., and *C* in 30 hrs. *A* works at it for 4 hrs. and then goes away ; *B* now comes, works at it for 5 hrs., and then leaves ; in what time will *C* finish it ?

7. *A* can do a piece of work in 4 days, *B* twice as much in 6 days, and *C* thrice as much in 8 days ; in what time can they together do four times the work ?

8. *A* and *B* can each do a piece of work in 24 days. They begin together, but after 5 days *B* goes away. After 6 days more *C* joins *A*, and they finish it in 4 days. In what time can *C* alone do it ?

9. *A* can do as much work in 2 hrs. as *B* can in 3 hrs., or as *C* can in 6 hrs. ; how long will it take *C* to complete a piece of work one-half of which has been done by *A* working 6 hrs. and *B* working 12 hrs. ?

10. *P*, *Q*, and *R* are three workmen ; *P* can do $\frac{2}{5}$ of a piece of work in 6 days and *Q* half as much again in the same time ; *P*, *Q*, and *R* can together do the whole in 3 days. How long will *R* take to do it alone ?

11. *A*, *B*, and *C* can perform a piece of work in 12 days ; *A* could have done it alone in 24 days, and with the assistance of *B* in 18 days. What part of the work can *C* do in 30 days ?

12. *A* does $\frac{1}{3}$ of a piece of work in 4 hrs., *B* $\frac{3}{4}$ of the remainder in 6 hrs., and *C* the remainder in 2 hrs. In what time can they together do it ?

13. *A* can do $\frac{1}{4}$ a piece of work in 20 days, and *B* can do as much in 6 days as *A* can do in 5 days. They work together for 20 days, when *A* leaves and *C* joins. After 15 days *A* returns, and they finish it in 10 days. How long would *C* take to do the work alone ?

***14.** *A* and *B* can do a piece of work in 6 and 9 days respectively. They begin together, but *A* leaves 3 days before the work is finished. How long did the work last ?

***15.** *A* can do a piece of work in 12 days, *B* in 8 days and *C* in 18 days. They all begin to work together. After two days *A* leaves, and *B* leaves 2 days before the work is done. When was the work finished ?

***16.** *A* can do a piece of work in 80 hrs., *B* in 60 hrs., and *C* in 40 hrs. They all begin together : but *B* leaves 10 hrs. and *A* 15 hrs. before the work is done. How long did the work continue ?

17. *M* and *N* reap a field of corn in 3 days, *M* and *P* in $3\frac{1}{3}$ days, and *N* and *P* in 4 days. In what time can they together finish it? In what time can *M* reap it alone?

18. *A* and *B* can do a piece of work in 16 days, *B* and *C* in 24 days, and *A* and *C* in 12 days. What part of the whole work can *A* do more than *B* in 6 days?

19. *M* and *N* can do a piece of work in 12 days; *M* and *P* in 18 days, and *N* and *P* in 30 days. They all work at it for 5 days; *M* then leaves, and *N* and *P* go on for 7 days; *N* then leaves. In how many days after *N* has left, will *P* finish the work?

20. If 30 men can reap 8 bighas in 14 hrs. and 36 others can reap 12 bighas in 18 hrs.: in what time can they jointly reap 130 bighas?

197. Simplification of concrete fractions.

Ex. 1. Simplify $\dfrac{13\frac{1}{3} \text{ of } \frac{7}{20} \text{ of } Rs.\ 10.\ 5a.}{3\frac{1}{2} \text{ of } \frac{11}{12} \text{ of } Rs.\ 5.\ 10a.}$.

The expression $= \dfrac{\frac{40}{3} \text{ of } \frac{7}{20} \text{ of } \frac{165}{16}}{\frac{7}{2} \text{ of } \frac{11}{12} \text{ of } \frac{45}{8}} = \dfrac{40\times7\times165}{3\times20\times16} \div \dfrac{7\times11\times45}{2\times12\times8}$.

$= \dfrac{55\times7}{8} \times \dfrac{2\times4\times8}{7\times11\times15} = \dfrac{8}{3} = 2\frac{2}{3}$. *Ans.*

Ex. 2. Simplify $\dfrac{\frac{2}{9} \text{ of } 1\frac{2}{3} \text{ of } £2.\ 8s.}{\frac{10}{81} \text{ of } \frac{4}{5} \text{ of } £2.\ 5s.} \times \dfrac{\frac{4}{11} \text{ of } 5 \text{ yds. } 1 \text{ ft. } 6 \text{ in.}}{\frac{9}{14} \text{ of } 7 \text{ yds } 2 \text{ ft. } 4 \text{ in.}} \times 15$ hrs.

The result $= \dfrac{\frac{2}{9}\times\frac{5}{3}\times\frac{12}{5}}{\frac{10}{81}\times\frac{4}{5}\times\frac{9}{4}} \times \dfrac{\frac{4}{11}\times\frac{11}{2}}{\frac{9}{14}\times\frac{70}{9}} \times 15$ hrs.

$= \frac{8}{9} \div \frac{2}{9} \times \frac{2}{5} \times 15$ hrs. $= \frac{8}{9} \times \frac{9}{2} \times \frac{2}{5} \times 15$ hrs $= 24$ hrs.

Examples 82.

Simplify:

1. $\dfrac{(\frac{3}{4}+\frac{4}{5}) \text{ of } £(\frac{5}{6}+\frac{7}{8})}{(\frac{4}{5}+\frac{3}{4}) \text{ of } £(\frac{7}{8}-\frac{5}{6})}$. **2.** $\dfrac{£(15\frac{4}{5}-1\frac{5}{8})}{(3\frac{1}{4}+\frac{9}{10}) \text{ of } £4} \times \dfrac{R(2\frac{7}{10}+5\frac{4}{9})}{4\frac{3}{20} \text{ of } R10}$.

3. $\dfrac{(R360+R26\frac{2}{3})\times 25 \text{ mds.}}{9\frac{2}{3} \text{ of } R100}$.

4. $\dfrac{\frac{37}{81} \text{ of } £1\frac{7}{9}}{9\frac{9}{11} \text{ of } £\frac{1}{14}} \times \dfrac{\frac{18}{5} \text{ of } 2\frac{5}{9} \text{ cwt.}}{\frac{21}{100} \text{ of } 4\frac{4}{7} \text{ cwt.}} \times 5$ ft. 6 in.

5. $\dfrac{\frac{1}{3} \text{ of } 1\frac{1}{8} \text{ of } 2\frac{1}{10} \text{ mi.}}{\frac{4}{15} \text{ of } 1\frac{1}{4} \text{ mi.}} \times \dfrac{3 \text{ ft. } 10 \text{ in.}}{7 \text{ ft. } 8 \text{ in.}} \times \dfrac{7 \text{ days } 15 \text{ hrs.}}{122 \text{ days}} \times 5$ cwt.

198. **Chain Rule.** The following Examples will illustrate the **chain rule.**

Ex. 1. If 8 mangoes cost as much as 14 apples, 9 apples as much as 12 oranges, 7 oranges as much 18 guavas: how many guavas may be obtained in exchange for 15 mangoes?

No. of guavas reqd.	=	15 mangoes.
8 mangoes	=	14 apples.
9 apples	=	12 oranges.
7 oranges	=	18 guavas.

[Place the several relations in the above manner, taking care that two numbers of the same kind may not stand in the same column. Then divide the continued product of all the numbers in the right-hand column by the continued product of all the numbers in the left-hand column, and the quotient will give the required result. Thus]

The reqd. number $= \dfrac{15 \times 14 \times 12 \times 18}{8 \times 9 \times 7} = 90.$ *Ans.*

Ex. 2. If $16\frac{1}{2}$ sr. of coffee can be had for 12 sr. of tea, and 10 sr. of sugar for $3\frac{2}{3}$ sr. of coffee; how much tea can be had for $3\frac{3}{4}$ mds. of sugar?

Reqd. quantity of tea	=	$3\frac{3}{4}$ mds. of sugar ;
(10 sr. or) $\frac{1}{4}$ md. of sugar	=	$3\frac{2}{3}$ sr. of coffee ;
$16\frac{1}{2}$ sr. of coffee	=	12 sr. of tea.

$\therefore$ the reqd. no. of seers $= \dfrac{3\frac{3}{4} \times 3\frac{2}{3} \times 12}{\frac{1}{4} \times 16\frac{1}{2}} = \dfrac{15 \times 11 \times 12 \times 4 \times 2}{33 \times 4 \times 3}$

$= 40$ *Ans.*

Examples 83.

1. If 4 cows cost as much as 320 geese, 11 cows as much as 4 horses, 2 sheep as much as 25 hens and 9 horses as much as 231 sheep; how many hens should be given for 144 geese?

2. If $2\frac{1}{2}$ mds. of ghee cost the same as $6\frac{2}{3}$ mds. of sugar, $\frac{5}{8}$ md. of sugar as $1\frac{3}{8}$ mds. of rice, $3\frac{1}{7}$ mds. of rice as $5\frac{1}{7}$ mds. of oats, and 11 mds. of oats as $6\frac{3}{4}$ mds. of wheat; what quantity of ghee may be had for 60 mds. of wheat?

3. If $2\frac{2}{3}$ rupees are worth 5 shillings, $2\frac{2}{3}$ shillings are worth $3\frac{1}{3}$ francs, 16 dollars are worth $87\frac{1}{2}$ francs, how many dollars are worth 42 rupees?

4. If 24 apples are worth 42 pears, 20 pears are worth 250 nuts, 1000 nuts are worth 35 peaches, 7 peaches are worth 100 cherries, and 500 cherries cost half a crown, find the price of 128 apples.

5. If $6\frac{1}{4}$ yds. of linen cost as much as 3 yds of silk, 18 yds. of silk as much as 30 yds. of flannel, 4 yds. of flannel as much as $1\frac{1}{4}$ yds. of velvet, 1 yd. of brocade as much as $6\frac{2}{3}$ yds. of velvet: how many yards of linen may be given in exchange for 15 yds. of brocade?

6. If 28 men can earn as much as 55 women, 33 women as much as 84 boys, and 15 boys earn Rs. 13*a*. per day; how much can 10 men in the same time?

7. If 12 turkeys cost as much as 21 geese, 14 geese as much as 36 ducks, 11 ducks as much as 22 chickens; how many chickens are worth 5 turkeys?

8. If A can do as much work in 5 hrs as B can in 6, and B can do as much work in 8 hrs. as C can in 15 : in what time can C do a piece of work which A can do in 40 hrs. ?

9. If P can do as much work in 7 hrs. as Q can in 10, and Q can do as much work in 15 hrs. as R can in 21 ; in what time can P do a piece of work which R can do in 16 hrs. ?

10. If $\frac{1}{6}$ of a sheep be worth £$\frac{2}{3}$, and $\frac{3}{7}$ of a sheep be worth $\frac{1}{14}$ of an ox, how much must be given for 100 oxen ?

Problems worked out.

Ex. 1. Compare the values of $1\frac{3}{4}$ of *Re.* 1. 2*a*. 6*p*., $1\frac{4}{5}$ of *Re.* 1. 1*a*. 6*p*. and $1\frac{5}{6}$ of *Re.* 1. 0*a*. 6*p*.

$1\frac{3}{4}$ of *Re.* 1. 2*a*. 6*p*.$=\frac{1}{4}$ of *Re.* 1. 2*a*. 6*p*.$\times$ 7 = *Rs.* 2. 0*a*. $4\frac{1}{2}$*p*.

$1\frac{4}{5}$ of „ 1. 1*a*. 6*p*.$=\frac{1}{5}$ of „ 1. 1*a*. 6*p*.$\times$ 9 = „ 1. 15*a*. 6*p*.

$1\frac{5}{6}$ of „ 1. 0*a*. 6*p*.$=\frac{1}{6}$ of „ 1. 0*a*. 6*p*.$\times$ 11 = „ 1. 14*a*. 3*p*.

∴ $1\frac{3}{4}$ of *Re.* 1. 2*a*. 6*p*. is the *greatest* and $1\frac{5}{6}$ of *Re.* 1. 0*a*. 6*p*., the *least.*

Ex. 2. Compare $\frac{2}{21}$ of £1 ; $\frac{1}{11}$ of a guinea ; and $\frac{1}{4}$ of 7*s*. 7*d*.

$\frac{2}{21}$ of £1 = £$\frac{2}{21}$; $\frac{1}{11}$ of 1 guinea $=\frac{1}{11}\times$ £$\frac{21}{20}$ = £$\frac{21}{220}$; $\frac{1}{4}$ of 7*s*. 7*d*. = £$\frac{91}{960}$.

[To compare £$\frac{2}{21}$, £$\frac{21}{220}$, £$\frac{91}{960}$, we should reduce them to equivalent fractions with the same denominator, and proceed as in ART. 166]

The L. C. M. of the denominators 21, 220, 960 is 73920.

$$\therefore\ £\frac{2}{21}=£\frac{2\times 3520}{21\times 3520}=£\frac{7040}{73920}\ ;\ £\frac{21}{220}=£\frac{21\times 336}{220\times 336}=£\frac{7056}{73920}\ ;$$

$$£\frac{91}{960}=£\frac{91\times 77}{960\times 77}=£\frac{7007}{73920}.$$

∴ $\frac{1}{11}$ of a guinea is the *greatest* and $\frac{1}{4}$ of 7*s*. 7*d*. is the *least.* *Ans.*

Ex. 3. What fraction of *Rs.* 5. 4*a*. together with *Re.* 1. 12*a*. is *Rs.* 3. 8*a*. ?

The above means,—what fraction of *Rs.* 5. 4*a*. is (*Rs.* 3. 8*a*. $-$ *Re.* 1. 12*a*.)

Rs. 3. 8*a*. $-$ *Re.* 1. 12*a*. = *Re.* 1. 12*a*. = *Rs.* $\frac{7}{4}$; also *Rs.* 5. 4*a*. = *Rs.* $\frac{21}{4}$;

∴ the required fraction $=\frac{7}{4}\div\frac{21}{4}=\frac{7}{4}\times\frac{4}{21}=\frac{1}{3}$. *Ans.*

Ex. 4. What fraction of £2. 2*s*. is the sum which being diminished by £1. 10*s*. is equal to 9*s*. ?

The meaning is,—what fraction of £2 2*s*. is (£1. 10*s*. + 9*s*)

£1. 10*s*. + 9*s*. = £1. 19*s*. = £$\frac{39}{20}$; also £2. 2*s*. = £$\frac{21}{10}$.

∴ the reqd. fraction $=\frac{39}{20}\div\frac{21}{10}=\frac{39}{20}\times\frac{10}{21}=\frac{13}{14}$. *Ans.*

Ex. 5. What sum is that of which *Rs.* 26. 10*a*. 8*p*. is $\frac{15}{28}$?

Rs. 26. 10*a*. 8*p*. = *Rs.* $26\frac{2}{3}$ = *Rs.* $\frac{80}{3}$.

∴ the reqd. sum = *Rs.* $\frac{80}{3}\div\frac{15}{28}$ = *Rs.* $\frac{80}{3}\times\frac{28}{15}$ = *Rs.* 49. 12*a*. $5\frac{1}{3}$*p*. *Ans.*

Ex. 6. A man bequeathed $\frac{5}{18}$ of his estate to his first son, $\frac{7}{18}$ of the remainder to his second son, and the remainder to his widow. The children's shares differ by *Rs.* 1320. 12*a*. 10*p*. ; find the widow's share.

After bequeathing $\frac{5}{12}$ to his first son, he had $(1-\frac{5}{12})$ or $\frac{7}{12}$ left;

$\therefore$ the second son received $\frac{7}{12}$ of $\frac{7}{12}$ or $\frac{49}{144}$ of the whole;

$\therefore$ the widow's share $=(\frac{7}{12}-\frac{49}{144})$ or $\frac{35}{144}$ of the whole.

Again, the son's shares differ by $\frac{5}{12}-\frac{49}{144}=\frac{11}{144}$ of the whole:

$\therefore$ the value of the estate $=$ *Rs.* 1320. 12*a.* 10*p.* $\div\frac{11}{144}=$ *Rs.* 17290. 8*a.*

$\therefore$ the widow's share $=$ *Rs.* 17290 8*a.* $\times\frac{35}{144}=$ *Rs.* 4202. 8*a.* 10*p.* *Ans.*

Ex. 7. By selling an article for £5. 15*s.* 6*d.*, I clear $\frac{3}{8}$ of what it cost me: what was the original cost?

Taking £1. for the original cost, the gain was £$\frac{3}{8}$, and the selling price £$(1+\frac{3}{8})$ or £$\frac{11}{8}$.

$\therefore$ the original cost $=$ £5. 15*s.* 6*d.* $\div\frac{11}{8}=$ £4. 4*s.* *Ans.*

Ex 8. By selling 25 yds. of brocade at £1. 10*s.* a yard, a trader clears $\frac{1}{4}$ of his outlay. He then raises the price, and sells 45 yds. at £1. 12*s.* a yard. How much does he gain altogether?

Taking £1 for the original cost, the selling price was £$1\frac{1}{4}$ or £$\frac{5}{4}$.

$\therefore$ original cost $=$ £1. 10*s.* $\div\frac{5}{4}=$ £1. 4*s.*;

$\therefore$ gain on the 25 yds $=$(£1. 10*s.* $-$ £1. 4*s.*) $\times$ 25 $=$ £7. 10*s.*

and " " " 45 yds. $=$(" 1 12*s.* $-$ " 1. 4*s.*) $\times$ 45 $=$ " 18.

$\therefore$ his whole gain $=$ " 25. 10*s.* *Ans.*

Ex. 9. A cloth-merchant buys for *Rs.* 325 a bale of cloth containing 100 pieces each, of cloths $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4 and $4\frac{1}{2}$ yds. in length. If the prices are to rise by 3*a.*, at what prices (per piece) must he sell them that he may clear *Rs.* 50 by the transaction?

Each 3-yd pieces should fetch 3*a.* more than a $2\frac{1}{2}$-yd. piece;
" $3\frac{1}{2}$ " " " 6*a.* " " " ; and so on

$\therefore$ 100 3-yd. pieces should fetch	3*a.* $\times$ 100	or *Rs.* 18. 12*a*	more;
" $3\frac{1}{2}$ " " "	6*a.* $\times$ 100	" " 37. 8*a*	"
" 4 " " "	9*a.* $\times$ 100	" " 56. 4*a.*	"
" $4\frac{1}{2}$ " " "	12*a.* $\times$ 100	" " 75. 0*a.*	"
$\therefore$ these 400 pieces " " "		" 187. 8*a.*	"

Leaving out this sum, the 500 pieces should be sold for *Rs.* (325 + 50) $-$ *Rs.* 187 8*a.*, or *Rs.* 187. 8*a.*; *i.e.*, at *Rs.* 187. 8*a.* $\div$ 500 or 6*a.* per piece.

Hence, the price of a $2\frac{1}{2}$-yd. piece	$=$		6*a.*	*Ans.*
" " 3 "	$=$	6*a.* + 3*a.* $=$	9*a.*	
" " $3\frac{1}{2}$ "	$=$	9*a.* + 3*a.* $=$	12*a.*	
" " 4 "	$=$	12*a.* + 3*a.* $=$	15*a*	
" " $4\frac{1}{2}$ "	$=$	15*a.* + 3*a.* $=$ *Re.* 1.	2*a.*	

Ex. 10 *A* and *B* engage to do a piece of work for £2. 8*s.*; *A* can do it alone in 6 days and *B* in 8 days; but *C* comes to help them, and the work is done in 3 days. What must they pay to *C* for his labour?

In 3 days *A* does $\frac{3}{6}$ or $\frac{1}{2}$ of the work, and *B* $\frac{3}{8}$; therefore *C* does in 3 days $1-(\frac{1}{2}+\frac{3}{8})=\frac{1}{8}$ of the work.

$\therefore$ *C* should get $\frac{1}{8}$ of £2. 8*s.* $=$ 6*s.* *Ans.*

Examples 84.

1. Compare the values of $\frac{5}{7}$ of R2. 3*a*., $\frac{5}{6}$ of Rs.2. 8*a*. and $\frac{3}{5}$ of R2. 5*a*.

2. Arrange $\frac{5}{7}$ of R5. 2*a*, $\frac{4}{7}$ of R4. 3*a*. and $\frac{6}{7}$ of R6. 4*a*. in descending order of magnitude.

3. What part of £7. 13*s*. 4*d*. together with £2. 5*s*. 2*d*. is £4. 11*s*. 10*d*. ?

4. What fraction of 3 mi. 4 fur. together with 5 mi. 3 fur. is equal to 7 mi. 1 fur. ?

5. What fraction of a ton is a weight which is 13 cwt. under 1 ton 11 cwt. 2 qr. ?

6. What fraction of a week is it, which when diminished by 3 days 12 hrs. is equal to 6 days 6 hrs. ?

7. What sum is that, $\frac{3}{7}$ of which is 6*s*. ?

8. What length is that, $\frac{3}{8}$ of which is 5 yds. 2 ft. ?

9. What sum is it, of which £8. 3*s*. is $\frac{3}{5}$?

10. Of what distance is $1\frac{1}{2}$ miles five-eighths ?

11. *A* bought $\frac{3}{8}$ of a certain estate, *B* bought $\frac{5}{12}$, and *C* the remainder. How much did *C* buy ? If *C*'s share be worth £3120. 11*s*. 8*d*, what is the value of *A*'s share ?

12. If $\frac{1}{4}$ of a sheep be worth $\frac{1}{3}$ of a rupee, and $\frac{3}{7}$ of a sheep be worth $\frac{1}{14}$ of a cow ; how much must be given for 106 cows ?

13. $\frac{3}{8}$ of an estate was given to *A*, $\frac{3}{8}$ of the remainder to *B* and the rest to *C* ; if *A*'s portion be worth £15001. 9*s*. 2*d*. more than *B*'s, find the value of *C*'s share.

14. I gave away $\frac{1}{10}$ of my money to one person, $\frac{5}{9}$ of the remainder to another, and have still R16 left. What had I originally ?

15. After selling $\frac{3}{4}$ of an article I sell $\frac{4}{5}$ of $\frac{7}{9}$ of the remainder for $\frac{3}{8}$ of $\frac{7}{8}$ of R20. 10*a*. ; what is the value of $\frac{2}{3}$ of it ?

16. *A* was owner of $\frac{3}{4}$ of an estate and sold $\frac{2}{3}$ of his share to *B*, who again sold $\frac{1}{3}$ of his share to *C* for £7500. 6*s*. 3*d*. : find the value of the portion left to *A* after the sale.

17. *A* and *B* can do a piece of work in 3 days, *A* and *C* in 4 days, and *B* and *C* in 6 days. The value of the work is R30 ; what should be their daily wages respectively ?

18. By selling a copy of a book for R36, a book-seller clears $\frac{1}{5}$ of the original cost ; what did he pay for the book ?

19. I have R100 in my pocket : out of this I pay off a debt of R7. 6*a*. 8*p*. What part of the whole have I still left ?

20. Into how many equal parts must a mile be divided, that 60 such parts may be equal to a mile and a half ?

21. By selling an article for 6*s*. 8*d*., I cleared $\frac{1}{4}$ of the outlay ; what is the value of $\frac{3}{4}$ of the cost ?

22. If $\frac{1}{8}$ of the prime cost be gained by selling an article for R36. 15*a.* $4\frac{1}{2}$*p.*, what is the prime cost ?

23. Buying $\frac{9}{16}$ of a mine, I sell $\frac{1}{3}$ of $\frac{4}{5}$ of this portion for £2130. 12*s.* ; what sum must I receive if I sell what is still left to me ?

24. Supposing the cargo of a ship to be worth £10000, and $\frac{4}{9}$ of $\frac{9}{16}$ of $\frac{2}{3}$ of the ship worth $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{7}{12}$ of the cargo ; what is the ship worth ?

25. *A* can finish a copy in 17 hrs. when he writes 3 lines per min. : *B* can finish the same in 21 hrs. : in what time can *B* write 476 lines ?

26. If by selling an article for R40, $\frac{1}{6}$ of the prime cost be lost ; what part of it would have been gained by selling it for R60 ?

27. A man bought 4 sorts of paper at an average price of R7 a ream. If the prices rise by 12*a.* per ream, find the cost of each sort per ream.

28. By selling a horse for R200, I cleared one-fourth of my purchase money. What would I have gained had I sôld it for R250 ?

29. *A* has R10. 9*a.* $9\frac{1}{3}$*p.* and *B* R7. 12*a.* $6\frac{6}{7}$*p.* *A* pays to *B* $\frac{2}{7}$ of $5\frac{1}{4}$ of R1. 8*a.* and *B* pays to *A* $\frac{3}{11}$ of $\frac{5}{6}$ of R10. How much more than *B*, *A* now has ?

30. A man left his property to his 5 sons. The eldest obtained $\frac{1}{3}$ of it and the other sons shared the remainder equally amongst them : if the eldest son got R2456. 13*a.* 10*p.* more than each of his brothers, what property did their father leave ?

MISCELLANEOUS EXAMPLES III.

1. Take from 1 successively its half, third and twenty-fourth parts ; add the product of these parts to the remainder ; and multiply the sum by $7\frac{1}{13}$.

2. Divide 4 by the sum of $5\frac{1}{5}$, $1\frac{3}{5}$ and 8 ; add $3\frac{1}{3} - 1\frac{5}{6}$ to the quotient ; and multiply the result by the difference of $10\frac{2}{5}$ and 5.

3. What number added to $\frac{5}{6}$ of itself, gives 22 as the sum ?

4. $\frac{1}{5}$ of a certain number is $7\frac{7}{18}$; what is $\frac{3}{5}$ of it ?

5. By how much does the sum of $2\frac{1}{3} \times 7\frac{1}{2}$, $5\frac{1}{3}$ of $3\frac{3}{8}$, and $51 \div \frac{19}{24}$ exceed the number, $\frac{7}{13}$ of which is 21 ?

6. What number divided by the product of $\frac{7}{8}$, $4\frac{3}{5}$, and $3\frac{1}{3}$ will give the excess of $2\frac{1}{3}$ over $1\frac{1}{8}$?

7. If $\frac{3}{8}$ of a sum of money be given to *A*, $\frac{4}{15}$ to *B*, $\frac{3}{5}$ of the remainder to *A*, and the rest to *B*, what are the respective shares ?

8. If $\frac{2}{3}$ of *A*'s money be equal to $\frac{4}{5}$ of *B*'s, and *A* s, money be R5 more than *B*'s, what is *A*'s money ?

9. *A* and *B* begin to play together for a certain stake, *A* with thrice as much money as *B*. *B* wins from *A* at the end of the first game $\frac{2}{5}$ of *A*'s money. What fraction of *B*'s original money must *A* win back in the second, that they may have equal sums ?

10. A person who possesses a nine-anna share in an estate, sells out first $\frac{5}{8}$ of his share and then $\frac{7}{10}$ of the remainder; what part of the whole estate does he still retain?

11. A tea-garden is bought by A, B, and C, for £5600: A paid $\frac{2}{7}$ of the capital, B $1\frac{1}{6}$ of what A paid, and C the remainder: find what sum was contributed by each.

12. A certain sum was divided amongst A, B, C, and D; A received $\frac{1}{5}$ of the whole sum, B $\frac{1}{3}$, C $\frac{1}{8}$, and D the remainder. If D's portion was R15, what sums did A, B, C respectively receive?

13. A can do a piece of work in $2\frac{2}{3}$ days, B in $3\frac{1}{3}$ days; and C in $5\frac{1}{3}$ days; in what time can they jointly do a piece of work 5 times as great?

14. After paying off R16 out of a debt, I have still $\frac{4}{5}$ of it to pay what was my debt?

15. A, B, C, and D are partners in a business which produces annually an income of R12500 A advances $\frac{1}{3}$ of the capital, B $\frac{1}{4}$, C $\frac{1}{5}$, and D the rest. Divide the profit of 4 years fairly between them.

16. A man after selling $\frac{5}{9}$ of his share in an estate, finds that he has $\frac{4}{15}$ of the estate left; what part of the estate had he at first.

17. Divide R100 among A, B, C and D so that B's share may be $\frac{1}{3}$ of A's share, C's share $\frac{1}{2}$ of what A and B together receive, and D's share $\frac{1}{2}$ of what A and C receive together.

18. By selling tea at 5*s.* 6*d.* a lb., a grocer clears $\frac{1}{21}$ of the prime cost of 1 lb.; find how much he paid for 200 lbs. of tea of the same quality.

19. Half of a piece of cloth is coloured red, one-third green, one-tenth blue, and the rest which is 52 yds. yellow; how long is the piece?

20. A certain sum of money was divided among four men A received $\frac{2}{3}$ of the whole, B $\frac{1}{4}$ of the remainder, C $\frac{4}{5}$ of what still remained, and D the rest which was R60. What did A, B and C get respectively?

21. By selling 30 seers of tea at R2. 9*a.* 8*p.* per seer, a grocer clears $\frac{1}{9}$ of the prime cost of a seer. He then raises the price to R2. 12*a.* per seer and sells 60 seers. Find the total profit.

22. If a piece of cloth measuring 56 yds. 2ft. 6 in. be divided into parts each 3 yds. 10 in. long: find how many such parts can be cut off and what fraction of the whole will be left.

23. A certain number of men can dig 24 cub. feet of earth in $4\frac{1}{2}$ hrs and a certain number of others 28 cub. feet in $4\frac{2}{3}$ hrs.: in what time can the two gangs jointly dig $188\frac{8}{9}$ cub. feet?

24. A wine merchant bought 48 gallons of wine; he kept 8 gallons for himself and sold the rest at R10. 10*a.* 8*p.* per gallon, gaining thereby $\frac{1}{15}$ of his outlay; what was the cost price per gallon?

***25.** A, B, C are three *serais* lying on the road. The distance from A to C is $23\frac{1}{3}$ miles: and B is $\frac{1}{14}$ of this distance nearer to A than to C. What is the distance between B and C?

26. If a man earn $\frac{2}{3}$ as much as 9 women, and a boy $\frac{1}{3}$ of $\frac{9}{10}$ as much as 2 women, what part of a man's earnings does a boy earn? Also find

the daily earnings of a man and a woman respectively, a boy earning 3s. $4\frac{1}{2}$d. a day.

27. Divide £59. 11s. 5d. between A and B, so that $\frac{7}{10}$ of A's share may be equal to $\frac{3}{4}$ of B's.

28. A person owes a guinea to each of 4 creditors; to one he pays $\frac{1}{3}$ of his debt, to another $\frac{3}{7}$, to the third $\frac{7}{9}$ and to the fourth $\frac{9}{21}$: what sum will he be still owing altogether?

29 A wheel makes 96 revolutions in a minute; if its speed were increased by $\frac{1}{12}$th, how many revolutions more would it make in 52 hrs.?

30. A party having to pay a bill of R123. 9a., one of them pays for himself and three friends R54. 14a. 8p.; how many were there?

31. Divide R150. 12a. 9p among A, B and C, so that A may receive thrice as much as B or twice as much as B and C together.

32. Three persons A, B, and C started together from the same place and began to walk round an island $52\frac{1}{2}$ miles round. They walked 21, 28 and 30 miles a day respectively. How soon were they together again at the point of starting, and how many rounds did each make?

33. £298. 3s. 11d. was divided among P, Q and R, so that Q's share was $\frac{7}{11}$ of P's and R's share $\frac{4}{5}$ of Q's: find the share of each.

34. 75 gal. of water being poured into a cistern which is two-thirds full, it becomes five-sixths full; how much water can the cistern hold?

35. A cistern can be emptied by two pipes in 25 and 30 min. respectively. Both the pipes are opened when the cistern is full. When should the first pipe be stopped, that the cistern might be emptied 12 min. later?

36. A and B engage to do a piece of work for R30. A could do the work alone in 4 days, and B in 5 days; with the help of C it is done in 2 days: how should the money be divided?

37. If a be $\frac{3}{8}$ of $3\frac{3}{5}$ of b and c $\frac{4}{5}$ of $2\frac{7}{10}$ of b, what fraction is a of c?

***38.** A man and a boy are to work every other day at a piece of work which would occupy the boy alone for 13 days If the man took up the work first, it would be finished in an exact number of days and half a day sooner than if the boy began first. Find how long it would take them to finish the work together.

***39.** A cloth merchant buys for R136. 4a. a bale of cloth containing 40 pieces each of cloths 2, 3, 4, 5 and 6 yards in length. If the prices are to ascend by 2a. 6p., at what price per piece must he sell each kind, that he may clear R30 by the transaction?

40. If A can do as much work in 10 days as B can in 13, and B can do as much work in 26 days as C can in 35; in what time can A and B together do the work which C can do in 105 days?

41. If X can do as much work in 3 days as Y can in 5, and Y can do as much work in 4 days as Z can in 9; in what time can X, Y, and Z jointly do a piece of work which Z can alone do in 45 days?

42. In which way is it best to buy sugar, at 6 guineas per cwt. or at £5. 16s. 8d. per 100 lbs.? How much would be saved in buying 1 ton?

43. A man walks a certain distance and rides back in 3 hrs. 45 min. He could walk both ways in $4\frac{1}{2}$ hours. How long would it take him to ride both ways ?

44. A man buys 10 maunds of tobacco for R82. 3*a*. 6*p*. He pays 2*a*. per seer for duty, and then sells, $\frac{1}{2}$ at 5*a*. 6*p*. a seer, $\frac{3}{7}$ at 6*a*. a seer, and the remainder at 4*a*. per seer. What profit does he make ?

45. The sum which will pay *A*'s wages for $61\frac{1}{4}$ days would pay *B*'s wages for $81\frac{2}{3}$ days. For how long would it pay *A*'s and *B*'s wages together ?

46. *P* and *Q* barter. *P* has 27 tons of coal worth £1. 13*s*. 9*d*. a ton, but he insists on a price of £1. 17*s*. 6*d*. a ton. *Q* has tea worth £3. 2*s*. 3*d*. a packet. How much must *Q* raise his price, to exchange 15 packets of tea for the 27 tons of coal ?

47. *A* and *B* together can do twice as much work as *A* and *C* together, and *B* can do three times as much work as *A* in the same time. *A* has to do a piece of work ; after 12 days he has done only $\frac{1}{4}$ of it. He then gets *B* to help him ; but after 8 days *B* leaves, and he gets *C* to help him. When will the work be finished ?

48. If a horse can be driven 3 times as fast as a man can walk, a man can walk half as fast again as a woman, and a woman twice as fast as a child ; how long will a horse take to be driven the same distance that a child can walk in 54 minutes ?

49. Divide R207. 2*a*. 4*p*. among 3 men, 4 women, 5 boys and 6 girls, so that each woman may get $\frac{3}{4}$, each boy $\frac{2}{7}$, and each girl $\frac{3}{14}$ of a man's share.

***50.** At an election where only $\frac{5}{6}$ of the whole number of voters voted, one of the two candidates received half as many votes again as the other and beat him by 795 votes. What was the whole number of votes ?

51. If 91 million of locusts weigh 1 ton, how many will weigh $1\frac{3}{13}$oz. ?

52. If 40 lbs. of standard gold, 30 lbs. of standard silver and 15 lbs. of copper are coined into 1869 sovereigns, 1980 shillings and 360 pennies respectively ; find the weight of a sovereign, a shilling and a penny.

***53.** When wheat was 2*s* a bushel, the weekly expenses of a family was £30. 12*s*. 8*d*. : and when the price fell to 1*s*. 9*d*. a bushel, the expenses were reduced by £2. 16*s*. 4*d*. ; what will the expenses be when the price rises to 2*s*. $2\frac{1}{2}$*d*. per bushel ?

54. A grocer mixes two kinds of tea which cost him $6\frac{1}{2}$*d*. and $8\frac{1}{4}$*d*. per lb. respectively, in the proportion of 3 lbs. of the former to 2 lbs. of the latter ; he also pays a duty of 6*d*. a lb. all round. If he sells 255 lbs. of the mixture at 1*s*. 6*d*. a lb., what is his profit ?

55. Two friends during a walk take steps of $2\frac{23}{48}$ ft. and $2\frac{23}{36}$ ft. ; if they start in step, how far will they have walked before they are in step again, and how many steps will each have taken ?

56. Divide R448 between 6 men, 8 women, 10 boys and 12 girls in such a way that each woman has $\frac{1}{4}$ less than a man, each boy $\frac{2}{7}$ as much as a man and a woman together, and each girl $\frac{1}{3}$ as much as a man, woman and boy together.

57. Divide £131 between A, B, and C so that as often as A gets 4*s*. 6*d*. B may get 7*s*. 6*d*., and as often as B gets 6*s*. C may get 3*s*. 6*d*.

58. A railway passenger counts the telegraph posts on the line as he passes them; if they are $58\frac{2}{3}$ yds. apart and the train is going 48 miles an hour, how many will he pass per minute?

***59.** When rice is 12 sr. per rupee, the expenses of a family amount to ₨50; but they amount to ₨47 only, when the price falls to 15 sr. per rupee. What will the expenses be, when rice is 18 sr. per rupee?

60. A person goes to France with £112 which he exchanges at the rate of $25\frac{1}{8}$ francs for a £. He stops there 120 days, spending at the rate of $18\frac{3}{4}$ francs a day, and changes what is left at the rate of 1 franc for $9\frac{1}{2}$*d*. Find what he will have in £. *s*. *d*.

61. Which is really cheaper,—home-made cloth which costs ₨1. 13*a*. per piece and lasts a year, or mill-made cloth which costs ₨1. 3*a*. per piece and lasts 7 months?

62. In a cricket match, A obtained $\frac{1}{4}$ of the total number of runs, B and C together $\frac{1}{2}$, the others $\frac{1}{6}$, and there were 8 byes; what was the total score?

63. There are 3 pumps that can discharge water in quantities which are as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$; how much water has been discharged by all, when the first has discharged 1000 gallons more than the third?

64. A besieged place garrisoned by 10000 men was victualled for 27 days; but after 9 days, 2500 men cut their way out. How long would the provisions last the others, if the men be put on three-quarter rations?

65. An estate is divided amongst 3 persons in the ratio of 4, 6, 7. Find the value of the estate when £270 added to the largest share would make it equal to half the whole.

66. A number of mangoes has to be divided amongst 5 persons in shares which are as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$; what should the number *at least* be, that this may be done without having to cut up any one mango?

***67.** A quantity of provisions serves 19250 men for 8 wks., at the rate of 24 oz. a day for each man: how many must leave in order that the same provisions may serve those that remain for 14 wks., at the rate of 16 oz. a day for each man?

68. A person at his death leaves $\frac{1}{4}$ of his property to his widow, and the remainder equally amongst his children. The widow's portion is thrice that of each child. How many children were there? If the share of each child be £2500, what is the value of the whole estate?

69 If 2 guineas make 3 Napoleons, 15 rix-dollars make 4 Napoleons, and 6 ducats make 7 rix-dollars; how many ducats are there in £490?

***70.** A garrison had provisions for 16 wks., at the rate of 25 oz. a day for each man; 750 men left, and the rations were reduced to 10 oz. a day, thus serving the remainder of the garrison for 28 weeks. Of how many men did the garrison originally consist?

CHAP. XXI. DECIMAL FRACTIONS.

199. It has been shewn in ART. 9, that as we proceed from the unit's place to the left, we come to *tens*, *hundreds*, *thousands*, &c., each of these being **ten** times the one preceding it ; *conversely*, as we pass *from left to right*, the local value of a figure *diminishes ten-fold* for each place through which it is moved. If this system be further carried out, a figure placed immediately to the right of the figure in the unit's place should indicate *tenths* of a unit ; a figure placed after this last, should indicate tenths of tenths, *i. e.*, *hundredths* of a unit ; the next figure, *thousandths* of a unit ; and so on

The figure in the unit's place, which has, therefore, to be distinguished from all others is usually known by a *dot* or *point* placed after it, this point being called the **decimal point.**

Thus, in 15·347, 5 is the figure in the unit's place ; 3 represents 3 *tenths* or $\frac{3}{10}$; 4 represents 4 *hundredths* or $\frac{4}{100}$; 7 represents 7 *thousandths* or $\frac{7}{1000}$; and $15·347 = 15 + \frac{3}{10} + \frac{4}{100} + \frac{7}{1000}$. Also 15·347 is read, 15 *decimal* 3, 4, 7.

200. Fractions denoted in the above way are called **Decimal Fractions** ; and the two parts to the left and the right of the decimal point, are called the *integral* and *decimal* parts respectively.

In the above, 15·347 is a *decimal fraction* ; 15 is the *integral* part, and 347 the *decimal* part.

201. Since 245·7891 = 2 *hundreds* + 4 *tens* + 5 *units* + 7 *tenths* + 8 *hundredths* + 9 *thousandths* + 1 *ten-thousandth*, it is obvious that the following would be a complete TABLE OF NOTATION.

&c.	&c.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths	Hundredths.	Thousandths.	Ten-thousandths.	&c.	&c.
...	...	5	4	3	2	1	2	3	4	5	...	...

202. Any whole number can be expressed as a decimal fraction by putting a decimal point at the right of the unit figure and adding cyphers to the decimal point : and the value of a decimal fraction will not be altered by adding any number of cyphers to the decimal part.

Thus, $152{\cdot}000 = 152 + \frac{0}{10} + \frac{0}{100} + \frac{0}{1000} = 152 + 0 + 0 + 0 = 152$ or $152 = 152{\cdot}000$.

Again, $15{\cdot}32 = 15 + \frac{3}{10} + \frac{2}{100} = 15 + \frac{32}{100}$; and $15{\cdot}3200 = 15 + \frac{3}{10} + \frac{2}{100} + \frac{0}{1000} + \frac{0}{10000} = 15 + \frac{32}{100}$; $\therefore$ $15{\cdot}32 = 15{\cdot}3200$.

203. If there be no integral part we may either place; a cypher at the left of the decimal point or begin at once with the decimal point.

Thus, $\frac{3}{10} + \frac{2}{100}$ may be denoted either as $0{\cdot}32$ or as ${\cdot}32$.

204. *To convert a decimal fraction into an equivalent vulgar fraction.*

RULE. Place in the numerator the given number omitting the decimal point, and in the denominator 1 followed by as many cyphers as there are figures in the decimal part.

Ex. 1. ${\cdot}247 = \frac{247}{1000}$; for $247 = \frac{2}{10} + \frac{4}{100} + \frac{7}{1000} = \frac{247}{1000}$.

Ex. 2. $32{\cdot}85 = \frac{3285}{100}$; „ $32{\cdot}85 = 32 + \frac{8}{10} + \frac{5}{100} = 32\frac{85}{100} = \frac{3285}{100}$.

Ex. 3. ${\cdot}0127 = \frac{127}{10000}$; „ ${\cdot}0127 = \frac{0}{10} + \frac{1}{100} + \frac{2}{1000} + \frac{7}{10000} = \frac{127}{10000}$.

205. *Conversely*, a vulgar fraction whose denominator is 10 or some power of 10 (*i.e.*, 1 followed by cyphers), can be mmediately converted into an equivalent decimal fraction.

RULE. Take the numerator, and mark in it the decimal point as many places (counting from right to left) to the left of the unit figure as there are cyphers in the denominator. If the number of figures in the numerator be less than the number of cyphers in the denominator, put as many cyphers to the left of the numerator as will make up the deficiency.

(1) $\frac{15621}{1000}$. There are 3 o's in the denominator; count 1, 2, 6; therefore the required decimal $= 15{\cdot}621$.

(2) $\frac{3257}{10000}$. Count, 7, 5, 2, 3 : $\therefore$ the required decimal $= {\cdot}3257$.

(3) $\frac{219}{100000}$. There are 5 o's in the denominator, but only 3 figures in the numerator; so put 2 cyphers at the left of 2.

$\therefore$ the required decimal $= {\cdot}00219$.

Examples 85.

1. Convert the following decimals into equivalent vulgar fractions:

(1) $3{\cdot}1$; $72{\cdot}19$; $947{\cdot}01$; $42{\cdot}001$; $576{\cdot}112151$; $67{\cdot}12346711$.

(2) ${\cdot}3$; ${\cdot}31$; ${\cdot}321$; ${\cdot}1461$; ${\cdot}01$; ${\cdot}003$; ${\cdot}00021$; ${\cdot}0000041$; ${\cdot}000000071$.

2. Convert the following decimals into equivalent vulgar fractions in their lowest terms:

(1) ${\cdot}5$; ${\cdot}2$; ${\cdot}4$; ${\cdot}8$; ${\cdot}12$; ${\cdot}05$; ${\cdot}02$; ${\cdot}04$; ${\cdot}005$; ${\cdot}000025$; $12{\cdot}75$.

(2) 67·004 ; 57·625 ; ·000725 ; 5·3725625 ; 6·00052.

(3) 15·725725 ; ·0000000025625 ; 441·000075125

3. Express the following decimals as mixed numbers :

(1) 72·628 ; 756·005 ; 7·5283 ; 27·0006 ; 2·1579.

(2) 67·005252 ; 141·0035 ; 478·070705 ; 30·000125125 ; 7521·00128.

4. Convert the following vulgar fractions into equivalent decimals :

(1) $\frac{1}{10}$; $\frac{3}{10}$; $\frac{6}{10}$; $\frac{11}{10}$; $\frac{251}{10}$; $\frac{3741}{10}$; $\frac{3}{100}$; $\frac{31}{100}$; $\frac{5}{1000}$; $\frac{511}{10000}$.

(2) $\frac{7}{10000}$; $\frac{5}{100000}$;

(3) $\frac{712}{10000}$; $\frac{21960}{1000}$;

(4) 17 tenths ; 25 hundredths ; 51 thousandths ; 621 ten-thousandths ; 451324 hundred-thousandths ; 72416 millionths ; 461 billionths.

206. *To multiply a decimal by* 10 *or some power of* 10.

RULE. Remove the decimal point as many places towards the *right* as there are cyphers in the multiplier, adding cyphers at the *end* of the decimal, if necessary.

Ex. Multiply ·0756 separately by 10, 100, 1000, 10000, 100000.

·0756 × 10 = ·756 ; ·0756 × 100 = 7·56 ; ·0756 × 1000 = 75·6 ;

·0756 × 10000 = 756 ; ·0756 × 100000 = 7560

[·0756 × 10 = $\frac{756}{10000}$ × 10 = $\frac{7560}{10000}$ = ·756 ; &c.]

Examples 86.

Multiply :

1. ·09 separately by 10, 100, 1000, 10000, 100000, 1000000.

2 ·0027 „ „ 100, 1000, 10000, 100000, 100000000.

207. *To divide a decimal by* 10 *or some power of* 10.

RULE. Remove the decimal point as many places towards the *left* as there are cyphers in the divisor, adding cyphers at the *beginning* of the decimal, if necessary.

Ex. Divide 2·05 separately by 10, 100, 1000.

2·05 ÷ 10 = ·205 ; 2·05 ÷ 100 = ·0205 ; 2·05 ÷ 1000 = ·00205.

[2·05 ÷ 10 = $\frac{205}{100}$ × $\frac{1}{10}$ = $\frac{205}{1000}$ = ·205 ; &c.]

Examples 87.

Divide :

1. 51 separately by 10, 100, 1000, 10000, 100000, 1000000.

2. ·057 „ „ 10, 1000, 10000, 100000, 1000000000.

3. 25·006 „ „ 1000, 10000, 10000000, 100000000.

208. As the operations of Addition, Subtraction, Multiplication and Division of decimal fractions can be performed in the same way as in the case of whole numbers, it is usually more advantageous to work out Examples of fractions by reducing them to decimals than to work them out as vulgar fractions.

ADDITION OF DECIMALS.

209. RULE. Place the numbers one under another so that all the unit figures may stand one under the other; then proceed to add as in Simple Addition, the unit figure of the sum being also under the column of units.

Ex. Add together 15·45, 216·946, ·0024, 1·01234, 21.

```
   15·45
  216·946
     ·0024
    1·01234
   21
  ---------
  254·41074
```

$$15{\cdot}45+216{\cdot}946+{\cdot}0024+1{\cdot}01234+21.$$

$$=\frac{1545}{100}+\frac{216946}{1000}+\frac{24}{10000}+1\frac{101234}{100000}+21$$

$$=\frac{1545000+21694600+240+101234+2100000}{100000}$$

$$=\frac{25441074}{100000}=254{\cdot}41074.$$

Examples 88.

1. Add together :

(1) 3·5, 4·15, 5·6, 9·146, 7·05, ·123, ·01, 2·401.

(2) 13·5, ·025, ·376, 12·4214, 1·961001, ·00002161, 4·62101.

(3) ·69621, ·8214, ·962, ·12, ·9, ·0091, ·00627, ·781416.

(4) ·576, ·4214, 7·4512, 30·000075, 375·01, 37·002, ·0125146.

(5) 11·146, ·256734, ·00723461, ·0000011, 9·24, ·00125.

(6) 3814·01, 257·253, ·027, 77·776, 34·792, ·002.

(7) 73·0014600071, ·07, ·0073, ·0003, 587·00011.

(8) 6711·3, 87·57, 19·00305, 52·07073, ·003, 37.

(9) ·471, ·049076, 550, ·705, 754·0461, ·50024.

(10) 17·016, 7414, ·004776, 97·000077051, 3·02, 7.

2. Find the values of :

(1) 11·215 + 96·317 + 702·021 + 243·123 + ·0213.

(2) 83·6 + 83·47916 + ·02976 + ·00313 + 20·000317.

(3) R201·1 + R3·7143 + R12 + R5905·16 + R·000176.

(4) £94·14 + £721 + £·0216 + £·00014 + £·030303 + £71.

(5) ·5763 md. + 741·2461 mds. + 1·002 mds. + 107·0001 mds. + 3 mds.

(6) 7·14 cwt. + ·0741 cwt. + ·0456 cwt. + ·00045 cwt. + ·000562 cwt.

(7) 721 ft. + 700·007 ft. + 5·005 ft. + ·006 ft. + 100·07 ft. + 23 ft.

(8) ·00021 da. + 1·000032 da. + 12·0012001 da. + 73·000000641 da.

SUBTRACTION OF DECIMALS.

210. RULE. Place the less number under the greater so that the unit figures may stand in the same column, and subtract as in Simple Subtraction.

N.B. When the number of digits in the decimal part of the lower line is greater than the number of digits in the decimal part of the upper line, put at the right end of the latter as many cyphers as will be required to make up the deficiency.

Ex. 1. Subtract 7·8146 from 8·014.

$$\begin{array}{r} 8{\cdot}0140 \\ \underline{7{\cdot}8146} \\ \underline{{\cdot}1994} \end{array} \qquad 8{\cdot}014 - 7{\cdot}8146 = \tfrac{8014}{1000} - \tfrac{78146}{10000} = \tfrac{1994}{10000} = {\cdot}1994.$$

Ex. 2. Which of the numbers 2·4889 and 2·4888, is nearer to 2·4888745?

2·4889 − 2·4888745 = ·0000255; 2·4888745 − 2·4888 = ·0000745.

∴ 2·4889 is nearer to 2·4888745 than 2·4888.

Examples 89.

1. Subtract:

(1) ·3807 from ·56441; 7·8491 from 15·005; ·0007 from 3·3.

(2) ·09 from ·1; ·009 from ·01; ·099 from ·1; ·999 from 1.

(3) ·942 from 1·0031; 5·00147 from 6; 4·00146 from 6·000041.

(4) 5·00040052 from 7·41; ·00641 from 572; ·99 from 9.

2. Find the difference between:

(1) ·0075 and ·01; ·519 and 13; 79·0468 and 80·14.

(2) 6·41 and 4·001; 21·2 and 30·02; 156·0001 and 158·00001.

(3) Six hundred and six hundredths; sixteen and sixteen millionths.

3. Find the values of:

(1) R7·416 − R4·8914; R·0256 − R·00941; R·05 − R·0124.

(2) £17·369 − £5·446; £·13096 − £·013096; £1 − £·99007765

(3) ·01 md. − ·009 md.; 1·06 cwt. − ·964 cwt.; 4·64 yds. − 3·947 yds.

4. Simplify:

(1) 14·8461 − 7·4146 − 4·0021 + ·3 + 11·461 − 2·0014.

(2) 50 − ·05 + ·005 − 7·0055 + ·0005 − 25·001444.

(3) 200 − 55·01 − 24·2761 − 42·1406 − 10·00146.
(4) 51·406 − 64·4621 − 13·891 + 80·041 − 37·001515.
(5) 21·46 − (15·24 − 9·0024) − (7·241 − 4·01) − 5·1.
(6) 5·345 − 3·728 − (3·24 − ·925) + 7 − ·007076.

5. Of 24·7892 and 24·7893, which is nearer in value to 24·7892874?

6. By which of the two, 57·8425 and 57·8424, will 57·8424681 be more nearly expressed?

MULTIPLICATION OF DECIMALS.

211. RULE. Multiply the numbers together as if they were whole numbers; and put down the decimal point before as many figures in the product (counting from right to left) as there are decimal places in the multiplicand and multiplier together, adding cyphers to the left where necessary.

Ex. 1. Multiply 7·35 by ·25.

```
 7·35
  ·25
 ----
 3675
1470
-----
18375
```

There are 4 places of decimals in the multiplicand and multiplier together; therefore insert the decimal point before 4 figures in the product, counting from right to left.

∴ the required product = 1·8375. *Ans.*

$[7\cdot35 \times \cdot25 = \frac{735}{100} \times \frac{25}{100} = \frac{18375}{10000} = 1\cdot8375.]$

Ex. 2. Multiply ·0425 by ·0044.

```
·0425
·0044
-----
 1700
1700
-----
18700
```

There are 8 decimal places in the multiplicand and multiplier together; therefore the decimal point must be inserted before 8 figures in the product (counting from right to left). But there are only 5 figures in the product; therefore prefix 3 cyphers.

∴ the product = ·00018700 = ·000187. *Ans.*

Examples 90.

1. Multiply:

(1) 42·75 separately by 3·5, 4·04, 47, ·47, ·0047.
(2) 2·5671 ,, ,, 15, ·015, 1·5, ·15, 2·7, ·27, ·00027.
(3) ·34565 ,, ,, 246, 24·6, 2·46, ·246, ·00246.
(4) ·00625 ,, ,, 46351, 46351000, ·0000046351.
(5) ·000128 ,, ,, ·0025, ·00075, ·0000125, ·0000625.
(6) ·009,843 ,, ,, ·004598, ·04598, 45·98.
(7) 464620·25 ,, ,, 55, ·0014, ·000242, 24·04896.
(8) 764·00225 ,, ,, 46 00578, 4600·578, ·004600578.

2. Find the values of:

(1) 5·319 × 10·1; 336·8 × 7·25; ·0151 × ·0564; 19·021 × 30·906.

(2) $3{\cdot}1206 \times 30000$; $190{\cdot}0901 \times {\cdot}229$; ${\cdot}00115 \times {\cdot}007266$.

(3) ${\cdot}4 \times 7{\cdot}6 \times {\cdot}55$; $7{\cdot}1 \times {\cdot}09 \times {\cdot}025$; $12{\cdot}02 \times 220 \times {\cdot}4405$.

(4) ${\cdot}5 \times {\cdot}016 \times {\cdot}974 \times {\cdot}0095$; $40{\cdot}05 \times 28{\cdot}055 \times {\cdot}00048 \times {\cdot}0075$.

3. Find the values of :

(1) $5{\cdot}714 \times 0{\cdot}5 - 75{\cdot}156 \times {\cdot}00000075 + 7{\cdot}046 \times 3{\cdot}005$.

(2) $5{\cdot}24 \times {\cdot}01 + 71{\cdot}006 \times {\cdot}005 - {\cdot}0025 \times {\cdot}00128 + 6{\cdot}4 \times 2{\cdot}05$.

(3) $(1{\cdot}524)^2 - ({\cdot}8901)^2$. (4) $(8{\cdot}74 - 5{\cdot}89) \times 6{\cdot}035$.

(5) $5{\cdot}794 - {\cdot}25 \times 2{\cdot}254$. (6) $(6{\cdot}45)^2 - {\cdot}426 \times 7{\cdot}41$.

DIVISION OF DECIMALS.

212. RULE I. Multiply both the dividend and the divisor by 10 or such a power of 10, as will make the divisor and the dividend both integers.

(i) If the dividend exceeds the divisor, divide as in Simple Division. If there be no remainder, the quotient will be an integer.

Ex. Divide $31{\cdot}5$ by ${\cdot}126$.

$31{\cdot}5 \times 1000 = 31500$; and ${\cdot}126 \times 1000 = 126$.

```
126)31500(250
    252
    ---
     630
     630
```

$$31{\cdot}5 \div {\cdot}126 = \tfrac{315}{10} \div \tfrac{126}{1000}$$
$$= 31500 \div 126$$
$$= 250.$$

When the division does not terminate as above, add cyphers to the successive remainders to make them greater than the divisor ; but place the decimal point in the quotient as soon as the first cypher is added, and continue the operation until either no remainder is left or the required number of decimal places in the quotient is obtained.

Note. If more than one cypher has to be affixed to any remainder to make it greater than the divisor, take care to put in the quotient a cypher for *each* of these *additional* cyphers.

Ex. 1. Divide $5{\cdot}2$ by ${\cdot}32$.

$5{\cdot}2 \times 100 = 520$ and ${\cdot}32 \times 100 = 32$.

```
32)520(16·25
   32
   ---
   200
   192
   ---
     80
     64
    ---
    160
    160
```

$$5{\cdot}2 \div {\cdot}32 = \tfrac{52}{10} \div \tfrac{32}{100}$$
$$= \tfrac{520}{32} = 16\tfrac{1}{4}$$
$$= 16 + \tfrac{25}{100}$$
$$= 16{\cdot}25.$$

Ex. 2. Divide 28·00028 by ·035.

28·00028 × 100000 = 2800028, and ·035 × 100000 = 3500.

```
3500)2800028(800·008.
     28000
     -----
        28000
        28000
```

$$28{\cdot}00028 \div {\cdot}035 = \frac{2800028}{100000} \div \frac{35}{1000}$$
$$= \frac{2800028}{3500} = \frac{400004}{500} = \frac{800008}{1000}$$
$$= 800{\cdot}008.$$

(ii) When the dividend is less than the divisor, the quotient will at once **begin** with the decimal point, and the operation of division should be carried on by affixing cyphers first to the dividend and then to the remainders successively, until either there is no remainder left or the required number of decimal places in the quotient is obtained.

Ex. Divide ·000324 by ·045, and ·0009 by 72.

·000324 × 1000000 = 324 ;
·045 × 1000000 = 45000 ;

```
45000)324000(·0072
      315000
      ------
        90000
        90000
```

·0009 × 10000 = 9 ;
72 × 10000 = 720000.

```
720000)9,00000(·0000125̇
       7,20000
       -------
       1800000
       1440000
       -------
        3600000
        3600000
```

Note. Sometimes the quotient may be required to a given number of decimal places, as in the following Examples.

Ex. Divide ·002 by ·0007 to 2, and 3·12 by 423·82 to 5 decimal places.

·002 × 10000 = 20,
·0007 × 10000 = 7 ;

```
7)20(2·85...
  14
  --
   60
   56
   --
    40
    35
```

3·12 × 100 = 312
423·82 × 100 = 42382 ;

```
42382)312,000(·00736...
      296 674
      -------
       153260
       127146
       ------
       261140
       254292
```

213. Rule II. Multiply the divisor and the dividend by the power of 10 which would make the divisor *only* an integer. Then proceed to divide as before, taking care to place the decimal point in the quotient just as the **first figure** of the decimal part is considered.

Ex. Divide (1) 429·408 by 59·64 : (2) ·325 by 8·7 to 3 places of decimals.

(1) 59·64 × 100 = 5964,
429·408 × 100 = 42940·8 ;

```
5964)42940·8(7·2
     41748
     -----
      11928
      11928
```

(2) 8·7 × 10 = 87,
·325 × 10 = 3·25 ;

```
87)3·25(·037...
    261
    ---
     640
     609
```

It is generally advantageous to employ the method of **short division** when the divisor does not exceed 20, or when it can be split up into factors none of which exceed 20.

Ex. Divide, (1) 457·2 by 2400, and (2) 23 by ·7 to five places of decimals.

```
100)457·2                    ·7 × 10 = 7.
  3)4·572                   ·23 × 10 = 2·3.
  8)1·524                   7)2·3
   ·1905                     ·32857...
```

214. RULE III. Divide as in Simple Division, neglecting the decimal points; and put the decimal point as many places to the left (adding cyphers if necessary,) as the number of decimal places in the dividend, together with the number of cyphers affixed, exceeds the number of decimal places in the divisor.

Ex. Divide 3·241 by 579·32 to 5 decimal places.

```
57932)3241000(559
      289660
       344400
       289660
        547400
        521388
```

There are 3 decimal places in the dividend and 4 cyphers have been added; also there are 2 places of decimals in the divisor.

$3+4-2=5$; therefore the decimal point should be put 5 places to the left of 9; hence the required quotient = ·00559.

215. A vulgar fraction can be converted into a decimal by dividing the numerator by the denominator, by RULE I.

Ex. Express (1) $\frac{35}{64}$ and (2) $76\frac{8}{625}$ as decimals.

```
64)350(·546875                         625)8,00(·0128
   320                                     6,25
    300                                    1750
    256                                    1250
     440       480                          5000
     384       448                          5000
      560       320
      512       320
```

∴ the decimal required = 76·0128.

Note. The student will do well to first prove and then commit to memory the following results:

$\frac{1}{2}$ = ·5; $\frac{1}{4}$ = ·25; $\frac{3}{4}$ = ·75. $\frac{1}{8}$ = ·125; $\frac{3}{8}$ = ·375;
$\frac{5}{8}$ = ·625; $\frac{7}{8}$ = ·875; $\frac{1}{16}$ = ·0625; $\frac{1}{5}$ = ·2.

Examples 91.

1. Divide:

(1) 49·44 by 41·2; 3408·255 by 11·1; ·11808 by 59·04; ·003051 by 3·051.

(2) 142·2 by 19·75; 146·6621 by 11·17; 211·06 by 692; 24·402 by ·014.

(3) ·380716 by ·00056; ·0001292 by ·0076; ·00011355 by ·3785.

(4) ·025 separately by ·2, 8, ·32, ·5, ·25, ·125, 1·25, 12500.
(5) 15·46875 separately by 12·5, 1·25, ·125, ·0125, ·00125.
(6) ·00081 by ·3, by ·003, by 3, by 300, by 3000, by ·00003.
(7) 2·16 separately by 1·2, 15, ·12, 1·5, ·00015, 12000.
(8) 3·5 by 700; ·00001 by ·0064; 589·12 by 46·025; ·00008 by 625.
(9) ·001771561 separately by 1·21, ·121, ·0121, 12·1, 12100.
(10) ·0001596 by ·0042; 3·72812 by ·916; ·03611 by 7·85.
(11) ·0006594 by ·314, also by ·0021; 173·889 by ·417, also by 417.
(12) 96·20328 by 3·012, also by 30·12; ·34 by 3·4, by ·05, also by ·002.

2. Find the value, to 4 places of decimals, of:
(1) 68·024÷7·91; 13·625÷3·25; 61·81473÷56·802.
(2) 362·7119÷191·181; ·439507÷59·64; 3247·07÷·036.
(3) 532·253÷21·56; ·09743÷·0046; 1·6÷·0031.
(4) 4·53739÷71·13; 5747·03÷·0063; 7·8508÷·00043.

3. Find the quotient, by *short division*, of:
(1) 5·7681 separately by ·8, ·12, ·16, 2·4, 3·2, ·064, ·072.
(2) 486·972 „ „ ·15, 4·5, 2·5, 12·8, 1·44, ·0256, ·075.

4. Divide by *short division* to 5 places of decimals:
(1) 4·721 separately by 7, ·07, ·007, ·14, ·0014, 1400.
(2) ·01256789451 by ·00000077; ·004521411 by ·00065.

5. Express as decimal fractions:
(1) $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{3}{4}$; $\frac{3}{16}$; $\frac{7}{25}$; $\frac{17}{20}$; $\frac{561}{5000}$; $\frac{37}{125}$.
(2) $5\frac{47}{64}$; $7\frac{91}{250}$; $45\frac{49}{128}$; $237\frac{59}{625}$; $37\frac{29}{32}$; $58\frac{94}{625}$; $729\frac{19}{25}$.

6. Compare the values (by reducing to decimals) of:
(1) $\frac{1}{2}, \frac{1}{3}, \frac{3}{5}$. (2) $\frac{3}{4}, \frac{5}{9}, \frac{4}{7}$. (3) $\frac{3}{10}, \frac{8}{11}, \frac{17}{21}$. (4) $\frac{3}{10}, \frac{2}{7}, \frac{5}{18}$.
(5) $\frac{4}{5}, \frac{7}{8}, \frac{8}{9}$. (6) $\frac{9}{11}, \frac{11}{13}, \frac{13}{15}$. (7) $\frac{2}{3}, \frac{7}{9}, \frac{11}{14}$. (8) $\frac{1}{4}, \frac{5}{9}, \frac{3}{11}$.

Examples worked out.

Ex. 1. Express $\frac{5}{16}$ of ·018 as a decimal.

$$\frac{5}{16} \text{ of } \cdot018 = \frac{5 \times \cdot018}{16} = \frac{\cdot090}{16} = \underline{\cdot005625}.$$

Ex. 2. Find the value of $\dfrac{7\cdot808 \times 7\cdot4339}{\cdot00128 \times \cdot0079}$.

$$\frac{7808 \times 74339}{128 \times 79} = 75401; \quad \frac{7\cdot808 \times 7\cdot4339}{\cdot00128 \times \cdot0079} = \frac{\frac{7808}{1000} \times \frac{74339}{10000}}{\frac{128}{100000} \times \frac{79}{10000}}$$

$$\therefore \frac{7\cdot808 \times 7\cdot4339}{\cdot00128 \times \cdot0079} = 7540100. \qquad = \frac{7808 \times 74339}{10000000} \times \frac{1000000000}{128 \times 79}$$

There are 7 decimal places in the numerator and 9 in the denominator; ∴ 2 cyphers have been added.

$$= \frac{7808 \times 7433900}{128 \times 79} = 7540100.$$

Examples 92.

Find the value of :

1. $\frac{7}{8}$ of ·0026 ; $\frac{9}{16}$ of 24·25 ; $\frac{11}{25}$ of ·00274 ; $7\frac{2}{5}$ of ·021487.

2. $\frac{9}{125}$ of ·00285 ; $\frac{77}{625}$ of ·000125 ; $9\frac{15}{64}$ of ·00000011.

Simplify, expressing as decimals when necessary :

3. $\dfrac{\cdot 0061 \times 198 \cdot 0198}{\cdot 00066}$ 4. $\dfrac{87\cdot 48 \times \cdot 00011}{\cdot 00108}$ 5. $\dfrac{3\cdot 6288}{1\cdot 89 \times \cdot 0025}$

6. $\dfrac{\cdot 065341 \times \cdot 0125}{\cdot 00475 \times \cdot 625}$ 7. $\dfrac{11\cdot 764 \times 1\cdot 68}{3\cdot 46 \times \cdot 012}$ 8. $\dfrac{14\cdot 3045 \times 7424}{4\cdot 27 \times \cdot 0032}$

216. *To find the G. C. M. and the L. C. M. of decimals.*

RULE. Express all the given numbers as integers, by multiplying each of them by 10 or *the same* power 10.

(i) For G. C. M., find the G. C. M. of the integers, and mark in it as many decimal places as there are cyphers in the multiplier, affixing cyphers at the left if necessary.

Ex. Find the G. C. M. of 1·6, 2·4, ·72.

1·6 × 100 = 160, 2·4 × 100 = 240, ·72 × 100 = 72.

The G. C. M. of 160, 240, 72 is 8 ; and the multiplier is 100.

∴ the G. C. M. of the given numbers = 8 ÷ 100 = ·08. *Ans.*

(ii) For L. C. M., find the L. C. M. of the integers, and mark in it as many decimal places as there are cyphers in the multiplier.

Ex. Find the L. C. M. of ·25, ·045, ·0075.

·25 × 10000 = 2500 ; ·045 × 10000 = 450 ; ·0075 × 10000 = 75.

The L. C. M. of 2500, 450 and 75 is 22500, and the multiplier is 10000.

∴ the L. C. M. of the given numbers = 2·2500 = 2·25. *Ans.*

Examples 93.

1. Find the G. C. M. of (1) 1·5, ·35, 1·25 ; (2) ·064, ·04, 4·8, ·32 ; (3) ·09, ·027, ·81 ; (4) 2·24, 33·6, ·728 ; (5) 6·25, ·075, 12·25.

2. Find the L. C. M. of (1) ·15, 1·6, 20, ·028 ; (2) ·024, ·32, 4·5, ·0025 ; (3) ·051, 1·87, 15·3 ; (4) ·22, ·033, 4·4, 55·44.

CHAP XXII. RECURRING DECIMALS.

217. In converting vulgar fractions into decimals, we find that the division terminates in some cases and does not terminate in others. Thus, $\frac{3}{4}$ = ·75, and here the division termi-

nates ; but $\frac{4}{3} = 1\cdot\dot{3}33...$, and in this case the division does not terminate and can be carried on to an unlimited length. The former is called a **terminating** decimal and the latter a **recurring, repeating,** or **circulating** decimal.

218. When a vulgar fraction in its lowest terms is reduced to a terminating decimal, it is practically converted into one which has 10 or some power of 10 in the denominator ; hence by ART. 204, the denominator must be a factor of 10 or some power of 10, and can therefore, consist only of 2, 5 and their powers. In order, therefore, that a given fraction may be converted into a terminating decimal, its denominator must not contain any prime factors except 2 and 5, or their powers.

Thus, (i) $\frac{3}{8} = \frac{3}{2 \times 2 \times 2}$; $\frac{14}{40} = \frac{2 \times 7}{2 \times 2 \times 2 \times 5}$;

∴ they can both be converted into terminating decimals

(ii) $\frac{15}{28} = \frac{3 \times 5}{2 \times 2 \times 7}$;

hence it cannot be converted into terminating decimals.

Examples 94.

1. Point out which of the following vulgar fractions can be reduced to terminating decimals ?

$\frac{5}{6}$; $\frac{7}{12}$; $\frac{21}{128}$; $\frac{4}{35}$; $\frac{7}{16}$; $\frac{5}{14}$; $\frac{27}{80}$; $\frac{93}{48}$; $5\frac{15}{19}$; $\frac{44}{110}$; $\frac{28}{175}$; $\frac{35}{130}$.

2. Which of the numbers between 19 and 65 will, when used as the denominators of fractions, produce terminating decimals ?

219. Decimals which do not terminate are called **circulating, repeating** or **recurring** decimals, because when a decimal does not terminate, certain figure or figures must *come round and round, i. e., recur*, or *be repeated.* For instance, when we reduce $\frac{5}{7}$ to a decimal, we can easily see that the only possible remainders are, 1, 2, 3, 4, 5, 6 ; so that after *six steps at least* there must occur a remainder which has occurred before ; therefore the remainders from that point must be repeated, and consequently also the figures in the quotient.

Ex. 1. Reduce $\frac{5}{3}$ to a decimal.

```
3)5(1·6
  3
  --
  20
  18
  --
  20
```

As there is a repetition of 2 in the remainder, there must be a repetition of the digit 6 in the quotient.

∴ $\frac{5}{3} = 1\cdot666666......$

Ex. 2. Reduce $\frac{7}{11}$ to a decimal.

$$\begin{array}{l} 11)70(\cdot 63 \\ \underline{66} \\ 40 \\ \underline{33} \\ 70 \end{array}$$

As the dividend 70 occurs once again, it is plain that there must be a repetition of the digits 6 and 3 in the quotient.

$\therefore \frac{7}{11} = \cdot 636363\ldots\ldots$

220. The recurring or repeated part is called the **period** or *repetend.* In expressing a recurring decimal, it is usual to avoid writing the recurring part repeatedly, by placing *dots* (˙) over the *first* and *last* digits of the period.

Thus $\frac{5}{3} = 1\cdot 6666\ldots = 1\cdot\dot{6}$; and $\frac{7}{11} = \cdot 636363\ldots = \cdot\dot{6}\dot{3}$.

So, $\cdot\dot{0}6\dot{4} = \cdot 064064064\ldots$; and $\cdot 5\dot{4}\dot{6} = \cdot 5464646\ldots$

The several **periods** in the above are 6, 63, 064, and 46

Examples 95.

Convert each of the following vulgar fractions into recurring decimals:

1. $\frac{1}{3}$; $\frac{5}{6}$; $\frac{2}{7}$; $\frac{4}{9}$; $\frac{10}{11}$; $\frac{9}{14}$; $\frac{11}{15}$; $2\frac{15}{28}$; $5\frac{3}{15}$; $4\frac{7}{17}$; $\frac{25}{33}$; $2\frac{23}{26}$.

2. $\frac{137}{4375}$; $\frac{245}{111}$; $\frac{81}{333}$; $\frac{25461}{999}$; $\frac{4231}{592}$; $\frac{281}{37}$; $\frac{4621}{999}$; $\frac{2846}{275}$;

3. $\frac{3431}{3330}$; $19\frac{451}{3307}$; $33\frac{56241}{999999}$; $44\frac{18}{70000}$; $11\frac{92}{444}$; $\frac{185}{999000}$.

221. A **pure circulating decimal** is one in which the period begins just from off the decimal point; as, $\cdot\dot{7}$, $\cdot\dot{0}6\dot{4}$

A **mixed circulating decimal** is one in which the period begins after some figures which do not recur; as $\cdot 7\dot{2}\dot{1}$, $\cdot 23\dot{4}\dot{2}$.

221 A. A vulgar fraction in its lowest terms whose denominator does not contain 2 or 5 as a factor can be reduced to a *pure circulating decimal,* whereas one whose denominator contains 2 or 5 and other prime factors can be reduced to a *mixed circulating decimal.* Thus,—

$\frac{2}{7} = \cdot\dot{2}8571\dot{4}$; $\frac{1}{41} = \cdot\dot{0}243\dot{9}$; $\frac{7}{15} = \cdot 4\dot{6}$; $\frac{3}{14} = \cdot 2\dot{1}4285\dot{7}$.

222. *To reduce a* pure circulating decimal *to a vulgar fraction.*

Rule. Place in the numerator the period (*i. e.*, the figures which recur), and in the denominator the number made up of as many *nines* as there are digits in the period; and reduce the fraction so obtained to its lowest terms.

Ex. Convert $\cdot\dot{3}$ and $\cdot\dot{7}\dot{5}$ into equivalent vulgar fractions in their lowest terms.

$\cdot\dot{3} = \frac{3}{9} = \frac{1}{3}$.

[$\cdot\dot{3} = \cdot 3333\ldots\ldots$

$10 \times \cdot\dot{3} = 3\cdot 3333\ldots\ldots$

$\therefore 9 \times \cdot\dot{3} = 3$, or $\cdot\dot{3} = \frac{3}{9}$.]

$\cdot\dot{7}\dot{5} = \frac{75}{99} = \frac{25}{33}$.

[$\cdot\dot{7}\dot{5} = \cdot 757575\ldots\ldots$;

$100 \times \cdot\dot{7}\dot{5} = 75\cdot 757575\ldots\ldots$

$\therefore 99 \times \cdot\dot{7}\dot{5} = 75$, or $\cdot\dot{7}\dot{5} = \frac{75}{99}$.]

223. *To reduce a* mixed circulating decimal *to a vulgar fraction.*

RULE. Subtract, for the numerator, the non-recurring part from the number made up of the non-recurring followed by the recurring figures; and put in the denominator as many 9's as there are figures in the recurring part, followed by as many o's as there are figures in the non-recurring part.

Ex. Convert $\cdot 2\dot{4}$, $\cdot 02\dot{4}\dot{3}$ and $\cdot 25\dot{7}6\dot{8}$ into equivalent vulgar fractions in their lowest terms.

$$\cdot 2\dot{4}=\frac{24-2}{90}=\frac{22}{90}=\frac{11}{45}; \qquad \cdot 02\dot{4}\dot{3}=\frac{243-2}{9900}=\frac{241}{9900};$$

$$\cdot 25\dot{7}6\dot{8}=\frac{25768-25}{99900}=\frac{25743}{99900}=\frac{8581}{33300}.$$

[$\cdot 2\dot{4}=\cdot 244444\ldots\ldots$
$10\times\cdot 2\dot{4}=2\cdot 44444\ldots\ldots$
$100\times\cdot 2\dot{4}=24\cdot 44444\ldots\ldots$
$\therefore 90\times\cdot 2\dot{4}=24-2$;
or $\cdot 2\dot{4}=(24-2)\div 90$.]

[$\cdot 02\dot{4}\dot{3}=\cdot 024343\ldots\ldots$
$100\times\cdot 02\dot{4}\dot{3}=2\cdot 434343\ldots\ldots$
$10000\times\cdot 02\dot{4}\dot{3}=243\cdot 434343\ldots\ldots$
$\therefore 9900\times\cdot 02\dot{4}\dot{3}=243-2$;
or $\cdot 02\dot{4}\dot{3}=(243-2)\div 9900$.]

224. When a pure or a mixed circulating decimal is preceded by a whole number, it can be converted into an equivalent vulgar fraction by the Rule given in ARTS. 222 and 223. It should be remembered, however, that the integral part has to be included in the non-recurring part when the operation of subtraction is performed, but should be left out of account when cyphers are affixed to the denominator.

Thus, (1) $2\cdot\dot{5}\dot{7}=10\times\cdot 2\dot{5}\dot{7}=10\times\frac{257-2}{990}=\frac{257-2}{99}.$

(2) $13\cdot 25\dot{7}\dot{2}=100\times\cdot 1325\dot{7}\dot{2}=100\times\frac{132572-1325}{990000}$

$$=\frac{132572-1325}{9900}.$$

[It will be seen that $\cdot\dot{9}=\frac{9}{9}=1$; $\cdot 0\dot{9}=\frac{9}{90}=\frac{1}{10}=\cdot 1$. So, $2\cdot 57\dot{9}=(2579-257)\div 900=25\cdot 8$. Thus when the recurring part consists of **nines only** we should leave it out and add 1 to the preceding figure.]

N.B.—Remember that $\cdot\dot{3}=\frac{1}{3}$; $\cdot\dot{6}=\frac{2}{3}$; $\cdot\dot{1}=\frac{1}{9}$; $\cdot\dot{0}\dot{9}=\frac{1}{11}$; $\frac{1}{27}=\cdot\dot{0}3\dot{7}$; $\frac{1}{37}=\dot{0}2\dot{7}$; $\cdot\dot{1}234567\dot{9}=\frac{10}{81}$; and *vice versa*.

Note 1. $\frac{3}{11}=\frac{3\times 9}{11\times 9}=\frac{27}{99}=\cdot\dot{2}\dot{7}.$

$$\frac{2}{27}=\frac{2\times 37}{27\times 37}=\frac{74}{999}=\dot{0}7\dot{4}.$$

$$\frac{4}{37}=\frac{4\times27}{37\times27}=\frac{108}{999}=\cdot\dot{1}0\dot{8}.$$

$$\frac{3}{101}=\frac{3\times99}{101\times99}=\frac{297}{9999}=\cdot\dot{0}29\dot{7}.$$

$$\frac{4}{15}=\frac{4\times6}{15\times6}=\frac{24}{90}=\frac{26-2}{90}=\cdot2\dot{6}.$$

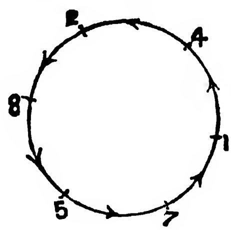
$$\frac{5}{12}=\frac{5\times75}{12\times75}=\frac{375}{900}=\frac{416-41}{900}=\cdot41\dot{6}.$$

Students should practice this method of converting vulgar fractions into recurring decimals in simple cases.

Note 2. When we reduce a vulgar fraction to a pure circulating decimal (Art 221 A) the possible remainders are 1, 2, 3, 4...up to a number which is less than the denominator of the fraction by unity.

(*i*) In some cases we get all these remainders, therefore in such cases all the proper fractions with the same denominator when reduced to decimals must have the same remainders but in different order; consequently the quotients have the same digits but in different order. Therefore all these fractions when reduced to circulating decimals have the same digits for their periods but in different order. As for example

$\frac{1}{7}=\cdot\dot{1}4285\dot{7}$; $\frac{2}{7}=\cdot\dot{2}8571\dot{4}$; $\frac{3}{7}=\cdot\dot{4}2857\dot{1}$; $\frac{4}{7}=\cdot\dot{5}7142\dot{8}$; $\frac{5}{7}=\cdot\dot{7}1428\dot{5}$; $\frac{6}{7}=\cdot\dot{8}5714\dot{2}$. Thus the same digits 1, 2, 4, 5, 7, 8 in different orders from the periods of the decimals equivalent to fractions having 7 for the denominators.

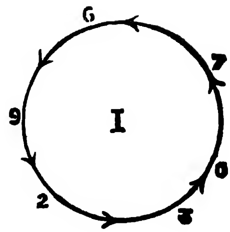


If these digits be placed round a circle and read off beginning with 1, 2, 4, 5, 7, 8 in turn with the other digits in the order as they are placed round the circle we get the periods of the decimals equivalent respectively to $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$.

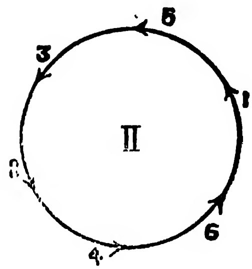
Students may take up the fractions with denominators 17, 19, 23, 29 .. &c. and see the relation between the periods of the decimals equivalent to vulgar fractions having the same denominator.

(*ii*) In other cases those remainders do not occur all at a time. So that same fractions with the same denominator when reduced to decimals have one set of digits for their periods and the remaining fractions have other sets. As for example, fractions with the denominator 13 may be divided into 2 groups. Fractions $\frac{1}{13}$, $\frac{3}{13}$, $\frac{4}{13}$, $\frac{9}{13}$, $\frac{10}{13}$, $\frac{12}{13}$ when reduced to decimals have the same digits 0, 7, 6, 9, 2, 3 in different orders for their periods whereas the decimals equivalent to fractions $\frac{2}{13}$, $\frac{5}{13}$, $\frac{6}{13}$, $\frac{7}{13}$, $\frac{8}{13}$, $\frac{11}{13}$ have the same digits 1, 3, 4, 5, 6, 8 for their periods. So that $\frac{1}{13}=\cdot\dot{0}7692\dot{3}$; $\frac{3}{13}=\cdot\dot{2}3076\dot{9}$; $\frac{4}{13}=\cdot\dot{3}0769\dot{2}$; $\frac{9}{13}=\cdot\dot{6}9230\dot{7}$; $\frac{10}{13}=\cdot\dot{7}6923\dot{0}$; $\frac{12}{13}=\cdot\dot{9}2307\dot{6}$; and $\frac{2}{13}=\cdot\dot{1}5384\dot{6}$; $\frac{5}{13}=\cdot\dot{3}8461\dot{5}$; $\frac{6}{13}=\cdot\dot{4}6153\dot{8}$; $\frac{7}{13}=\cdot\dot{5}3846\dot{1}$; $\frac{8}{13}=\cdot\dot{6}1538\dot{4}$; $\frac{11}{13}=\cdot\dot{8}4615\dot{3}$.

Thus if the digits 0, 7, 6, 9, 2, 3 and the digits 1 5, 3, 8, 4, 6 be placed round the circle I and II and read off beginning with 0, 2, 3, 6, 7, 9 and 1 3, 4, 5, 6, 8 in turn with the other digits in order as they are placed round the circle, we get decimals equivalent to $\frac{1}{13}$, $\frac{3}{13}$, $\frac{4}{13}$, $\frac{9}{13}$, $\frac{10}{13}$, $\frac{12}{13}$ and $\frac{2}{13}$, $\frac{5}{13}$, $\frac{6}{13}$, $\frac{7}{13}$, $\frac{8}{13}$, $\frac{11}{13}$.



Students should find out the 2 groups of fractions with the denominator 31, 3 groups of fractions with 21 as denominator and 8 groups of fractions with the denominator 41, each group having the same digits for the periods of their equivalent decimals.



Generally the number of digits in the periods of the decimals equivalent to different groups of fractions in their lowest terms having the same denominator, is the same.

Examples 96.

1. Reduce the following recurring decimals to equivalent vulgar fractions in their lowest terms:

(1) $\cdot 1\dot{8}$; $\cdot 0\dot{1}\dot{8}$; $\cdot\dot{0}2\dot{1}$; $\cdot\dot{0}1\dot{5}$; $\cdot 2\dot{1}\dot{7}$; $\cdot 0\dot{0}3\dot{7}$; $\cdot 00\dot{3}\dot{7}$; $\cdot 5\dot{4}$; $\cdot 1\dot{6}$; $\cdot\dot{0}1\dot{6}$.

(2) $\cdot 00\dot{3}$; $\cdot\dot{7}26\dot{3}$; $\cdot\dot{8}1\dot{9}$: $\cdot 81\dot{2}$; $\cdot 1\dot{5}\dot{6}$: $\cdot\dot{1}4285\dot{7}$; $\cdot\dot{7}1428\dot{5}$; $\cdot 60\dot{5}$.

(3) $\cdot 10\dot{1}8\dot{5}$; $\cdot 3\dot{5}7142\dot{8}$; $\cdot\dot{3}8461\dot{5}$; $\cdot\dot{8}5714\dot{2}$; $\cdot 2\dot{1}4285\dot{7}$; $7\cdot\dot{7}1428\dot{5}$.

(4) $4\cdot\dot{3}7606\dot{8}$; $3\cdot\dot{6}9230\dot{7}$; $\cdot 504\dot{5}$; $72\cdot 004\dot{2}857\dot{1}$: $\cdot 473\dot{5}121\dot{9}$.

2. Express the following as terminating decimals:

$3\cdot\dot{9}$; $4\cdot 2\dot{9}$; $7\cdot 0\dot{9}$; $19\cdot\dot{9}9\dot{9}$; $15\cdot\dot{9}\dot{9}$; $\cdot 42\dot{9}\dot{9}$; $49\cdot\dot{9}\dot{9}$; $17\cdot 4\dot{9}$.

ADDITION OF RECURRING DECIMALS.

225. (i) A recurring decimal may be expressed in different forms by marking the period from any figure after the first circulating figure.

Thus, $\cdot\dot{3}614\dot{5}$ **may be written** $\cdot 36\dot{1}45\dot{6}$, $\cdot 361\dot{4}56\dot{1}$, $\cdot 3614\dot{5}61\dot{4}$, &c.

(ii) The period can be repeated as often as we like.

Thus, $\cdot 2\dot{7}\dot{5} = \cdot 2\dot{7}5757\dot{5}$.

226. **Similar** recurring decimals are those in which the number of the non-recurring digits and of the circulating digits are the same respectively.

Thus, $\cdot 34\dot{5}6\dot{3}$ and $7\cdot 46\dot{2}8\dot{1}$ are *similar* recurring decimals.

227. Different circulating decimals can be made similar by applying methods (i) and (ii), ART. 225.

Ex. Convert $5\cdot\dot{7}$, $\cdot 24\dot{7}\dot{5}$, and $\cdot 421\dot{5}6\dot{8}$ into similar recurring decimals.

$$5\cdot\dot{7} = 5\cdot 777\dot{7}7777\dot{7}$$
$$\cdot 24\dot{7}\dot{5} = \cdot 247\dot{5}7575\dot{7}$$
$$\cdot 421\dot{5}6\dot{8} = \cdot 421\dot{5}6856\dot{8}$$

The largest number of non-recurring figures occurs in the last, and $=3$. Also the L. C. M. of the number of digits in the 3 periods $=6$. Therefore 3 figures should be taken in the non-recurring, and 6 in the recurring, part of each.

228. RULE (for addition.) Place the decimals under one another making them all similar, as in the preceding ART.; and proceed to add as in Simple Addition, taking care to add to the last figure the number of tens to be *carried* to the recurring part and remembering that the sum is a recurring decimal similar to those to which the given decimals have been reduced.

Note. It is usual to take 2 or 3 figures *more*, to find the numbers of *tens* to be *carried*, as in the following Example.

Ex. Add together $2\cdot 003\dot{5}\dot{8}$, $\cdot 08\dot{3}889\dot{4}$, $35\cdot 1612$, $1\cdot 00\dot{6}$.

$$\begin{array}{r|l}
2\cdot 0035\ 858585 & 85 \\
\cdot 0838\ 899438 & 89 \\
35\cdot 1612 & \\
1\cdot 0066\ 666666 & 66 \\
\hline
38\cdot 2553\ \dot{4}2469\dot{1} &
\end{array}$$

Examples 97.

Add together :

1. $9\cdot 992\dot{8} + 37\cdot 4\dot{0}\dot{1} + 88\cdot 063 + \cdot 4\dot{0}7\dot{1} + 12\dot{3}\dot{6} + 1\cdot 006$.
2. $\cdot 0\dot{3}1\dot{3} + 3\cdot 60\dot{6} + 32\cdot 0\dot{1}01\dot{1} + 3\cdot 0216 + 2\cdot 81\dot{4} + \cdot 0\dot{5}\dot{9}$.
3. $2\cdot 188\dot{1} + 6\cdot 7671 + \cdot 00\dot{0}31\dot{4} + 5\cdot 9\dot{1}6\dot{2} + \cdot 40\dot{7}\dot{1} + \cdot\dot{0}7\dot{4}$.
4. $3\cdot 7671 + \cdot 00\dot{5}42\dot{6} + 8\cdot 125\dot{5}\dot{4} + 5\cdot 9\dot{1}6\dot{2} + \cdot 3021\dot{6}$.
5. $\cdot 4\dot{1}2\dot{3} + 4\cdot 1\dot{2}0\dot{3} + \cdot 03\dot{7}4\dot{1} + \cdot 0\dot{1}\dot{2} + 0\cdot 03\dot{7} + \cdot 001\dot{2}\dot{3} + 24\cdot 042$.

229. *To add together given circulating decimals correctly to* any number of decimal places.

RULE. Put down the numbers one under another after extending each of them by repetition of its period to a number

of decimal places which should be 2 or 3 places more than the number of decimal places required in the sum. Then add as in decimal Addition.

Ex. Add together $3{\cdot}428\dot{1} + {\cdot}00\dot{2}7\dot{3} + 1{\cdot}5612\dot{0}0\dot{7} + 2{\cdot}00\dot{2}8\dot{6} + {\cdot}\dot{2}\dot{9}$ to *seven* places of decimals.

$$\begin{array}{r|l}
3{\cdot}4281111 & 111 \\
{\cdot}0027327 & 327 \\
{\cdot}5612007 & 007 \\
2{\cdot}0028628 & 628 \\
{\cdot}2929292 & 929
\end{array}$$

If we had taken only 2 columns more instead of 3, we would have got $6{\cdot}287836697$ or $6{\cdot}2878366$ instead of the correct answer $6{\cdot}2878367$. When in fact, it is found that the first of the figures left out is 9, it is always advisable to take one or two more columns than at first.

Examples 98.

Add together, correct to 6 places of decimals :

1. $3{\cdot}25\dot{6}2\dot{1} + 43{\cdot}0002\dot{6} + {\cdot}0129\dot{6} + 57{\cdot}12\dot{3}6\dot{7} + {\cdot}000234\dot{8}$.

$23{\cdot}4615 + 3621 + 59{\cdot}\dot{1}24\dot{6} + 10{\cdot}89\dot{4}6\dot{1} + {\cdot}00729\dot{4}16\dot{2}$.

$3{\cdot}56\dot{9}2\dot{4} + 69{\cdot}002\dot{1}\dot{4} + {\cdot}02\dot{1}4691\dot{5} + {\cdot}0012\dot{8}\dot{7} + 46$.

$9{\cdot}28\dot{9}\dot{1} + 42{\cdot}\dot{3}\dot{1} + {\cdot}00\dot{2}\dot{1} + {\cdot}7\dot{2}1\dot{3} + 94{\cdot}64\dot{1}2\dot{4} + 42{\cdot}4$.

$789{\cdot}21\dot{4}\dot{6} + {\cdot}002\dot{1}4\dot{5} + 7{\cdot}12\dot{4}61\dot{4} + {\cdot}6\dot{2}145\dot{6} + {\cdot}\dot{0}0021\dot{4}$.

SUBTRACTION OF RECURRING DECIMALS.

230. The RULE given for addition should also be followed in the Subtraction of circulating decimals.

Ex. 1. Subtract $3{\cdot}2\dot{9}\dot{6}$ from $15{\cdot}00\dot{2}1\dot{3}$

$$\begin{array}{rl|l}
15{\cdot}00 & \dot{2}1321\dot{3} & 2 \\
3{\cdot}29 & \dot{6}9696\dot{9} & 6 \\
\hline
11{\cdot}70 & \dot{5}1624\dot{3} &
\end{array}$$

The periods have 2 and 3 figures; the L.C.M. is 6 ; therefore the period of the difference has 6 figures.

Note. In Subtraction it is never necessary to take more than one column of figures, unless the figures in the additional column are the same.

Ex. 2. Find (1) the difference of $5{\cdot}60\dot{6}$ and $4{\cdot}\dot{5}6\dot{4}$ correct to [illegible] places of decimals ; (2) the **complement** of ${\cdot}7\dot{3}408\dot{2}$, *i.e.*, its difference from 1.

$$\begin{array}{r|l}
5{\cdot}60666 & 66 \\
4{\cdot}56464 & 64 \\
\hline
1{\cdot}04202 &
\end{array}
\qquad\qquad
\begin{array}{r|l}
1{\cdot}000000 & 0 \\
{\cdot}7\dot{3}408\dot{2} & 3 \\
\hline
\end{array}$$

Examples 99.

1. Find the difference of :

(1) $56{\cdot}24\dot{1}\dot{4} - 34{\cdot}2\dot{8}\dot{1}$ (2) $7{\cdot}8\dot{1}46\dot{2} - 4{\cdot}\dot{5}2\dot{1}4\dot{3}$ (3) ${\cdot}5\dot{1}67 - {\cdot}28\dot{6}3\dot{4}$

(4) $1{\cdot}2 - 1{\cdot}1\dot{7}6\dot{9}$ (5) ${\cdot}\dot{1} - {\cdot}0\dot{9}$ (6) $25{\cdot}\dot{8}\dot{2} - 16{\cdot}3\dot{0}6\dot{7}$

2. Find the value, correct to 6 places of decimals, of :

(1) $45{\cdot}621\dot{4} - 41{\cdot}44\dot{6}2\dot{8}$ (2) ${\cdot}36\dot{7} - {\cdot}03\dot{1}4\dot{2}$ (3) ${\cdot}82\dot{9}60\dot{3} - {\cdot}\dot{5}6\dot{3}2$

(4) $9{\cdot}0\dot{5}2\dot{6} - 7{\cdot}0\dot{6}8\dot{6}$ (5) $18{\cdot}06\dot{3}\dot{4} - 15{\cdot}89\dot{6}2\dot{4}$ (6) $5{\cdot}21 - 3{\cdot}82\dot{6}41\dot{7}$

3. Find the complements of ${\cdot}327\dot{4}26\dot{0}$, ${\cdot}\dot{4}256\dot{8}$, and ${\cdot}9820\dot{3}82\dot{1}$.

MULTIPLICATION OF RECURRING DECIMALS.

231. A recurring decimal may be multiplied by a whole number or by a terminating decimal.

(*a*) *To multiply by an integer.*

RULE. Extend the multiplicand 2 or 3 places beyond the period, in order to ensure the correctness of the last figure retained ; and make the product *similar* to the multiplicand.

Ex. Multiply $5{\cdot}64\dot{2}\dot{7}$ by 7, and by 11.

$5{\cdot}64\dot{2}\dot{7}|27$ $5{\cdot}64\dot{2}\dot{7}|27$

$39{\cdot}4$ $62{\cdot}06\dot{9}\dot{9} = 62{\cdot}07$ (See ART. 224.)

(*b*) *To multiply by a terminating decimal.*

RULE. Extend 2 or 3 places beyond the end of the first period, in order to ensure the correctness of the last figure retained, and take care to have as many decimal places in the product as there are decimal places in the multiplicand and multiplier together ; the period consisting of as many digits as there are figures in the period of the multiplicand.

Ex. Multiply $5{\cdot}42\dot{2}1\dot{8}$ by $2{\cdot}345$.

$5{\cdot}42\dot{2}1\dot{8}|218$
$2{\cdot}345$

$2711\dot{0}9\dot{1}$
$[illegible]68\dot{8}7\dot{2}$
$162\dot{6}65\dot{4}$
$108\dot{4}43\dot{6}$

$2711\dot{0}9\dot{1}|091$
$21688728|728$
$162665465|465$
$1084436436|436$
$12{\cdot}71501\dot{7}2\dot{1}$

232. *To multiply one circulating decimal by another.*

RULE. Reduce the decimals to equivalent vulgar fractions, multiply as in ART. 176, and reduce the product to a decimal.

Ex. Multiply ${\cdot}0\dot{1}\dot{2}$ by ${\cdot}0\dot{8}$.

${\cdot}0\dot{1}\dot{2} = \frac{12}{990} = \frac{2}{165}$; ${\cdot}0\dot{8} = \frac{8}{90} = \frac{4}{45}$.

$\therefore$ the product reqd. $= \frac{2}{165} \times \frac{4}{45} = \frac{8}{7425} = {\cdot}00\dot{1}0774\dot{4}$. *Ans.*

Examples 100.

Multiply :

1. $4\cdot\dot{5}\dot{7}$ by 5, by 8.
2. $7\cdot 0\dot{5}$ by 12, by 16.
3. $1\cdot 12\dot{5}\dot{6}$ by 18, by 24.
4. $\cdot 00\dot{8}\dot{1}$ by 32, by 36.
5. $\cdot 5\dot{7}\dot{8}$ by 44, by 48.
6. $\cdot 00\dot{3}2\dot{5}$ by 56, by 60.
7. $41\cdot 05\dot{6}\dot{3}$ by $2\cdot 06$.
8. $\cdot 04\dot{2}\dot{1}$ by $2\cdot 12$.
9. $\cdot 0004\dot{2}\dot{7}$ by $4\cdot 708$.
10. $49\cdot 56\dot{2}$ by $\cdot 00081$.
11. $47\cdot 25\dot{1}$ by $8\cdot 44\dot{5}$.
12. $\cdot 041\dot{3}$ by $\cdot 0\dot{2}\dot{7}$.
13. $3\cdot\dot{9}8\dot{1}$ by $\cdot 0\dot{4}$.
14. $30\cdot\dot{2}\dot{7}$ by $3\cdot\dot{7}7\dot{2}$.
15. $\cdot\dot{8}8\dot{9}$ by $\cdot\dot{1}2\dot{6}$.
16. $\cdot 005$ by $\cdot 00\dot{5}$.
17. $\cdot 09\dot{9}\dot{0}$ by $\cdot 7163$.
18. $\cdot 01\dot{1}1\dot{2}$ by $4\cdot 0\dot{3}\dot{6}$.

DIVISION OF RECURRING DECIMALS.

233. *To divide a circulating decimal by a whole number or by a terminating decimal.*

RULE. Proceed to divide as in ordinary Decimal Division : but instead of affixing cyphers at every successive step, bring down in succession the several digits of the period.

Ex. Divide $4\cdot 2\dot{3}$ by $\cdot 06$.

$$
\begin{array}{l}
6)42\dot{3}\dot{3}\backslash 3\ldots(70\cdot\dot{5} \\
\underline{42} \\
33 \\
\underline{30} \\
33
\end{array}
$$

234. *To divide one circulating decimal by another.*

RULE (i) Reduce the decimals to equivalent vulgar fractions, divide as in ART. 178, and reduce the quotient to a decimal.

Ex. Divide $1\cdot 2\dot{7}$ by $\cdot\dot{0}3\dot{7}$.

$$1\cdot 2\dot{7}=\frac{127-12}{90}=\frac{115}{90}=\frac{23}{18};\quad \cdot\dot{0}3\dot{7}=\frac{37}{999}=\frac{1}{27};$$

$$\therefore \text{ the quotient}=\frac{23}{18}\div\frac{1}{27}=\frac{23\times 27}{18}=\frac{69}{2}=\underline{34\cdot 5}.$$

(ii) Make both similar and proceed as in ART. 224.

$$\frac{1\cdot 2\dot{7}}{\cdot\dot{0}3\dot{7}}=\frac{1\cdot 2\dot{7}7\dot{7}}{\cdot\dot{0}37\dot{0}}=\frac{12777-12}{9990}\div\frac{370}{9990}=\frac{12765}{370}=\underline{34\cdot 5}.$$

Examples 101.

Divide :

1. $\cdot\dot{3}$ by 4, by 6.
2. $\cdot 665\dot{5}\dot{0}$ by 8, by 25.
3. $1\cdot 21$ by 8.
4. $4125\cdot 78\dot{5}$ by 560.
5. $\cdot\dot{5}8755\dot{3}$ by 834.
6. $2\cdot\dot{3}$ by 21.

7. $3.9914\dot{2}74\dot{8}$ by 7.62. **8.** $.2\dot{0}0\dot{1}$ by 100.1, by 1.001.

9. $8.6\dot{5}$ by $.0101$, by 10.1. **10.** $99.\dot{6}$ by 6.4, by $.46$.

11. $.11\dot{2}\dot{7}$ by $.41\dot{3}$, by $.\dot{2}\dot{7}$. **12.** $1.04\dot{3}2\dot{7}$ by $31.2\dot{9}8\dot{1}$, by $.0\dot{3}$.

13. $1.\dot{7}6\dot{9}$ by $3.\dot{9}8\dot{1}$. **14.** $.281\dot{3}$ by $5.0\dot{6}\dot{3}$, by $.0\dot{5}$.

15. $161.\dot{8}5\dot{1}$ by $15.\dot{3}$, by $10.\dot{5}$. **16.** $.32\dot{7}$ by $26.\dot{8}\dot{1}$, by $.012$.

Examples 102.

Simplify, expressing the results as decimals :

1. $\dfrac{2.4-1.7}{5.4-5.12}$ **2.** $\dfrac{2.233-3.41+2.86}{13.31-10.142}$ **3.** $\dfrac{42.55\times.0056}{.00035}$

4. $\dfrac{3-.15}{35+3.5}\times\dfrac{21+5.6}{17.2-5.8}\div\dfrac{.125}{1.25}$ **5.** $\dfrac{4.75+8.\dot{5}\dot{4}}{.8\dot{6}\div10.5}\times\dfrac{6.27\times.\dot{4}}{4+2.\dot{3}}$

6. $\dfrac{15.\dot{6}+7-.\dot{3}}{7.\dot{4}\times.25}$ **7.** $\dfrac{2.2\dot{5}}{2.\dot{6}}+\dfrac{2.5+5.2}{3.\dot{3}+\dot{9}.5}+\dfrac{1.\dot{3}}{2.\dot{6}}$

8. $\dfrac{31.35\times5.5}{\frac{15}{7}\text{ of }.875\times91.96}\times\dfrac{\frac{35}{6}\text{ of }.\dot{6}+1.\dot{0}\dot{1}}{\frac{3}{10}\text{ of }1.75\times7\frac{1}{6}\text{ of }.75}$

9. $\left\{.\dot{3}+\dfrac{.\dot{4}2857\dot{1}}{1.5}+\dfrac{.7\dot{8}5714\dot{2}+.5}{.\dot{2}\dot{7}}-1\right\}\div(.\dot{5}\times.625)\times\frac{3}{20}$

10. $\dfrac{12.5-8.75}{6.\dot{6}\times4+.8\dot{3}\times18}\div\dfrac{.5+\frac{2}{5}}{5-.\dot{5}}+.\dot{5}$

11. $\dfrac{1+2.5+3.\dot{3}}{.\dot{6}+.\dot{8}5714\dot{2}+12\frac{1}{7}}\div\dfrac{5.125-2.\dot{6}}{3.25-2.5}$

12. $\dfrac{\frac{3}{8}+.25}{4-\frac{5}{8}\text{ of }5.5}\times\dfrac{17}{7+\dfrac{3}{4-2.75}}\div\dfrac{.4+.25}{.7\times4.5-2.5}\times9.4$

13. $\dfrac{1.25}{.08\dot{3}}\times.\dot{6}+\dfrac{2.5-1.8\dot{3}}{2.5+1.8\dot{3}}-\dfrac{7.2}{7.8}$

14. $\dfrac{\frac{4}{5}\text{ of }1.\dot{6}}{1\frac{4}{11}\text{ of }1.\dot{8}}+\dfrac{1\frac{6}{7}\text{ of }1.9}{1\frac{3}{8}\text{ of }1.875}\times\dfrac{.05\text{ of }\frac{1}{17}}{\frac{1}{10}\text{ of }.0\dot{5}}+\dfrac{\frac{1}{5}\text{ of }.\dot{0}7692\dot{3}}{\frac{1}{8}\text{ of }.\dot{0}\dot{9}}$

15. $\dfrac{3}{5}\text{ of }.8125-\dfrac{1\frac{3}{4}}{6\dot{6}}\times.95+\dfrac{3}{7}\text{ of }\dfrac{6.41\dot{6}}{3.\dot{6}}$

16. $\dfrac{.3\times.3\times.3+.03\times.03\times.03}{.6\times.6\times.6+.06\times.06\times.06}$

17. $\dfrac{5.375+4.6}{8.6-5.[illegible]}+\left\{\dfrac{.875-.\dot{8}5714\dot{2}+.8\dot{3}}{1.\dot{1}4285\dot{7}-.\dot{7}+1.2}\text{ of }1\frac{3}{11}+3.9\right\}$

18. $\dfrac{(\cdot325)^2-(\cdot275)^2}{\cdot325-\cdot275}$ **19.** $\dfrac{(2\cdot75)^2-(2\cdot25)^2}{2\cdot75+2\cdot25}$

20. $\dfrac{(\cdot02)^3+(\cdot002)^3}{(\cdot04)^3+(\cdot004)^3}$ **21.** $\dfrac{(\cdot05)^3-(\cdot005)^3}{(\cdot005)^3-(\cdot0005)^3}$

22. $\cdot03125\times\cdot\dot{7}804\dot{8}$ of $5\cdot\dot{8}5714\dot{2}\times\cdot\dot{5}3846\dot{1}$ of $3\frac{1}{4}$.

23. $\dfrac{\{\cdot\dot{2}682\dot{9}+\cdot\dot{3}658\dot{5}\}+5\cdot2}{\cdot\dot{1}219\dot{5}}-\cdot\dot{2}8571\dot{4}$

24. $\dfrac{3\cdot375\times\cdot\dot{0}3\dot{7}}{5\frac{1}{2}\text{ of }2\cdot\dot{3}\dot{4}}+\{\cdot\dot{4}+\cdot\dot{1}2345679\dot{9}\times\cdot015625\}+$

$$\left\{\frac{2\cdot6\text{ of }5\frac{3}{11}}{11\cdot6\times1\cdot\dot{1}\dot{8}}\div\cdot\dot{1}5384\dot{6}+\frac{1}{2}\right\}\text{ of }\frac{1}{14\cdot5}$$

REDUCTION OF DECIMALS.

235. The following are Examples of simple and compound quantities expressed in decimals.

Ex. 1. Reduce *Rs.* 3·75 to *pies*, and find the value of *Rs.* 15·855.

$$\begin{array}{r} Rs.\,3\cdot75 \\ 16 \\ \hline a.\ 60\cdot00 \\ 12 \\ \hline 720p. \end{array} \qquad\qquad \begin{array}{r} Rs.\,15\cdot855 \\ 16 \\ \hline a.\ 13\cdot680 \\ 12 \\ \hline p.\ 8\cdot16 \end{array}$$

The reqd. result $=$ 720*p*. The reqd. value $=$ *Rs.* 15. 13*a.* 8·16*p*.

Ex. 2. Find the value of 25·275 of £3, and 2·725 of *Rs.* 7.

$$\begin{array}{r} 25\cdot275 \\ 3 \\ \hline £75\cdot825 \\ 20 \\ \hline s.\ 16\cdot500 \\ 12 \\ \hline d.\ 6\cdot0 \end{array} \qquad\qquad \begin{array}{r} 2\cdot725 \\ 7 \\ \hline Rs.\,19\cdot075 \\ 16 \\ \hline a.\ 1\cdot200 \\ 12 \\ \hline p.\ 2\cdot4 \end{array}$$

The reqd. result $=$ £75. 16*s*. 6*d*. The reqd result $=$ *Rs.* 19. 1*a.* 2·4*p*.

Ex. 3. Find the value of ·375 of £7. 9*s*. 8*d*.

(*a*) ·375 of £7. 9*s*. 8*d*. $=$ ·375 of £7 + ·375 of 9*s*. + ·375 of 8*d*.

$$\begin{array}{r} \cdot375 \\ 7 \\ \hline £2\cdot625 \\ 20 \\ \hline s.\ 12\cdot500 \\ 12 \\ \hline d.\ 6\cdot0 \end{array} \qquad \begin{array}{r} \cdot375 \\ 9 \\ \hline s.\ 3\cdot375 \\ 12 \\ \hline d.\ 4\cdot500 \\ 4 \\ \hline 2\cdot0q. \end{array} \qquad \begin{array}{r} \cdot375 \\ 8 \\ \hline d.\ 3\cdot000 \end{array}$$

∴ ·375 of £7 $=$ £2. 12*s*. 6*d*.
·375 of 9*s*. $=$ 3*s*. $4\frac{1}{2}$*d*.
·375 of 8*d*. $=$ 3*d*.
∴ reqd. value $=$ £2. 16*s*. $1\frac{1}{2}$*d*.

(*b*) £7. 9*s*. 8*d*. × ·375 = 1796*d*. × ·375 = 673·5*d*. = £2. 16*s*. 1½*d*.

(*c*) ·375 of £7. 9*s*. 8*d*. = $\frac{375}{1000}$ of £7. 9*s*. 8*d*. = $\frac{3}{8}$ of £7. 9*s*. 8*d*.
= 18*s*. 8½*d*. × 3 = £2. 16*s*. 1½*d*.

N.B.—The last two methods are always found more advantageous.

Ex. 4. Find the value of 3·5$\dot{6}$ of *Rs*.9. 6*a*.

3·5$\dot{6}$ of *Rs*.9. 6*a*. = $3\frac{51}{90}$ of *Rs*.9. 6*a*. = *Rs*.9. 6*a*. × 3 + *Rs* 9. 6*a*. × $\frac{17}{30}$
= *Rs*.28. 2*a*. + *Rs*.5 5*a*. = *Rs*.33. 7*a*.

Ex. 5. Express *Rs*.3. 7*a*. 8*p*. in its highest dinomination as a decimal.
Rs.3. 7*a*. 8*p*. = *Rs*. $3\frac{23}{48}$ = *Rs*.3·4791$\dot{6}$

Examples 103.

1. Reduce each of the following to its lowest denomination :

(1) R5·875 ; R24·000275 ; R17·1575 ; R0·000175.

(2) £7·15 ; £·002575 ; £3·16875 ; £·28125.

(3) R·$\dot{3}$; R·332$\dot{5}$; R37·$\dot{0}$8$\dot{7}$; R·2$\dot{0}$0$\dot{1}$.

(4) £9·1$\dot{4}$; £4·1$\dot{6}$; £11·$\dot{2}\dot{5}$; £5·$\dot{5}$; £32·$\dot{1}$8$\dot{1}$.

(5) 4·275075 Mds. ; 5·0375 mi. ; ·025 cwt. ; ·3865 da.

2. Express as compound quantities :

(1) R3·35 ; R4·725 ; R2·0625 ; R3·071$\dot{6}$.

(2) £3·6125 ; £2·21375 ; £·0275 ; £·$\dot{6}$.

(3) 2·575 mds. ; 7·375 kros. ; ·2075 da. ; ·512 yd.

(4) 4·256 ac. ; 1·9725 mi. ; 5·3725 cwt. ; 4·9725 guineas.

3. Find the value of :

(1) 1·00625 of R2, of R5 ; ·775 of R14, of R16 ; ·57475 of R24 ; 6·6975 of R21 ; 5·01625 of R32.

(2) 3·3474 of £3, of £8, of £13 ; 13·1375 of £17, of £24 ; 2·775 of £64 ; 2·236 of £80.

(3) [illegible]125 of 2 mds. ; ·46875 of 1 ton ; 3·035 of 5 mi. ; ·025 of 7 days.

(4) 7·32$\dot{5}$ of R9 ; 3·5$\dot{7}$0$\dot{2}$ of £37 ; ·$\dot{5}\dot{2}$ of 198 mi. ; 3·572 of 22 days.

(5) ·25 of R2. 4*a*. 6*p*. ; ·325 of R4. 6*a*. 10*p*. ; ·0032 of R8. 10*a*. 8*p*.

(6) ·325 of R11. 12*a*. 9*p*. ; 5·275 of R8. 4*a*. 4*p*. ; 5·06 of R5. 6*a*. 5*p*.

(7) 3·6825 of £3. 2*s* 6*d*. ; 5·75 of £13. 12*s*. 8*d*. ; 22·0175 of £15. 8*s*.

(8) ·724 of 15*s*. 10*d*. ; ·325 of 11*s*. 9½*d*. ; 7·00275 of £12. 16*s*. 8*d*.

(9) ·012$\dot{7}\dot{5}$ of R1. 3*a*. 4*p*. ; ·01$\dot{6}$ of £2. 3*s*. 6*d*. ; ·$\dot{6}$2$\dot{5}$ of R37. 3*a*. 1*p*.

(10) ·7325 of 4 mds. 8 sr. 12 ch. ; 8·235 of 2 tons. 16 cwt. 2 qr.

(11) 2·325 of R3 + 3·35 of R8 − 2·285 of R8 − 2·7275 of R4.

(12) ·0125 of £28 + 12·345 of £11 − ·3775 of £5 − 8·245 of £12.

(13) 4·57 of R1. 7*a*. 8*p*. + 6·825 of R7. 11*a*. 6*p*. − ·9325 of R8. 4*a*. 4*p*. − 8·45 of R3. 12*a*. 8*p*.

(14) $\dot{6}$ of R4. 11*a*. 9*p* + ·$\dot{8}\dot{1}$ of R17 8*a*. 6*p*. − ·$2\dot{0}0\dot{1}$ of R52. 0*a*. 6*p*. − 1·$\dot{3}125$ of R1. 14*a*. 8*p*.

(15) 1·5 of £2. 11*s*. 4*d*. + 3·25 of £4. 4*s*. 6*d*. − 4·75 of £1. 4*s*. 8*d*. − 7·00625 of £1. 4*s*. 2*d*.

(16) ·00125 of 100 guineas + 2·000625 of £5. 16*s*. 8*d*. − 11·225 of 13*s*. 4*d*. + ·0285 of £6. 5*s*.

(17) ·$07\dot{1}\dot{6}$ of £11. 11*s*. + ·$\dot{0}\dot{9}$ of £1. 13*s*. − ·$0\dot{6}$ of 15*s*. + 1·$\dot{6}$ of £2.

4. Reduce to decimals of the highest denomination :

(1) 4563*p*. ; 4623*d*. ; 427299*q*. ; 37281*ps*.

(2) 281468 ch. ; 74011 lbs. Avoir ; 64572 grs. Troy ; 927639 in.

(3) 2*a*. 8*p*. ; 5*a*. 4*p*. ; 10*a*. 8*p*. ; 7*a*. 6*p*. ; R3. 4*a*. 8*p*.

(4) 3*s*. 4*d*. ; 6*s*. 8*d*. ; 13*s*. 4*d*. ; 7*s*. 6*d*. ; 17*s*. 6*d*. ; £3. 9*s*. 4*d*.

5. What is that sum, ·375 of which is R48. 15*a*. 9*p*. ?

6. ·0625 of a sum of money is £16. 18*s*. 8*d*. ; what is ·9375 of it ?

7. ·$3\dot{6}$ of a sum of money is £15. 3*s*. 5*d*. : find the sum.

236. *To reduce a simple or a compound quantity to the decimal of another of the same kind.*

RULE. Proceed as in ART. 193 ; and then reduce the vulgar fraction thus found to an equivalent decimal.

Ex. 1. Reduce 4*s*. 8*d*. to the decimal of £1.

4*s*. 8*d*. = £$\frac{7}{30}$; and $\frac{7}{30} \div 1 = \frac{7}{30}$.

∴ the required decimal $= \frac{7}{30} = \cdot 2\dot{3}$. *Ans.*

Ex. 2. Reduce ·625 of *Rs*.3. 7*a*. 4*p*. to the decimal of 2·75 of *Rs* 6. 14*a*. 8*p*.

·625 of *Rs*.3. 7*a*. 4*p*. = $\frac{5}{8}$ of *Rs*.3. 7*a*. 4*p*. = *Rs*.$2\frac{31}{192}$.

2·75 of *Rs*.6. 14*a*. 8*p*. = *Rs*.$19\frac{1}{48}$.

$2\frac{31}{192} \div 19\frac{1}{48} = \frac{415}{192} \times \frac{48}{913} = \frac{5}{44}$.

∴ the required decimal $= \cdot 1\dot{1}\dot{3}6$. *Ans.*

Examples 104.

1. In the following Examples, reduce the first of the two given quantities to the decimal of the second :

(1) R2. 5*a*. 3*p*. ; R4.

(2) R4. 7*a*. 6*p*. ; R10.

(3) R3. 8*a*. 3*p*. ; R16.

(4) R10. 18*a*. 4*p*. ; R20.

(5) R17. 7*a*. 4*p*. ; R22.

(6) R15. 9*a*. 10*p*. ; R25.

(7) R10. 11*a*. 8*p*. ; R8. 9*a*. 4*p*.

(8) R10. 5*a*. 4*p*. ; R20. 10*a*. 8*p*.

(9) £1. 2*s*. 6*d*. ; £4.

(10) £10. 17*s*. 6*d*. ; £15.

(11) £2. 7s. $10\frac{1}{2}d.$; £6. 7s. 8d. (12) £3. 17s. 3d. ; £16. 13s. 4d.

(13) £2. 9s. $10\frac{1}{4}d.$; £8. 6s. 3d. (14) £3. 6s 8d. ; £18. 13s. 4d.

(15) 1 md. 35 sr. ; 5 mds. (16) 5 md. 4 sr. ; 15 md. 16 sr.

(17) 7 cwt. 2 qr. 10 lbs. ; 5 tons. (18) 3 mi. 220 yds. ; 9 mi. 660 yds.

(19) 5 days 20 hrs. ; 16 days. (20) 3 ac. 2 ro. 15 po. ; 5 ac. 3 ro.

(21) 7 yds. 0 ft. 8 in. ; 14 yds. 1 ft. 4 in.

(22) 2 tons. 6 cwt. 1 qr. 4 lbs. ; 3 tons 1 cwt. 2 qr. 24 lbs.

(23) $\frac{5}{8}$ of R4. 8a. 8p. ; $\frac{3}{5}$ of R15. 12a. 6p.

(24) $\frac{7}{9}$ of £12. 13s. $1\frac{1}{2}d.$; $\frac{5}{8}$ of £21. 13s. $1\frac{1}{2}d.$

(25) $\frac{15}{28}$ of 3 guineas ; $4\frac{3}{8}$ of a noble.

2. What decimal of R5. 8a. is R1. 2a. ?

3. " " is 3s. $3\frac{3}{4}d.$ of £5. 6s. 8d. ?

4. " " is 1·75 of R$16\frac{9}{10}$, of $\frac{1}{10}$ of R1. 1a. 1p. ?

5. " " is $\frac{7}{12}$ of £4. 13s. 9d., of $\frac{5}{16}$ of £20. 16s. 8d. ?

6. " " is $\frac{5}{8}$ of 2 mds. 18 sr., of $\frac{5}{16}$ of 24 mds. 20 srs. ?

7. " " of a pole is a foot ?

8. " " of a day is ·375 of 28 minutes ?

9. Express ·0125 of £1 + ·0625 of 1s. + ·5 of 1d. as the decimal of 7s. 1d.

10. Express ·625 of £143. 12s + ·625 of £71. 16s. as the decimal of $\frac{3}{8}$ of £200.

11. Express $\frac{5}{6}$ of R2. 8a. + $\frac{3}{8}$ of R4. 11a. + 2·05 of R5 as the decimal of R60. 9a. 4p.

12. Express $\frac{3}{25}$ of R17. 7a. 2p. + $3\frac{3}{7}$ of R12. 5a. 9p. + R6·46583 as the decimal of R30.

13. Express $\frac{7}{8}$ of R16. 14a. − ·875 of R5. 0a. 8p. + $\frac{7}{8}$ of R20. 10a. 8p. as the decimal of R100.

14. Express $\frac{4}{7}$ of $\frac{2}{11}$ of $\frac{49}{64}$ of £100. 16s. 8d. − $\frac{3}{5}$ of $\frac{5}{8}$ of $\frac{4}{25}$ of £31. 5s. as the decimal of £10. 8s. 4d.

15. Express ·375 of 5s. + ·7 of 12s. 6d. + ·6 of £5 as the decimal of 5 guineas.

16. Express $\cdot\dot{6}$ of $\frac{3}{8}$ of £15 + $\frac{3}{4}$ of $\cdot 8\dot{3}$ of 5s. 4d. − 8·5 of $\frac{1}{4\frac{1}{4}}$ of 5s. 3·75d. as the decimal of 2s. $1\frac{1}{2}d.$

17. What decimal of £10 must be added to £16. 9s. 8d. to make it equal to £30 ?

18. What decimal of R15 must be added to R27. 15a. 6p. to make it R31. 11a. 6p. ?

19. What decimal of a mile is the distance which being diminished by 440 yds. is 550 yds. ?

20. Reduce the least of the quantities in each set to the decimal of the greatest :

(1) ·125 of R16. 8*a*. ; 1·25 of R1. 4*a*. ; 12·5 of 2*a*. 3*p*.

(2) ·625 of £3 ; ·0625 of £20 ; ·00625 of £128.

(3) ·75 of 3 mds. 20 sr. ; ·075 of 28 mds. 20 sr. ; ·0075 of 300 mds.

(4) ·325 of 11 cwt. 2 qr. ; ·0325 of 100 cwt. 3 qr. ; ·00325 of 100 tons.

CHAP. XXIII APPROXIMATION.

237. In many cases it is neither necessary nor practicable to find the value of a certain quantity *exactly*. For instance, the sum of R3 has to be divided equally amongst 7 persons, each share should be 6*a*. 10$\frac{2}{7}$*p*. ; but as there is no coin by which $\frac{2}{7}$*p*. can be paid, each share may be roughly or **approximately** put down as 6*a*. 10*p*. Then, again, the distance between Calcutta and Hugli may be roughly given as 24 miles, the number of yards, feet and inches by which the actual distance is greater or less, being left out of account.

The principle generally adopted in such cases is, that *the quantity omitted must be very small in comparison with the quantity retained.*

If a sum of money is R3. 7*a*. 8$\frac{1}{2}$*p*. or if a given weight is 2 qr. 3 lbs. 14$\frac{5}{8}$ oz. exactly, their values are said to be R3. 7*a*. 8*p*. or 2 qr. 3 lbs. 15 oz., to the *nearest pie* or the *nearest ounce*. See. *Ex*. 3, ART. 96.

238. In the case of decimals, it is generally sufficient to find the value to 3 places ; in some cases, however, where greater accuracy is required, the value has to be found correctly to 5 places or more.

The value of $\frac{5}{13}$ = ·384615384...... : but correct to 7 places, it should be given as ·3846154, to 6 places as ·384615, to 5 places as ·38462, &c., *1 being added to the last figure retained, when the first of the figures omitted is 5 or more than 5.*

Taking 5 places, the value ·38461 would make the error equal to ·000005384, but ·38462 would make it ·000004616 ; hence the error made would be smaller in the latter case.

239. **Abbreviated Addition** and **Subtraction.** The methods of contracted Addition and Subtraction have already been explained. See ARTS. 229 and 230.

240. **Abbreviated** or **Contracted Multiplication.** The method will be best understood from a consideration of the solutions of the following Examples.

Ex. 1. Multiply 32·159832 by 5·6708 and 4·284 by 521·43, each correctly to 3 places of decimals.

```
32·159,8            4·284,00
80·765               34125
------              --------
160799              2142·000
 19295                85·680
  2251                 4·284
    00                 1·714
    26                  ·128
------              --------
182·371             2233·806
```

RULE—Mark the figure in the 3rd decimal place of the multiplicand *Reserving* the multiplier, place it so that its unit figure may fall under the figure marked. Retain in the multiplicand (adding 0's if necessary) one more figure than we have in the multiplier (*for approximation*), except when the last figure is 0 Multiply by each figure of the multiplier only that part of the multiplicand which extends up to its column, *carrying*, however, the number of tens in the product by the figure to the right, (taking *one more ten* when the unit figure in the product is 5 or more than 5), and place the unit figure of each product in the column of the figure marked.

***Ex.* 2.** Find, to 6 places of decimals, the value of 489·46284 × ·04268 and of ·0028756$\dot{3}$ × 501·4006.

·04268 may be written as 0·04268.

```
489·462840          ·002875,635
   862400            6004105
----------          ----------
19·578514           1437817
  978926               2876
  293677               1150
   39157                  1
----------          ----------
20·890274           1·441844
```

241. Abbreviated or Contracted Division. This method will also be best understood from a consideration of the solutions of the following Examples.

***Ex.* 1.** Divide 42·48 by 2·341721 correctly to 3 decimal places.

```
23417,2)42480(18·140
        23417
2341,7)19063
       18734
234,1)329
      234
23,4)95
     94
2,3)1
```

Find by inspection how many figures there will be in the *integral part* of the quotient, here 2; 3 places of decimals are to be retained; and 1 figure is required for *approximation.* Hence 6 figures are retained in the divisor. 23417 will not go into 4248, but will go into 42480; therefore a cypher is added In the next stage, the divisor is 2341, 7 being retained for approximation only; in the next, 234; and so on.

***Ex.* 2.** Divide 3400·6123 by 862·53451 correctly to 5 decimal places.

```
862534,5)3400612(3·94258
         2587604
86253,4)813008
        776281
8625,3)36727
       34501
862,5)2226
      1725
86,2)501
     431
8,6)70
    69
```

The integral part will have 1 figure; 5 decimal places are required: therefore the divisor should contain 6 figures, and is 862534 The additional figure is retained for approximation only. 862534 would go into 3400612, which is therefore taken for the dividend It should be noticed that one figure in the divisor is struck off after every division, and that nothing is brought down and affixed to the remainder.

Ex. 3. Divide ·00262581 by 56·289461482, correct to eight decimal places.

5,6,2,8,9)26258(·0000466
    22516
     3742
     3377
      365
      337
       28
       28

It can be found by inspection that there will be 4 cyphers after the decimal point in the quotient; hence practically only (8 − 4) or 4 figures are wanted in the quotient. As 1 figure is required for approximation, 5 figures are retained in the divisor.

The rest as before.

242. **Series.** The value of a **series** has often to be found correctly to a certain number of decimal places. In such cases, we have to proceed as in the following Example.

Ex. Find the value, to five decimal places, of

$$\frac{1}{2}+\frac{1}{2.3}+\frac{1}{2.3.4}+\frac{1}{2.3.4.5}+\&c.$$

$\frac{1}{2}$			$=·5$	
$\frac{1}{2.3}$	$=\frac{1}{3}\times\frac{1}{2}$	$=\frac{1}{3}\times·5$	$=·16666$	67
$\frac{1}{2.3.4}$	$=\frac{1}{4}\times\frac{1}{2.3}$	$=\frac{1}{4}\times·1666667$	$=·04166$	67
$\frac{1}{2.3.4.5}$	$=\frac{1}{5}\times\frac{1}{2.3.4}$	$=\frac{1}{5}\times·0416667$	$=·00833$	33
$\frac{1}{2.3.4.5.6}$	$=\frac{1}{6}\times\frac{1}{2.3.4.5}$	$=\frac{1}{6}\times·0083333$	$=·00138$	89
$\frac{1}{2.3.4.5.6.7}$		$=\frac{1}{7}\times·0013889$	$=·00019$	84
$\frac{1}{2.3.4.5.6.7.8}$		$=\frac{1}{8}\times·0001984$	$=·00002$	48
$\frac{1}{2.3.4.5.6.7.8.9}$		$=\frac{1}{9}\times·0000248$	$=·00000$	28
$\frac{1}{2.3.4.5.6.7.8.9.10}$		$=\frac{1}{10}\times·0000028$	$=·00000$	03
			1·828	91

The next and the following terms will all give 0's only; therefore they need not be considered.

Examples 105.

1. Multiply the first two Examples correctly to 3 places, and the last Example correctly to 5 places of decimals:

(1) 462·46894 by 63·8946814 ; 78·94281314 by ·89648214.

(2) ·78921468 by ·00289461 ; ·7862428 by 67·28.

(3) ·00628146 by 52·460568 ; 9·752891406 by ·214028061.

2. Divide:

(1) $57{\cdot}8942$ by $48{\cdot}69842$ correctly to 5 places of decimals;

(2) $128{\cdot}36281$ by $\cdot08936481$ „ 6 „ „ „ ;

(3) $\cdot002891$ by $462{\cdot}8904$ „ 8 „ „ „ ;

(4) $\cdot02\dot{8}\dot{9}$ by $\cdot024\dot{8}$ „ 7 „ „ „ ;

3. Find the values of:

(1) $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots$ to 6 places of decimals;

(2) $1+\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\frac{1}{625}+\ldots\ldots$ 7 „ „

(3) $1+\frac{1}{1.3}+\frac{1}{1.3.5}+\frac{1}{1.3.5.7}+\ldots\ldots$ 8 „ „

(4) $\frac{1}{7}+\frac{1}{7\times5}+\frac{1}{7\times5^2}+\frac{1}{7\times5^3}+\ldots$ 6 „ „

CHAP. XXIV. THE METRIC SYSTEM.

243. In the **Metric system,** which is in use in France, Germany, &c., the TABLES are formed by taking as *auxiliary* units, 10 times, 100 times and 1000 times the *principal* unit for multiples, and tenths, hundredths, and thousandths of this unit for sub-multiples.

244. The multiples, are all denoted *in order* by the Greek prefixes **deka, hecto, kilo,** and the sub-multiples by the prefixes **deci, centi, milli.**

245. The unit of **length** is the **metre.** It was originally taken as one *ten-millionth* of the distance of the Pole from the Equator, though subsequent calculations have proved this to be incorrect. 1 metre $=39{\cdot}37\ldots$in. or $39\frac{3}{8}$ in. approximately.

10 Millimetres (m.m.)	= 1 Centimetre	10 metres	= 1 Dekametre
10 centimetres (c.m.)	= 1 Decimetre	10 dekametres	= 1 Hectometre
10 decimetres	= 1 **Metre**	10 hectometres	= 1 Kilometre.

246. The unit of **surface** is the **Are,** with like multiples and sub-multiples. It measures a dekametre each way, and contains 100 sq. metres. It is equal to $119{\cdot}6\ldots$sq. yds., or rather less than 4 poles.

247. The unit of **capacity** is the **Litre,** with like multiples and sub-multiples. It is a cubic decimetre, and contains about 22 gallons.

248. The unit of **weight** is the **Gram,** with like multiples and sub-multiples. It is the weight of a cubic centimetre of pure water at 4°C, and weighs about $15{\cdot}43$ grains.

249. The unit of **money** is the **Franc**, which is about 9½*d*. 25 francs = £1. Its sub-multiples are the *decime* (never used) and the *centime* (cent.).

250. **Proposed Decimal Coinage.** It has been proposed to introduce into England a decimal coinage, in which the unit of money should be £1, with sub-multiples, *florins*, *cents*., and *mils*.

10 Mils. = 1 Cent.
10 cents. = 1 Florin
10 florins. = £1.

In this system, in a decimal of £1 the first figure in the decimal place denotes florins, the second cents., and the third mils.

Ex. 1. Reduce £3. 6*s*. 10*d*. to £, fl., &c., and £3. 8fl. 5c. 6m. to £. *s*. *d*.

£3. 6*s*. 10*d*.
$= £(3 + \frac{6}{20} + \frac{10}{240})$
$= £(3 + \cdot3 + \cdot04166\ldots)$
$= £3\cdot34166\ldots$
= £3. 3fl. 4c. 2m. nearly.

£3. 8fl. 5c. 6m. = £3·856
20
s. 17·12
12
d. 1·44
the sum = £3. 17*s*. 1½*d*. nearly.

Ex. 2. Find the value of £5. 3fl. 4c. 2m. − £1. 4fl. 8c. 7m. + £12. 8fl. 1c. 3m. − £2. 6fl. 5c. 1m. − £3. 2fl. 8c. 8m. − £4. 8fl. 3c. 3m.

£5. 3fl. 4c. 2m. = £5·342
£12. 8fl. 1c. 3m. = £12·813
18·155
12·259
5·896 = £5. 8fl. 9c. 6m.

£1. 8fl. 8c. 7m. = £1·487
,, 2. 6 ,, 5 ,, 1 ,, = ,, 2·651
,, 3. 2 ,, 8 ,, 8 ,, ,, 3·288
,, 4. 8 ,, 3 ,, 3 ,, ,, 4·833
12·259

Ex. 3. Divide £15. 8fl. 7c. 5m. × 12 by 20.

£15. 8fl. 7c. 5m. = £15·875
12
£190·500

20) 190·500
9·525 = £9. 5fl. 2c. 5m.

Examples 106.

1. Reduce to *decimal money*: 13*s*. 4*d*.; 9*s*. 3¾*d*.; £11. 9*s*. 1½*d*.

2. Express in £. *s*. *d*.; £1·2302; £10·6523; £12. 8fl. 9c. 2½m.; £120. 5fl. 7c. 0¼m.

3. Reduce 21 kilogram 8 dekagram 5 grm. to *centigrm*. 25 kilolit. 9 dekalit. to *litres*: and 8 hectomet. 9 met. 5 decimet. 8¼ c. m. to *metres*.

4. Multiply £32. 5fl. 8c. 2½m. by 1·20, and divide £30763. 7fl. 1c. 1½m. by (1) 99; (2) by £1. 5fl. 8c. 4m.

Problems worked out.

Ex. 1. *A* and *B* begin business together with equal sums. At the end of the year *A* has gained £500 and *B* lost ·3 of his capital. *A* has then twice as much as *B*. How much had each at first?

If each of A's and B's original money be denoted by 1, B's money at the end of the year would be denoted by $(1 - \cdot\dot{3})$ or $\frac{2}{3}$, and A's money by $2 \times \frac{2}{3}$ or $\frac{4}{3}$; $\therefore$ A's money is increased by $(\frac{4}{3} - 1)$ or $\frac{1}{3}$;

$\therefore$ $\frac{1}{3}$ of the reqd. sum $=$ £500;

$\therefore$ the reqd. sum $=$ £1500. *Ans.*

Ex. 2. A man has a certain number of apples; he sells ·5 of the number and 1 more to one person, ·375 of the remainder to a second, $\frac{2}{3}$ of what still remain to a third, and has only 15 left. How many had he at first?

After disposing of $\frac{2}{3}$ of the second remainder he had $(1 - \frac{2}{3})$ or $\frac{1}{3}$ of it left. Therefore $\frac{1}{3}$ of the second remainder $= 15$; $\therefore$ the second remainder $= 15 \div \frac{1}{3} = 45$.

·375 or $\frac{3}{8}$ of the first remainder being sold, $\frac{5}{8}$ remained; $\therefore$ $\frac{5}{8}$ of the first remainder $= 45$, or the first remainder $= 45 \div \frac{5}{8} = 72$.

Again, after selling $\frac{1}{2}$ of what he now had and 1 more, he had 72 left; $\therefore$ $\frac{1}{2}$ of the number is 73.

$\therefore$ the reqd. number of apples $= 73 \times 2 = 146$. *Ans.*

Ex. 3. A owns ·4 of a shop and B the remainder. They take in a third partner C, who pays *Rs.* 1000 on the understanding that they are all to have equal shares. How should the money be divided between A and B?

B was the owner of ·4 or $\frac{2}{5}$ of the shop, and is in future to have only $\frac{1}{3}$; $\therefore$ he parts with $(\frac{2}{5} - \frac{1}{3})$ or $\frac{1}{15}$.

B had $(1 - \frac{2}{5})$ or $\frac{3}{5}$, and is to have only $\frac{1}{3}$; $\therefore$ he parts with $(\frac{3}{5} - \frac{1}{3})$ or $\frac{4}{15}$.

$\therefore$ B should have 4 times as much money as A;

$\therefore$ A's share $= \frac{1}{5}$ of *Rs.* 1000 $=$ *Rs.* 200; }
and B's „ $= \frac{4}{5}$ of „ 1000 $=$ „ 800. } *Ans.*

MISCELLANEOUS EXAMPLES IV.

1. How often is ·75 contained in 45·3675?

2. What number divided by ·625 will give $12 \times 1{\cdot}2 \times {\cdot}12 \times {\cdot}012$?

3. Into how many equal parts must $\cdot 41\dot{6}$ be divided, that four of such parts may amount to $\cdot\dot{3}$?

4. What circulating decimal multiplied by $2\frac{3}{5} \times \frac{4}{7}$ will give 2 for the product?

5. Find the product of the sum and difference of ·0421 and ·029, and divide $\frac{1}{10}$ of the product by 30 times the continued product of ·02, ·03, and ·07.

6. What decimal added to the sum of 2·0015, 5·0000261, 4·56948, 5·93421, and ·00072114178 will make the least possible integer?

7. Divide 75 into two parts, such that one may exceed the other by 7·251.

8. A person having ·5 of an estate, sells ·0075 of his share: what decimal of the whole has he left?

9. The height of a dwarf is ·1875 of that of a giant who is 8·$\dot{8}$ ft. high : find the height of the dwarf.

10. If, by selling a book for R12, ·$\dot{3}$ of the cost price be cleared, what would be gained if the book were sold for R15·5 ?

11. A person sells ·75 of a mine to *A* and ·125 of what he sells to *A* to *B*; what decimal of the whole is still left to him ?

12. *C* gives to *B*, ·875 of what *B* gives to *A* ; if *B* gives to *A* R12, what does *C* give to *B* ?

13. *A* and *B* have equal sums of money ; if *A*'s money is increased by ⅛th and *B* spends away R140, then *B*'s money becomes ·6 of *A*'s money ; what sum had each originally ?

14. If ·3125 of a md. be worth R45, what is the price of ·3 of 1 md. ?

15. A butcher bought an equal number of calves and sheep for £265 ; for the calves he gave £3·75 a head and for the sheep £2·875 a head ; how many did he buy of each kind ?

16. A man owns ·1875 of a house, and sells ·$\dot{1}3\dot{5}$ of his share ; what decimal of the whole house does he still own ?

17. Divide R3900 among *A*, *B* and *C*, *B*'s share being ·4 of *A*'s and *C*'s ·4 of *B*'s.

18. What is the value of ·09 of ·08$\dot{3}$ of an estate, if a person who owns ·$\dot{2}\dot{7}$ of it, sells ·1 of 0·875 of his share for R7000 ?

19. If ·$\dot{4}2857\dot{1}$ of a house be worth R1800, for what sum of money must a person sell ·$\dot{6}$ of his share, which is ·375 of the whole ?

20. ·$\dot{6}$ of a cistern is full of water. 35 seers being drawn off, it becomes only ·375 full ; how much can the cistern hold ?

21. ·25 of the number being away at an examination, and 5 absent, a class is only ⅔rds full ; find the strength of the class.

22. What number is that which when multiplied by 6·25 and divided by ·815 gives 7·25 ?

23. A school of 250 children is divided into 6 classes. The first class contains ·36 of the whole number, the second class ·28, and the third class ·22 If the fourth class contains 17, and the fifth class twice as many as the sixth class, find the number of children in each class.

24. A gentleman distributed a certain sum of money amongst 15 orphans, two of whom received ·125 of the whole each, each of three others ·$\dot{1}$, each of four others ·041$\dot{6}$, and each of the remaining six R3 ; what was the sum of money distributed ?

25. The captain of a ship worth R51611·875, is himself owner of ·$\dot{3}$ of ·6 of her. If she be sold for only 6$\dot{3}$ of her value, what loss must the captain sustain ?

26. In a cricket match one player made ·25 of the score, each of three others ·1, each of two others ·0625, and the rest 39 amongst them ; find the total number of runs.

27. A man gained ·1$\dot{6}$ of his capital during the first year and ·$\dot{3}$ during the second year, but spent ·5 of his profits every year. At the end of the second year he found that his capital had increased by R800: what sum did he start with?

28. *A* can do ·5 of a piece of work in one day, *B* can do ·75 of the remainder in one day, and *C* can finish it in ·$\dot{3}$ of a day; in what time can they jointly do it?

***29.** A woman had a certain number of eggs: she sold ·$\dot{3}$ of the number and 2 more to one person, 3 of the remainder to a second, and ·5 of what still remained to a third, when she had only 14 left. How many had she at first?

30. A certain sum of money was divided amongst four men. A got ·6 of the whole, *B* ·75 of the remainder, *C* ·8 of what still remained, and *D* the rest which was £180. What did *A*, *B*, *C* get respectively?

31. If by selling an article for R10. 8*a*, ·$\dot{3}$ of its cost price is gained, what decimal of the cost price would be gained if it were sold R8?

32. *A* can reap ·$\dot{4}$ of a field in 2·$\dot{6}$ days, and *B* ·6 of it in 4·5 days. If *A* earns 5*a*. a day, what ought *B* to earn in the same time?

33. The waste-pipe of a cistern empties in a certain time ·$\dot{3}$ of what the supplying pipe can fill in the same time. The latter can fill the empty cistern in 9·3375 hrs.; and after it has been opened for 2·8125 hrs., the waste-pipe is also set to work. When will the cistern be filled?

34. Of a regiment of soldiers, ·0$\dot{3}$ are killed in the first battle, ·175 of the remainder in the second, ·$\dot{2}\dot{7}$ of the remainder in the third, and 580 are left. How many were there at first?

35. A person walks 71·1 miles in 9 days travelling 5·082 hrs. a day how many hours per day must he travel so as to walk 252·8 miles in 30·72 days?

36. Find to 9 places of decimals the value of

(1) $\frac{1}{9}+\frac{1}{3\cdot 9^3}+\frac{1}{5\cdot 9^5}+\frac{1}{7\cdot 9^7}+\ldots$ (2) $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 3^3}+\frac{1}{5\cdot 3^5}+\frac{1}{7\cdot 3^7}+\ldots$

37. Eight bells which toll at intervals of 1·2, 2·4, 3·6, 4·8, ·96, ·112, 7·2, 8·4 seconds respectively, begin tolling simultaneously; how long must elpase before they all toll simultaneously again?

38. Find to the nearest farthing the price per lb., in English money, of tea which costs in France 3·5 francs per kilogramme; £1 being equivalent to 25·3$\dot{6}$ francs and 1 kilogramme to 2·204 lbs.

***39.** *A*, *B*, *C* can do a piece of work together in 6 days: after they have worked together a day, *A* is withdrawn, and *B* and *C* work together at the same rate for 2 days more; *B* then leaves, and *C* completes the work in 9·6 days more, working ·$\dot{3}$ longer each day. Working at his former rate *C* alone could do the work in 22·2 days; find how long *A* and *B* would each take to do it separately.

*40. A merchant after trading for 6 years died worth R73426. 1*a.*; the first year he had increased his capital by $\cdot 1\dot{6}$, the second by ·2, the third by ·25, the next two years by $\cdot\dot{3}$ and ·5, and the last year he had doubled his capital. Find the original capital.

41. If 1 lb. of coffee cost as much as ·625 lbs. of tea, or $5\cdot\dot{7}1428\dot{5}$ lbs. of sugar; what is the price of each, when 1 lb. of tea, 1 lb. of coffee, and 1 lb. of sugar, together cost R2. 5*a.*?

42. The circumference of the earth is 40,000,000 metres, a metre being equal to 39·37079 in.; calculate the diameter of the earth in miles, assuming the circumference of a circle to be $\frac{355}{113}$ times diameter.

43. Subtract $\frac{15}{16}$ of £16. 2*s* 4*d* from ·0125 of £1626. 15*s.*: and find by what decimal the result must be multiplied to produce £1. 6*s.* 1½*d.*

*44. A warehouse consists of 7 floors: the rent of each floor is ·875 times that of the floor below; the rent of the middle floor is £120. 1*s.*: compare the rents of the highest and the lowest floors, and find that of the lowest.

45. One clerk has $24\cdot\dot{4}2857\dot{1}$ and another $38\frac{4}{7}$ sheets to engross; they call in a third clerk and agree to divide the work equally among the three and to pay the third clerk $\cdot 2430\dot{5}$*s.* per sheet: how much will he receive from each of them?

46. A merchant buys teas as follows: 11·5 lbs. at R1 per lb.; 12·25 lbs. at R1. 1*a.* per lb; 13·75 lbs. at R1. 2*a.* per lb.; 14 lbs. at R1. 4*a.* per lb.; 15 lbs. at R1. 5*a.* per lb.; and mixes the whole. If he sells the mixture at R1. 3*a.* per lb., what profit does he make on every rupee invested?

47. Two trains start at the same time from two stations 60 mi. apart and travel towards each other, the one at the rate of 32·125 mi. an hour and the other 26·375 mi. an hour. After what time will they meet, and how far will the faster train have gone?

*48. In a certain town, the number of children under 5 years of age who died in a month was 675. This was ·36 of the total deaths for the month, and the death-rate, *i. e.*, the number of deaths per thousand inhabitants in a year, was 19·2. What is the estimated population?

*49. A grocer mixes tea of a certain quality (A) with ¾ its weight of tea which cost him half as much and sells the whole at $\cdot 8\dot{3}$ of the price of (A) per lb. If his outlay is R600, find his profit.

*50. Two settlers in New Zealand own adjoining farms of 3000 and 5000 acres respectively. They unite their farms, taking at the same time an additional partner, who pays them £8000 on the understanding that $\cdot\dot{3}$ of the land will in future belong to each. How is the £8000 to be divided between the original owners?

CHAP. XXV. PRACTICE.

291. That part of a quantity which being taken an *integral* number of times makes up the quantity, is called an **Aliquot**

Part. Its relation with the whole can be expressed by a simple fraction having unity for its numerator.

Table of Aliquot Parts.

OF A Rupee.		OF A £.		OF A Maund.	
1*a*.	$=\frac{1}{16}$R	1*s*.	$=\frac{1}{20}$£	1 sr.	$=\frac{1}{40}$ md.
2*a*.	$=\frac{1}{8}$R	1*s*. 8*d*.	$=\frac{1}{12}$£	1 sr. 4 ch.	$=\frac{1}{32}$ md.
4*a*.	$=\frac{1}{4}$R	2*s*.	$=\frac{1}{10}$£	2 sr.	$=\frac{1}{20}$ md.
8*a*.	$=\frac{1}{2}$R	2*s*. 6*d*.	$=\frac{1}{8}$£	2 sr. 8 ch.	$=\frac{1}{16}$ md.
1*a*. 4*p*.	$=\frac{1}{12}$R	3*s*. 4*d*.	$=\frac{1}{6}$£	4 sr.	$=\frac{1}{10}$ md.
2*a*. 8*p*.	$=\frac{1}{6}$R	4*s*.	$=\frac{1}{5}$£	5 sr.	$=\frac{1}{8}$ md.
5*a*. 4*p*.	$=\frac{1}{3}$R	5*s*.	$=\frac{1}{4}$£	8 sr.	$=\frac{1}{5}$ md.
		6*s*. 8*d*.	$=\frac{1}{3}$£	10 sr.	$=\frac{1}{4}$ md.
		10*s*.	$=\frac{1}{2}$£	20 sr.	$=\frac{1}{2}$ md.

OF AN Anna.		OF A Shilling.		OF A Seer.	
1*ps*.	$=\frac{1}{4}a$.	1*d*.	$=\frac{1}{12}s$.	1 ch.	$\frac{1}{16}$ sr.
2*ps*.	$=\frac{1}{2}a$.	1⅓*d*.	$=\frac{1}{9}s$.	2 ch.	$\frac{1}{8}$ sr.
1*p*.	$=\frac{1}{12}a$.	1½*d*.	$=\frac{1}{8}s$.	4 ch.	$\frac{1}{4}$ sr.
2*p*.	$=\frac{1}{6}a$.	2*d*.	$=\frac{1}{6}s$.	8 ch.	$\frac{1}{2}$ sr.
3*p*.	$=\frac{1}{4}a$.	3*d*.	$=\frac{1}{4}s$.		
4*p*.	$=\frac{1}{3}a$.	4*d*.	$=\frac{1}{3}s$.		
6*p*.	$=\frac{1}{2}a$.	6*d*.	$=\frac{1}{2}s$.		

OF A lb. AVOIR.	
1 oz.	$=\frac{1}{16}$ lb.
2 oz.	$=\frac{1}{8}$ lb.
4 oz.	$=\frac{1}{4}$ lb.
8 oz.	$=\frac{1}{2}$ lb.

OF A Ton.		OF A Quarter.		OF AN oz. AVOIR.	
1 cwt.	$=\frac{1}{20}$ ton	1 lb.	$=\frac{1}{28}$ qr.	1 dr.	$=\frac{1}{16}$ oz.
1 cwt. 1 qr.	$=\frac{1}{16}$ ton	1 lb. 12 oz.	$=\frac{1}{16}$ qr.	2 dr.	$=\frac{1}{8}$ oz.
2 cwt.	$=\frac{1}{10}$ ton	2 lb.	$=\frac{1}{14}$ qr.	4 dr.	$=\frac{1}{4}$ oz.
2 cwt. 2 qr.	$=\frac{1}{8}$ ton	3 lb. 8 oz.	$=\frac{1}{8}$ qr.	8 dr.	$=\frac{1}{2}$ oz.
4 cwt.	$=\frac{1}{5}$ ton	4 lb.	$=\frac{1}{7}$ qr.		
5 cwt.	$=\frac{1}{4}$ ton	7 lb.	$=\frac{1}{4}$ qr.		
10 cwt.	$=\frac{1}{2}$ ton	14 lb.	$=\frac{1}{2}$ qr.		

252. **Practice** is an easy method of finding the value of a simple or a compound quantity when the value of a unit is given. Practice is of two kinds, **simple** and **compound.**

253. When it is required to find the value of a simple quantity by means of aliquot part, it is called *Simple Practice.* It is *Compound Practice* when it is required to find by means of aliquot parts the value of a compound quantity.

Thus to find the value of 15 things at R3. 4*a*. each, is *Simple Practice*: and to find the value of 63 mds. 15 sr. at R4. 15*a*. 6*p*. per maund, is *Compound Practice*.

SIMPLE PRACTICE.

254. The RULE for Simple Practice will be illustrated by the following Examples.

Ex. 1. Find the value of 56 things at *Rs.* 5. 14*a.* 8*p.* each.

If the price of 1 thing be *Re.* 1. 56 things will cost *Rs.* 56.

	Rs.	*a.*	*p.*			
8*a.* = $\frac{1}{2}$ of *Re.* 1.	56	0	0	@ *Re.* 1 each.		
			5			
	280	0	0	@ *Rs.* 5		each.
4*a.* = $\frac{1}{2}$ of 8*a.*	28	0	0	@	8*a.*	"
2*a.* = $\frac{1}{2}$ of 4*a*	14	0	0	@	4*a.*	"
8*p.* = $\frac{1}{2}$ of 2*a.*	7	0	0	@	2*a.*	"
	2	5	4	@	8*p.*	"
	Rs. 331	5*a.*	4*p.*	@ *Rs.* 5. 14*a.* 8*p.*		"

Or better *thus* :—

Rs. 5. 14*a.* 8*p.* is less than *Rs.* 6 by 1*a.* 4*p.* The calculation may, therefore, be considerably shortened thus :

1*a.* 4*p.* = $\frac{1}{12}$ of *Re.* 1	*Rs.* 56.	0*a.*	0*p.*	@ *Re.* 1		ea
			6			
	Rs. 336.	0	0	@ *Rs.* 6		
	4	10	8	@	1*a.* 4*p.*	,
	Rs. 331	5*a.*	4*p.*	@ *Rs.* 5. 14*a.* 8*p.*		,

Ex. 2. Find the cost of 85 articles at £12. 8*s.* 10*d.* each.

10*s.* = $\frac{1}{2}$ of £1.	£85.	0*s.*	0*d.*	@ £1		each.
			12			
	£1020	0	0	@ £12		
5*s.* = $\frac{1}{2}$ of 10*s.*	42	10	0	@	10*s.*	
2*s.* 6*d.* = $\frac{1}{2}$ of 5*s.*	21	5	0	@	5*s.*	
1*s.* 3*d.* = $\frac{1}{2}$ of 2*s.* 6*d.*	10	12	6	@	2*s.* 6*d.*	
1*s.* = $\frac{1}{15}$ of 1*s.* 3*d.*	5	6	3	@	1*s.* 3*d.*	
		7	1	@	1*d.*	
	£1100	0*s*	0*d*	@ £12. 18*s.* 10*d.*		

Otherwise thus :—As £12. 18*s.* 10*d.* is less than £13. by 1*s.* 2*d.*, we may simplify the process thus :—

1*s.* = $\frac{1}{20}$ of £1	£85.	0*s.*	0*d.*	at	£1		each.
			13				
	£1105	0	0	at £13			"
2*d.* = $\frac{1}{6}$ of 1*s.*			£4. 5*s.* 0*d.*	at	1*s.*		"
			14*s.* 2*d.*	at		2*d.*	"
	4	19	2	at	1*s.*	2*d.*	"
	£1100	0*s.* 10*d.*		at £12. 18*s.* 10*d.*			"

Ex. 3. Find the value of 24¾ mds. at *Rs.* 11 11*a.* 9*p.* per md.

8*a.* = ½ of *Re.* 1	*Rs.* 24	12*a.*	0*p.*	at *Re.* 1.	each.
			11		
	*Rs.*272	4*a.*	0	at *Rs.* 11.	,,
2*a.* = ¼ of 8*a.*	12	6	0	at 8*a.*	,,
1*a.* = ½ of 2*a.*	3	1	6	at 2*a.*	,,
6*p.* = ½ of 1*a.*		8	9	at 1*a.*	,,
3*p.* = ½ of 6*p.*		12	4½	at 6*p.*	,,
		6	2¼	at 3*p.*	,,
	Rs 290.	6*d.*	9¾*p.*	at *Rs.* 11. 11*a.* 9*p.*	,,

Ex. 4. Find the value of 216¾ cwt at £15 16*s.* 11¾*d.* per cwt.

10*s.* = ½ of £1.	£216.	17*s.*	6*d.*	at £1	each.
			15.		
	£3253	1*s.*	6*d.*	at £15	,,
5*s.* = ½ of 10*s.*	108	8	9	at 10*s.*	,,
1*s.* 3*d.* = ¼ of 5*s.*	54	4	4½	at 5*s.*	,,
7½*d.* = ½ of 1*s.*3*d.*	13	11	1⅛	at 1*s.* 3*d.*	,,
1¼*d.* = ⅙ of 7½*d.*	6	15	6 9/16	at 7½*d.*	,,
	1	2	7 3/32	at 1¼*d.*	,,
	£3437.	4*s.*	10 9/32 *d.*	at £15. 16*s.* 11¾*d.*	,,

Or thus:

10*s.* = ½ of £1	£216.	17*s.*	6*d.*	at £1	each.
			15		
	3253	2	6	at £15	each.
5*s.* = ½ of 10*s.*	108	8	9	at 10*s.*	,,
1*s.* 3*d.* = ¼ of 5*s.*	54	4	4·5	at 5*s.*	,,
7½*d.* = ½ of 1*s.*3*d.*	13	11	1·125	at 1*s.* 3*d.*	,,
1¼*d.* = ⅙ of 7½*d.*	6	15	6·5625	at 7½*d.*	,,
	1	2	7·09375	at 1¼*d.*	,,
	£3437	4*s.*	10·28125*d.*	at £15. 16*s.* 11¾*d.*	,,
	=£3437	4*s.*	10¼*d.* ⅛*q.*		

Examples 107.

Find by Practice the value of the following articles:

1. 57 at 5*a.* 10*p.* **2.** 88 at 6*a.* 4*p.* **3.** 105 at 11*a.* 7*p.*
4. 137 at 5*a.* 4*p.* **5.** 427 at R25. 12*a.* **6.** 299 at 13*a.* 11*p.*
7. 587 at R11. 3*a.* 8*p.* **8.** 642 at R17. 7*a.* 9*p.* **9.** 697 at R9. 15*a.* 3½*p.*
10. 1027 at 15*a.* 10½*p.* **11.** 1110 at R18. 14*a.* **12.** 1215 at R12. 2*a.* 8¾*p.*
13. 1347 at R4. 7*a.* 8¾*p.* **14.** 2140 at R5. 13*a.* 7½*p.* **15.** 5121 at R17. 11*a.* 9*p.*

16. 2469 at R11.14a.6p. **17.** 4621 at R1. 9a. 9p. **18.** 6411 at R19. 13a. 7p.
19. 478¾ at R4. 13a. 7p. **20.** 741¼ at R11. 5a. 9p. **21.** 462·875 at R11.9a.8p.
22. 437 at 7s. 8d. **23.** 518 at £1. 6s. 8d. **24.** 619 at 17s. 6d.
25. 519 at £1. 2s. 6d. **26.** 457 at £1. 8s. 8d. **27.** 789 at £3. 13s. 4d.
28. 611⅜ at £2. 16s. 8d. **29.** 142⅝ at £4. 11s. 3½d. **30.** 947375 at £17.11s.8d.
31. 274·875 at £5.9s.10d. **32.** 351$\frac{7}{20}$ at £22.8s.6¾d. **33** 2634·74 at £11.19s.8d.
34. 4621 at £11. 18s.7d. **35.** 365 at £12.14s. 9¾d. **36.** 374·85 at £7.18s.7½d.
37. 478$\frac{7}{10}$ at £2.11s. 9d. **38.** 748·75 at £3.8s. 4½d. **39.** 941·75 at £14.13s.8d.

COMPOUND PRACTICE.

255. The following Examples will illustrate the method of Compound Practice.

Ex. 1. Find the value of 27 mds. 37 sr. 13 ch. at *Rs.*14. 13*a.* 8*p.* per md.

20s. = ½ of 1 md.	*Rs.* 14.	13*a.*	8*p.*	is the price of		1 md.
			3			
	Rs. 44.	9*a.*	0*p.*	,,	,,	3 mds.
			9			
	Rs. 401.	1*a.*	0*p.*	,,	,,	27 mds.
10 sr. = ½ of 20 sr.	7	6	10	,,	,,	20 sr.
5 sr. = ½ of 10 sr.	3	11	5	,,	,,	10 sr.
2 sr.8ch. = ½ of 5 sr.	1	13	8½	,,	,,	5 sr.
5 ch. = ⅛ of 2s. 8 ch.		14	10¼	,,	,,	2 sr. 8 ch.
		1	10$\frac{9}{32}$	,,	,,	5 ch.
	*Rs.*415.	1*a.*	8$\frac{1}{32}$*p.*	,,	,,	27mds. 37sr.13 ch.

Ex. 2. Find the value of 7 mds. 18 sr. 9 ch. at 5*a.* 4*p.* per seer.

8 sr. = 1 sr. × 8	*Rs.*	*a.*	*p.*	
10 sr. = 1 sr. × 10		5	4	is the price of 1 sr.
8 ch. = ½ of 1 sr.			8	
	2	10	8	is the price of 8 sr.
			5	
	13	5	4	is the price of 1 md.
			7	
	93	5	4	is the price of 7 mds.
	2	10	8	,, ,, 8 sr.
1 ch. = ⅛ of 8 ch.	3	5	4	,, ,, 10 sr.
		2	8	,, ,, 8 ch.
			4	,, ,, 1 ch.
	*Rs.*99.	8*a.*	4*p.*	is the price of 7 mds. 18 sr. 9 ch.

Ex. 3. Find the cost of 5 miles 6 fur. 206 yds. 0 ft. 9 in. of iron rail at £32. 18*s*. 8*d*. per furlong.

6 fur = 1 fur. × 6. 110 yds. = ½ of 1 fur.	£32.	18*s*.	8*d*.	= cost of 1 furlong.
			8	
	233			= cost of 1 mile.
	1317	3	8	= cost of 5 mi.
	197	12	0	" " 6 fur.
55yd. = ½ of 110yd.	16	9	4	" " 110 yd.
27yd. 1ft. 6in. = ½ of 55yd.	8	4	8	" " 55 yd.
13yd. 2ft. 3in. = ½ of 27yd.	4	2	4	" " 27 yd. 1 ft. 6 in.
1 ft. 6in.	1	1	2	" " 13 yd. 2 ft. 3 in.
	£1545	16*s*	2*d*.,	cost of 5mi. 6fur. 206yd. 0ft. 9in.

Ex. 4. Find the price of 16 boxes of tea each containing 2 mds. 25 sr. 10 ch. at *Rs.* 100. 12*a*. per maund.

20 sr. = ½ of 1 md.	*Rs.*	100	12*a*.	0*p*.	price of 1 md.
				2	
	Rs.	201		0*p*.	price of 2 mds.
5 sr. = ¼ of 20 sr.		50		0	" " 20 sr.
10 ch. = ⅛ of 5 sr		13		6	" " 5 sr.
		1		2¼	" " 10 ch.
	Rs.	266	0*a*.	8¼*p*.	" 2 mds. 25 sr. 10 ch.
				16	or of 1 box.
	Rs.	4256	11*a*.	0*p*.	= price of the 16 boxes.

Ex. 5. Find the value of 211 cwt. 3 qr. 25 ℔. at £15. 16*s*. 9½*d*. per cwt.

2 qr. = ½ of 1 cwt.	£ 15	16*s*.	9½*d*.	= price of 1 cwt.
			10	
	158	7	11	= price of 10 cwt.
			4	
	633	11	8	= price of 40 cwt.
			5	
	3167	18	4	= price of 200 cwt.
	158	7	11	" 10 "
1 qr. = ½ of 2 qr.	15	16	9½	" 1 "
14 ℔. = ½ of 1 qr.	7	18	4¾	" 2 qr.
7 ℔. = ¼ of 1 qr.	3	19	2⅜	" 1 "
4 ℔. = ⅐ of 1 qr	1	19	$7\frac{3}{16}$	" 14 ℔.
		19	$9\frac{19}{32}$	" 7 "
		11	$3\frac{43}{64}$	" 4 "
	£3357.	11*s*.	$4\frac{39}{64}$*d*.	" of 211 cwt. 3qr. 25℔.

Examples 108.

Find by Practice the value of:

1. 17 mds. 29 sr. 12 ch. sugar at R13. 14*a*. 8*p*. per maund.

2. 14 sacks of flour, each containing 5 mds. 27 sr. 8 ch., at R4. 12a. 6p. per maund.

3. 37 lbs. 8 oz. 18 dwt. of silver at £4. 7s. 6d. per lb.

4. 110 cwt. 3qr. 21 lbs. at £2. 17s. 6d. per cwt.

5. 17 tons 19 cwt 3qr. 14 lbs. at £4. 13s. 10d. per cwt.

6. 73 bighas 17 kathas 13 ch. at R245. 13a. 8p. per bigha.

7. 247 yds. 2 ft. 8 in. of cloth at R5. 7a. 10p. per yard.

8. 561 tons 15 cwt. 2 qr. 13 lbs. at £3. 7s. 6d. per ton.

9. 41 mi. 7 fur. 144 yds 1 ft. $1\frac{1}{2}$ in. of railings at R1100. 12a. per furlong.

10. 57 ac 3 ro. 37 po. of land at £257. 18s. 8d. per acre.

11. 21 ac. 3 ro. 12 po. of land at £125. 7s. 6d. per acre.

12. 25 bales of cotton, each weighing 7 mds. 38 sr. 15 ch., at R18. 10a. 8p. per maund.

13. 215 boxes of tea, each containing 3 cwt. 3 qr. 25 lbs., at £14. 13s. 4d. per cwt.

14. 56 mi. 1000 yds of wire at £33. 14s. 8d. per mile.

15. 27 lbs. 10 oz. 19 dwt. 8 gr. of silver at £4. 7s. 6d. per lb.

16. 73 qr. 5 bus. 3 pk. of barley at £3. 14s. 10d. per quarter.

17. 33 tons 15 cwt. 3 qr. 21 lbs. at £23. 13s. $11\frac{1}{2}d$. per ton.

18. 2756 mds. 31 sr. 8 ch. of rice at R4. 10a. 8p. per maund.

19 715 sq. yds. 7 sq. ft. 110 sq. in. at £1. 6s. per square yard.

20. 39 hhd. 15 gal. 3 qts. at £3. 5s. 6d. per hogshead.

21. 45 ac. 3 ro. 24 po. at £96. 2s. $2\frac{1}{2}d$. per. acre.

22. 26 *pans* $17\frac{1}{2}$ *gandas* of mangoes at R3. 2a. per *pan*.

23. 93 cwt. 3 qr. 27 lbs. 12 oz. at R46. 10a. 8p. per cwt

24. Find the produce of 516 bighas 18 kathas 15 ch. of paddy land at 15 mds. 30 sr per bigha.

25. Find the value of 64 mds 35 sr. 12 ch. at R5·5625 per maund.

26. If the produce per acre be 13·625 cwt., what will be the produce of 36 ac. 3 ro. $37\frac{1}{2}$ po.?

27. Find the rent of 25 bighas 17 kathas 8 ch. at R4·325 per bigha.

28. The debts of a bankrupt amount to R6745. 10a. 8p, and he can pay 13a. 8p. in the rupee. What sum do his creditors receive?

29. Find the dividend on £758. 17s 6d. at 16s. 8d. in the £.

30. Find the amount of a tax on 464 bighas 12 kathas 12 ch. at 3a. 4p. per bigha.

31. A bankrupt can pay 17s. 7d. in the £, on a debt of £2644. 17s. 6d.; find his assets.

32. Find the tax on an estate worth R5629. 12a. 10p. per annum, at the rate of 3a. in the rupee.

33. A bankrupt can pay £·625 in the £; how much can he pay on a debt of £7625 12*s*. 6*d*.?

34. Find the cost of enclosing a field that will require a fence 7 mi. 6 fur. 121 yds. long at R1694. 13*a*. 4*p*. per mile.

35. Find the value of 408 butts 1 hhd. 37 gal. of beer at £33. 1*s*. 6*d*. per butt.

36. Find the value of 2464 mds. 37 sr. 8 ch. at R25·4375 per maund.

37. A man has R7560. 13*a* 8*p*. in Railway shares which pay a dividend of $5\frac{3}{4}$*p*. in the rupee. Find his income from them.

38. If the Government Revenue is 4*a*. 4*p*. in the rupee, what ought a zamindar to pay, the annual value of whose estates is R25000 14*a*. 6*p*.?

39. If the annual rent of a tenanted house be £30. 8*s*. 4*d*. per annum, what will it amount to in 8 years 219 days?

INVOICE.

256. A tradesman has usually two kinds of sale, **cash** and **credit.** Both these kinds have to be **entered** in his **Day-book,** which gives each day's transactions in the order in which they occur.

The transactions with those customers who are allowed credit, are at short intervals **posted** in the **Ledger,** in which are given *all* the transactions which have taken place with that particular customer. For facility of reference, the ledger contains an alphabetical list of the customers names, with a reference to the particular page or pages in which their accounts appear.

An **Invoice** is a detailed statement sent to the buyer, *at the time of delivery* of the goods sold, of their quantity and price. Each separate entry is called an **Item.**

A copy of each customer's account is sent to him after the term of his credit is over. In such a case, the **Account** is said to be **rendered.** If the details of the articles are also given, it is called a **Detailed Account** or **Bill of Parcels.**

(i) SPECIMEN OF AN INVOICE.

INVOICE, Calcutta, *19th March*, 1887.
FROM MESSRS. THACKER, SPINK & CO.,
5, Government Place, North.

		Rs.	*a.*	*p.*
15 copies of Barnard's Arithmetic	at *Rs*. 2. 8*a*. 6*p*.	37	15	6
12 copies of Todhunter's Algebra .	" " 4. 3*a*. 6*p*.	50	10	0
15 copies of Novels	" " 2. 6*a*.	35	10	0
		124	3	6

(ii) SPECIMEN OF AN ACCOUNT.

DHUR & Co., Calcutta, *May 15th*, 1890.
Bought of DAW & Co , Clive Street.
Calcutta.

1890			Rs.	a.	p.
January	17	To goods as per invoice	57	12	6
February	1	To ditto	63	9	3
March	3	To ditto.	104	15	3
			226	5	0

(iii) SPECIMEN OF A DETAILED ACCOUNT.

J. WILLIAMS ESQ., Calcutta, *July 16th*, 1890.
Bought of JOHN REID & Co ,
Esplanade Row.

1890.		at	Rs.	a.	p.	Rs.	a.	p.
April 15	15 yds. Irish Linen	*Re.* 1. 10*a.* 9*p.*	25	1	3			
	5 dozen Stockings	*Rs.* 5. 4*a.*	26	4	0			
	25 yds. of Flannel	*Re.* 1. 7*a.*	35	15	0	87	4	3
May 17	12 yds. Calico	6*a.*	4	8	0			
"	15 yds Brussels Carpet	*Rs.* 2. 12*a.*	41	4	0	45	12	0
June 21	20 yds. of Flannel	*Re.* 1. 7*a.*	28	12	0	28	12	0
						161	12	3

Examples 109.

Make out invoice for the following :

1. 73 mds. of coal at 6*a.* 6*p.* per md. ; 56 mds. 30 sr. at 5*a.* 4*p.* per md. ; 37 mds. at 7*a.* 8*p.* per md. ; and 19 mds. 25 sr. at 8*a.* per md.

2. 27 yds. of calico at 5*a.* 4*p* per yd. ; 40 yds. of carpet at R2. 8*a.* per yd. ; 95 yds of matting at 4*a* per yd ; 57 yds. of damask at R2. 4*a.* per yd. ; and 58 yds. of chintz at 10*a.* 6*p.* per yd.

3. 75 lbs. of tea at R1. 2*a.* ; 50 lbs. of sugar at 3*a.* ; 25 lbs. of coffee at 10*a.* 6*p.*; and 24 lbs. of butter at 7*a.* 6*p.*

4. 24 mds. of rice at R4. 13*a.* ; 20 mds of flour at R4. 14*a.* ; 12 mds. of rice at R4. 10*a.* ; and 15 mds. of ghee at R31. 8*a.* The cost of carriage per md. is 3*a.*

5. 42 yds. of flannel at 1*s.* 4*d.* ; 52 yds. of calico at $7\frac{1}{2}$*d.* ; $10\frac{1}{2}$ dozen stockings at £1. 8*s.* 6*d.* per dozen ; 13 pairs of gloves at 5*s.* 8*d.* ; 20 collars at 1*s.* $7\frac{1}{4}$*d.*

6. $28\frac{1}{2}$ yds. of flannel at 2*s.* 6*d.* ; 44·5 yds. of calico at 10*d.* ; $32\frac{3}{4}$ yds. of linen at 4*s.* 8*d.* ; 26 yds. of carpet at 5*s.* 10*d.* ; 31 pairs of stockings at 2*s.* : and 15 pairs of gloves at 3*s.* 6*d.*

7. Supposing that Examples 5 and 6 refer to purchases made by the same party on two different dates, make out a detailed account.

8. Sheffield, March 23, 18—. Mr David Johnson bought of J. Rodger & Sons. 7 dozen razors at 1*s.* $5\frac{1}{4}$*d.* each, 5 dozen ditto at 1*s.* $2\frac{3}{4}$*d.*, 8 dozen ditto at 9*d*, 8 dozen knives and forks at £1. 2*s.* 6*d.* per dozen, 3 dozen ditto at 18*s.* 6*d*, 8 dozen scissors at 7*s.* 6*d.* per dozen, 3 dozen knives at 6*s.* $8\frac{1}{2}$*d.* per dozen, and 10 cases of lancets at 16*s.* $3\frac{1}{4}$*d.* per case. Required a copy of the bill and its amount.

CHAP. XXVI. SQUARE ROOT.

257. The **square root** of a given number is the number whose *square* is the given number. A number is therefore the square of its square root.

Thus, $6^2=36$; ∴ 6 is the *square root* of 36. The square root of 36 is denoted by $\sqrt{36}$ or $36^{\frac{1}{2}}$.

258. Since the square of every number must have the same unit figure as the square of its unit figure, it is obvious that every square number must end with 1, 4, 5, 6, 9 or an even number of o's. Hence *no number can be a perfect square* which ends with (i) the digits 2, 3, 7, 8: (ii) an odd number of o's.

Examples 110.

Find by inspection which of the following may be square numbers:

1. 23, 86, 772, 1890, 32571, 4362000, 35777, 243969, 324788.

2. 300, 90000, 56725, 893427, 59323, 46828, 62524.

259. The square root of a square number less than 400 can be found immediately from the Multiplication Tables.

Thus, $7\times7=49$, and $18\times18=324$; ∴ $\sqrt{49}=7$ and $\sqrt{324}=18$.

260. When a number can be easily reduced to its prime factors, its square root can be found by inspection thus:

Ex. 1. Find the square root of 1225.

$$1225=5\times5\times7\times7=35\times35=35^2;$$

$$\therefore \quad \sqrt{1225}=35. \quad \textit{Ans.}$$

Ex. 2. Determine the square root of 435600.

$$435600=10\times10\times6\times6\times11\times11=10^2\times6^2\times11^2=(10\times6\times11)^2$$

$$\therefore \quad \sqrt{435600}=10\times6\times11=660. \quad \textit{Ans.}$$

Ex. 3. What is the square root of 298116?

$$298116=4\times9\times7\times7\times13\times13=2^2\times3^2\times7^2\times13^2;$$

$$\therefore \quad \sqrt{(298116)}=2\times3\times7\times13=546. \quad \textit{Ans.}$$

[It should be noted that each prime factor must occur an *even* number of times in order that the square root may be found in this way.]

Ex. 4. What is the *least number* which will make 8316 a perfect square?

$8316 = 11 \times 756 = 11 \times 7 \times 108 = 11 \times 7 \times 3 \times 9 \times 4 = 11 \times 7 \times 3 \times 6^2$

$\therefore$ the required no. $= 11 \times 7 \times 3 = 231$. *Ans.*

Examples 111.

Find the square root of:

1. 36, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 361, 400.

2. 1296, 4225, 211600, 2924100, 451584, 99225, 2340900.

Find the *least numbers* which will make the following perfect squares:

3. 216, 1512, 16632, 34125, 580125.

4. What must be the least number of soldiers in a regiment, that will allow it to be drawn up 15, 18 or 30 deep, and also to be formed into a solid square?

261. It can be easily seen that $\sqrt{(100)} = 10$, $\sqrt{(10000)} = 100$, $\sqrt{(1000000)} = 1000$, &c.; thus the square roots of all numbers of 1 or 2 figures only contain 1 figure, those of numbers of 3 or 4 figures contain 2 figures, those of numbers of 5 or 6 figures contain 3 figures, and so on.

Again, $23^2 - 20^2 = 529 - 400 = 129 = 2 \times 20 \times 3 + 3^2$.

$\therefore (23^2 - 20^2) \div (2 \times 20) = 3 +$ a proper fraction.

From the above, the following RULE for extracting the square root of an integer is obtained:

262. *To extract the square root of an integer.*

RULE. Place a dot over the unit figure and over every alternate figure to the left, thus dividing the integer into periods of 2 figures each, each period consisting of the figure pointed and the figure to its left. (The first period may, however, consist of one figure only).

Find the number whose square is immediately below the number in the first period; place it in the root, and subtract its square from the first period.

To the remainder bring down the next period; find how often twice the figure in the root called the (*trial divisor*) is contained in this number with its last figure omitted; set down the quotient as the second figure in the root, and also annex it to the trial divisor; multiply the divisor so obtained by the second figure in the root and subtract.

[It will often happen that the product is greater than the dividend. In such a case the figure taken in the root *must be diminished.* It might, however, happen that the remainder is greater than the divisor]

To this remainder bring down the next period; take twice the number in the root, and find how often this *trial divisor* is contained in the dividend with its last figure omitted; and proceed as before.

Continue thus, until *all* the periods have been brought down.

N.B.—When in any case the dividend with its last figure omitted is found to be *less* than the trial divisor, place a cypher in the root, affix it also to the trial divisor, and bring down the next period.

Ex. 1. Find the square root of 1764.

```
   1764 | 42
   16
82 | 164
   | 164
```

Place dots over 4 and 7, so that the number is divided into two periods, 17 and 64.

The number whose square is immediately below 17 is 4; ($5^2=25$ which is greater than 17). Hence **4** is put in the root and 16 subtracted from 17.

To the remainder 1 is brought down the next period 64; the number 164 is thus obtained. $2\times4=8$, is the *trial divisor* which goes into 16 (164 with 4 omitted) 2 times. Hence **2** is put after 4 in the root and also annexed to 8.

Reason for the process: Since the number has 4 figures, its root has 2 figures and the number should be divided into 2 periods.

$4^2=16$, but $5^2=25$; hence $\sqrt{1764}$ lies between 40 and 50, and therefore 4 is put in the root. $1764-40^2=164$; therefore the next figure of the root should be the integral part of $164\div(2\times40)$ or 2.

Ex. 2. Extract the square roots of 28772496, 444155625 and 3236401.

```
      28772496 | 5364        444155625 | 21075        3236401 | 1799
      25                     4                        1
  103 | 377             41 | 44                  27 | 223
      | 309                | 41                     | 189
 1066 | 6824           420 | 31556              349 | 3464
      | 6396               | 29449                  | 3141
10724 | 42896        42145 | 210725            3589 | 32301
      | 42896              | 210725                 | 32301
```

263. When an integer (which is a perfect square) ends with an even number of cyphers, the operation may be stopped as soon as the last significant figure is brought down, *one* cypher being added to the root for every *two* cyphers in the integer.

Ex. Extract the square root of 81000000.

```
81000000 | 9000
81
```

There are 6 cyphers in the given number; therefore 3 cyphers are added to 9.

264. Since the square of a number with one decimal place contains 2 decimal places, that of a number with two decimal places contains 4 decimal places, and so on, it is plain that the square root of a decimal cannot be exactly found unless it contains an even number of decimal places and ends with 1, 4, 5, 6 or 9; also the number of decimal places in the root must be half the number of decimal places in the decimal.

265. The RULE for the pointing of decimals is similar to that of integers. Point the unit figure and every alternate figure *both to its right and left*; but if there be no integral part place the first dot over the *second* decimal figure.

The integral and decimal parts of the root should have as many figures respectively as there are periods in the two parts of the given number.

```
      499·9696|22·36              ·0000603729|·00777
      4                                  49
   42 99                            147 1137
      84                                1029
  443 1596                         1547 10829
      1329                              10829
 4466|26796
     |26796
```

Examples 112.

1. Find the square root of :

(1) 11025, 53361, 63504 379456, 531441, 998001.

(2) 824464, 29506624, 2480625, 12446784, 28547649.

(3) 236144689 998876025, 300155625, 5294745225.

(4) 930677049, 823862209, 546121000000, 91536502500.

2. Extract the square root of :

(1) 54·6121, 126·1129, 1483·7904, 6048·1729.

(2) 20615·2164, 2824·29536481, 529474·5225.

(3) ·00022801, ·0000253009, ·000182493081, ·000000063001.

3. A General having 106276 men under him, arranges them into a solid square. Find the number of men in the front.

4. In a meeting a subscription was raised amounting to R365. 12*a*. 1*p*., each person paying as many pies as there were members present in the meeting. Find the number of members present.

5. In a square plantation there are 1500625 cabbages; find the number of cabbages in each line.

6. A General intending to arrange his men, who were 1205625 in number, into a solid square, found there were 21 men over. How many men were there in the front?

266. If the number is not an exact square, we may find its square root to any required number of decimal places by adding cyphers and bringing down periods of 2 cyphers each.

```
5·00 00 00 00 | 2·236...          3·20 00 00 | 1·788...
4                                 1
42  | 100                         27  | 220
    |  84                             | 189
443 | 1600                        348 | 3100
    | 1329                            | 2784
4466| 27100                       3568| 31600
    | 26796                           | 28544
```

N. B. When the given number is a recurring decimal, the decimal part should be extended by a repetition of its periods.

267. Where the square root is wanted to a *large number* of decimal places, we may get just more than half the number of figures required in the root in the usual way, and then the remaining figures by dividing the last remainder by the last divisor, as in ART. 241.

Ex. Extract the square root of 10 to 8 places of decimals.

```
10       | 3·16227766          6,3,2,4,4)49116
 9                                       44271
61    | 1·00                              4845
      |   61                              4427
626   | 3900                               418
      | 3756                               379
6322  | 14400                               39
      | 12644                               38
63242 | 175600
      | 126484
         49116
```

The first 5 figures are obtained in the usual way and the last 4 by Abbreviated Division.

Examples 113.

1. Find the square root of (each to 4 places of decimals):

(1) 2, 3, 5, 7, 11, 18, 24, 32, 57, 72, 80.

(2) 223, 242, 3727747, 921568, 2728122, 79684147.

2. Extract the square root of (each to 3 places of decimals)

(1) ·1, ·2, ·4, ·9, 1·6, 2·2, 4·9, $4\cdot\dot{5}$, $7\cdot\dot{4}$, $\cdot\dot{7}$, $\cdot\dot{1}\dot{8}$, $6\cdot\dot{4}$.

(2) ·024, 864·9, 41·162, 288·369, 1015·69, 1008·61849.

(3) ·0010201, ·0000961, ·0005184, 40·6506244, 26044·69156.

(4) $4\cdot\dot{3}2\dot{1}$, $2\cdot\dot{3}6\dot{1}$, $5\cdot4\dot{1}3\dot{2}$, $\cdot0\dot{2}98\dot{1}$, $\cdot001\dot{7}2\dot{8}$, $46\cdot42\dot{5}4\dot{6}$, $300\cdot\dot{3}24\dot{1}$.

3. Find to 9 decimal places the square roots of :

321·73025, 64·34, ·001728, 12, 13, 1·57, 2·221, 44·284.

268. The square root of a fraction is the square root of its numerator divided by the square root of its denominator.

Thus, $\sqrt{\frac{121}{16}} = \sqrt{121} \div \sqrt{16} = \frac{11}{4}$;

$\sqrt{45\frac{31}{121}} = \sqrt{\frac{5476}{121}} = \sqrt{5476} \div \sqrt{121} = 74 \div 11 = \frac{74}{11} = 6\frac{8}{11}$

269. When the numerator and the denominator are not both square numbers, we may either convert the given fraction into an equivalent fraction with the denominator equal to a square number and proceed as below, or convert it into a decimal and proceed as in ART. 265.

Ex. Extract the square root of $3\frac{3}{7}$ (to 3 places of decimals).

$3\frac{3}{7} = \frac{24}{7} = \frac{168}{49}$.

$\sqrt{\frac{168}{49}} = \frac{1}{7}\sqrt{168}$

$= \frac{1}{7} \times 12·9614...$

$= 1·851...$

$3\frac{3}{7} = 3·428571...$

$\sqrt{3·428571...} = 1·851...$

This latter method is, as a rule, the more convenient of the two.

270. If a recurring decimal is a perfect square, it is better to convert it into a vulgar fraction and proceed as in ART. 268.

Thus, $\sqrt{·\dot{4}} = \sqrt{\frac{4}{9}} = \frac{2}{3} = ·\dot{6}$.

Examples 114.

1. Find the square root of :

(1) $\frac{81}{100}$, $\frac{121}{144}$, $\frac{169}{225}$, $\frac{256}{361}$, $\frac{7056}{9216}$, $\frac{1369}{6084}$, $\frac{38642}{31752}$, $\frac{22801}{44100}$.

(2) $148\frac{213}{1681}$, $19\frac{30}{361}$, $43\frac{34}{529}$, $39\frac{1}{16}$, $566\frac{11}{25}$, $27666\frac{7}{9}$.

2. Extract the square root of (to 3 places of decimals) :

(1) $5\frac{7}{9}$, $4\frac{22}{25}$, $\frac{2467}{121}$, $\frac{8448}{169}$, $421\frac{13}{225}$, $84621\frac{43}{6084}$.

(2) $\frac{7321}{15}$, $\frac{28112}{111}$, $41\frac{7}{8}$, $37\frac{2}{5}$, $2814\frac{9}{11}$, $462\frac{73}{99}$.

3. Find the square root of : $13·\dot{4}$, $40·\dot{1}$, $13611·\dot{1}$, $802·\dot{7}$.

CHAP. XXVII. SQUARE MEASURE.

271. (*a*) **A rectangle** is a four-sided figure whose opposite sides are equal, and whose angles are right angles.

(*b*) A **square** is a rectangle whose sides are all equal.

(*c*) The quantity of surface contained in a figure is called its **area**.

In **Arithmetic** we treat of the areas of Rectangles only.

272. Suppose that a rectangular surface, as that of a table, is **4 feet** long and **3 feet** broad; its sides can be divided into 4 and 3 equal parts respectively, each part being 1 foot. Through the points of division draw lines parallel to the sides. It is obvious that the figure is thus divided into 4×3 or 12 equal squares, the area of each being *one square foot*. Hence the **area** of the surface of the table is 4×3 or **12 square feet.**

Hence, **area** in sq. ft. = **length** in ft. × **breadth** in ft. *i.e.*, the product of two adjacent sides.

∴ length = area ÷ breadth, and breadth = area ÷ length.

N.B.—(1) The length and breadth must both be reduced to the same denomination, the area being obtained in square units of that denomination.

(2) The length and breadth are called the **dimensions**.

(3) The distinction between **5 feet square** and **5 square feet** should be remembered. By 5 *feet square* is meant the area of a square each of whose sides is 5 feet and consequently whose area is 5×5 or 25 sq. ft. By 5 *square feet* is meant the area which is equal to 5 squares, each of which is a foot square. The latter is therefore only *one fifth* of the former.

(4) The student should now be able to understand the relations between the measures in tables III and IV. Thus, a square foot being a square whose side is 12 in., its area = 12×12 or 144 sq. in.; so a square yard has each of its sides = 3 ft., and therefore its area = 3×3 or 9 sq. ft.; and so on.

273. The length of a square being equal to its breadth, its side will be obtained by extracting the square root of its area.

If the length of a rectangle be *twice* or *thrice* the breadth, divide the given area by 2 or 3, and then extract the square root, which will give the breadth.

[**Matting** and **carpeting** always refer to the **floor** only.]

Ex. 1. Find the area of a floor 16 ft. 8 in. long by 13 ft. 4 in. broad.
Length = 16 ft. 8 in. = $16\frac{2}{3}$ or $\frac{50}{3}$ ft.; breadth = 13 ft. 4 in. = $13\frac{1}{3}$ or $\frac{40}{3}$ ft.
∴ the area of the floor = $(\frac{50}{3} \times \frac{40}{3})$ sq. ft. = $\frac{2000}{9}$ sq. ft. = $222\frac{2}{9}$ sq. ft.
= 24 sq. yds. 6 sq. ft. 32 sq. in.

Ex. 2. Find the length of a rectangular court-yard which contains $233\frac{1}{3}$ sq. ft. and is 13 ft. 4 in. broad.
Area = $233\frac{1}{3}$ sq. ft. = $\frac{700}{3}$ sq. ft.; breadth = 13 ft. 4 in. = $\frac{40}{3}$ ft.
∴ length = $(\frac{700}{3} \div \frac{40}{3})$ ft. = $(\frac{700}{3} \times \frac{3}{40})$ ft. = $17\frac{1}{2}$ ft. = 17 ft. 6 in.

Note. A sq. yd. or a sq ft. &c. when divided by a linear yard or a linear foot &c., gives as quotient a linear yard or a linear foot &c.

Ex. 3. A square field contains 12345 sq. yds. 6 sq. ft. 16 sq. in.: find the length of fence required to enclose it.

Area $=$ 12345 sq. yds. 6 sq. ft. 16 in. $= 111111\frac{1}{9}$ or $\frac{1000000}{9}$ sq. ft.;

$\therefore$ a side $= \sqrt{(\frac{1000000}{9})}$ ft $= \frac{1000}{3}$ ft.;

$\therefore$ length of fence required. $= \frac{1000}{3}$ ft. $\times 4 =$ 444 yds. 1 ft. 4 in.

Ex. 4. Find the number of paving stones each measuring 4 ft. 6 in. by 1 ft. 4 in. that are required to pave a verandah 70 ft. by 9 ft. broad.

Area of a paving stone $= (4\frac{1}{2} \times 1\frac{1}{3})$ sq. ft. $= 6$ sq. ft.

Area of the verandah $= (70 \times 9)$ sq. ft. $= 630$ sq. ft.

$\therefore$ no. of paving stones reqd $= (630 \div 6) = 105$. *Ans.*

Ex. 5. How many marble tiles each 2 ft. 8 in. square will be required to pave a court-yard 100 ft. long and 64 ft. broad?

Area of a tile $= (2\frac{2}{3})^2$ or $\frac{64}{9}$ sq ft.: area of the court $= (100 \times 64)$ sq. ft.

$\therefore$ no. of tiles $= 100 \times 64 \div \frac{64}{9} = 900$. *Ans.*

Ex. 6. Find the cost of a carpet 17 ft. 6 in. long and 13 ft. 4 in. broad, at *Rs.* 2. 13*a.* per square yard.

Area of the carpet $= 17\frac{1}{2} \times 13\frac{1}{3}$ sq. ft $= \frac{700}{3}$ sq. ft. $= \frac{700}{27}$ sq. yds.

$\therefore$ reqd. cost. $=$ *Rs.* $2\frac{13}{16} \times \frac{700}{27} =$ *Rs.* 72 14*a.* 8*p.*

Ex. 7. The cost of carpeting a room twice as long as it is broad at *Rs.* 3. 7*a.* per square yard, is *Rs.* 336. 14*a.*; find its length and breadth.

Rs. 3. 7*a.* $=$ *Rs.* $3\frac{7}{16} =$ *Rs.* $\frac{55}{16}$; *Rs.* 336. 14*a.* $=$ *Rs.* $336\frac{14}{16} =$ *Rs.* $\frac{5390}{16}$.

$\therefore$ area of the room $= (\frac{5390}{16} \div \frac{55}{16})$ sq. yds. $= 98$ sq. yds.

$\therefore$ breadth $= \sqrt{(\frac{98}{2})}$ yds. $= 7$ yds. $= 21$ ft.;
and length $= 21$ ft. $\times 2 = 42$ ft. } *Ans.*

Ex. 8. Find the length of mats required for the floor of a room 20 ft. 3 in. square, the mats being $\frac{9}{8}$ of a yard wide; also find the cost of the mats at 2*a.* 8*p.* per yard.

Area of matting $= (20\frac{1}{4})^2$ sq. ft. $= \frac{81 \times 81}{4 \times 4}$ sq. ft. $= \frac{9 \times 81}{16}$ sq. yds.

$\therefore$ length of matting $= \left(\frac{9 \times 81}{16} \div \frac{9}{8}\right)$yds. $= \frac{81}{2}$ yds. $= 40\frac{1}{2}$ yds.
Also, the cost of mats $=$ 2*a.* 8*p.* $\times \frac{81}{2} =$ *Re.* $\frac{1}{6} \times \frac{81}{2} =$ *Rs.* 6. 12*a.* } *Ans.*

Ex. 9. The cost of carpeting a room 12 ft. 6 in. wide at *Rs.* 2. 8*a.* per sq. yd., is *Rs.* 187. 8*a.*; find the length of the room.

Rs. 2. 8*a.* $=$ *Rs.* $\frac{5}{2}$; *Rs.* 187. 8*a.* $=$ *Rs.* $\frac{375}{2}$.

$\therefore$ area of the room $= (\frac{375}{2} \div \frac{5}{2})$ or 75 sq. yds. $= 675$ sq. ft.;

$\therefore$ length of the room in ft. $= 675 \div 12\frac{1}{2} = 675 \times \frac{2}{25} = 54$. *Ans.*

Ex. 10. A roller is 7 ft. 6 in. in circumference and 2 ft. 4 in. in breadth; how much area does it pass over in 12 revolutions?

In each revolution, the roller covers a strip of ground whose area is equal to the product of the breadth and the circumference.

$\therefore$ area rolled over in one revolution $= (7\frac{1}{2} \times 2\frac{1}{3})$ sq. ft. $= \frac{35}{2}$ sq. ft.;

$\therefore$ the whole area rolled $= \frac{35}{2}$ sq. ft. $\times 12 = 210$ sq. ft. *Ans.*

Examples 115.

1. Find the area of the following rectangles :

(1) 25 ft. by 16 ft. (2) 34 ft. by 23 ft.

(3) 125 ft. 6 in. by 98 ft. (4) 30 ft. 8 in. by 11 ft. 3 in.

(5) 33 ft. 4 in. by 18 ft. 9 in. (6) 37 ft. 8 in. by 31 ft. 2 in.

(7) 14 ft. 9 in. by 9 ft. $10\frac{1}{2}$ in. (8) 15 ft. $11\frac{1}{4}$ in. by 11 ft. $0\frac{3}{4}$ in.

2. What quantity of matting will be required for each of these rooms ?

(1) 32 ft. by 21 ft. $4\frac{1}{2}$ in. (2) 58 ft. 6 in. by 34 ft. 8 in.

(3) 17 ft. 6 in. by 14 ft. 8 in. (4) 7 yds. 1 ft. 6 in. by 4 yds.

3. Find the length or breadth of the following rooms :

(1) area = 85 sq. ft. 36 sq. in. and breadth = 8 ft. 3 in.

(2) " = 238 " 90 " " " = 13 " 10 "

(3) " = 137 " 72 " " length = 16 " 8 "

(4) " = 98 " $15\frac{3}{4}$ " " " = 11 " $4\frac{1}{2}$ "

4. Find the sides of the following squares :

32400 sq. in.; 37 sq. yds. 3 sq. ft. 16 sq. in. ; 18 sq. yds. 81 sq. in.

5. Find the area of a square field whose side is 459 yds., in acres, roods, square yds. &c.

6. The length of a field is 7 chains 11 yds. and its breadth 5 chains ; find its area in acres.

7. How many mats each 4 ft. 6 in. by 2 ft. 4 in. will be required for a floor measuring 112 ft. 6 in. by 23 ft. 4 in. ?

8. How many pieces of carpet each 3 ft. 4 in. long by 2 ft. 8 in. wide will cover the floor of a hall 53 ft. 4 in. long by 21 ft. 4 in. broad ?

9. How many stone tiles, each 1 ft. 9 in. by 1 ft. 2 in., will be required to pave a court-yard 131 ft. 3 in. long and 29 ft. 2 in. broad ?

10. How many planks, each measuring 8 ft. 8 in. by 1 ft. 4 in., will be required for the floor of a room 17 ft. 4 in. long by 14 ft. 8 in. broad ?

11. Find the number of paving stones each 1 ft. 6 in. long and 8 in. broad, required to pave a court 37 ft. 6 in. long and 20 ft. 8 in. broad.

12. How many postage stamps, each $\frac{3}{4}$ in. long and $\frac{5}{8}$ in. wide, will cover a wall 16 ft. 3 in. long and 13 ft. 9 in. high ?

13. How many bricks (the exposed surface of each of which is 11 in. by 3 in.) will pave a court-yard 57 ft. 9 in. long and 30 ft. wide ?

14. A hall is 48 ft. long and 22 ft. wide ; how many chairs can it hold, supposing each to occupy 3 sq. ft. ?

15. If 8960 tiles, each 18 in. square, are required to pave a foot-path 6 ft. wide, what must be its length ?

16. How many turfs each 2 ft. 10 in. long and 1 ft. 3 in. wide, will be required for a piece of ground 141 ft. 8 in. long and 112 ft. 6 in. wide ?

17. A barrack is 120 ft. long and 24 ft. broad; how many soldiers will it accommodate, allowing to each soldier 48 sq. ft.?

18. Find the cost of carpeting the following rooms:

(1) 16 ft. 8 in. long and 12 ft. 6 in. wide at R2. 12*a*. per sq. yd.

(2) 17 ft. 6 in. „ „ 14 ft. 8 in. „ „ 5*s*. 6*d*. per sq. yd.

(3) 24 ft. 8 in. „ „ 18 ft. 4 in. „ „ R10. 5*a*. for 15 sq. ft.

(4) 15 ft. long by $11\frac{1}{3}$ ft. wide with carpet $2\frac{1}{2}$ ft. wide at 3*s*. a sq. ft.

19. Find the cost of matting the following rooms:

(1) 37 ft. 6 in. by 14 ft. 8 in. with mats $\frac{5}{8}$ yd. wide at 4*a*. per yd.

(2) 47 ft. 6 in. „ 22 ft. 6 in. „ „ $\frac{5}{7}$ yd. „ „ 6*a*. per yd.

(3) 26 ft. 8 in. „ 20 ft. 3 in. „ „ $\frac{10}{11}$ yd. „ „ 2*s*. $1\frac{3}{4}$*d* per yd.

(4) 21 ft. 4 in. „ 15 ft. 9 in. „ „ $\frac{7}{8}$ yd. „ „ 1*s*. 5*d*. per yd.

20. The cost of matting a square room at 6*p*. per sq. ft. is R19. 8*a*. 6*p*.; what is its length?

21. What are the length and breadth of a hall which requires carpets worth £36. 6*s*. at 2*s*. per sq. ft., the length being thrice the breadth?

22. The cost of matting a room at 7*a*. per sq. yd. is R5. 7*a*. 6*p*. If the room be half as wide as it is long, find its length.

23. The length of a room is 24 ft. 3 in. and the cost of carpeting it at 9*s*. a sq. yd is £18. 19*s*. 11*d*.: find the breadth of the room.

24. The breadth of a room is 14 ft. 8 in. and the cost of carpeting it at R3. 12*a*. per sq. yd. is R137. 8*a*.; find the length of the room.

25. How many sheets of paper each measuring 2 ft. 9 in. by 1 ft. $4\frac{1}{2}$ in. will just cover one acre?

26. What length must be cut off from a field 88 yards wide, in order to get an area of 6 acres?

27. A house has 56 windows, each being 10 ft. high and 4 ft. 6 in. wide; if a pane be 15 in. by $13\frac{1}{2}$ in., find the number of panes, and also their cost at 5*a*. 4*p*. per sq. ft.

28. A table-cloth 8 ft. 4 in. by 5 ft. 4 in. is divided into inch squares, and a half-pice laid on each; how much money will lie on it?

29. A street-roller is 20 ft. round and 7 ft. 6 in. wide: what area will it roll over in 216 revolutions?

30. A roller used for rolling a bowling-green is 8 ft. 4 in. in circumference and 3 ft. 4 in. wide and makes 32 turns in passing over from one end of the green to the other. Find the area rolled when it has passed 36 times over the whole length of the green.

31. There is a foot-path 9 ft. wide on each side of a street a mile long; find the cost of paving them with pebbles at 7*a*. 6*p*. per sq. yd.

***32**. A square field contains 90 bighas 5 kathas. A person riding a bicycle starts to go round it. If the circumference of the bicycle be 9 ft. 6 in. and if it turn 48 times per minute, how many minutes must pass before he comes back to the starting point?

33. Find the cost of enclosing a square field containing 16 ac. 401 sq. yds., at 2*s.* 3½*d.* per yard.

34. A square field containing 12 acres 484 sq. yds., is to be enclosed by a fence in which the palisades are put in a span apart; find the number of palisades.

274. **Painting, papering &c.** To find the area of a **wall** we have to multiply the length of the wall by its height.

In a room there are 4 walls, and the two that are opposite are equal.

The area of a wall running length-wise = length (of the room) × height, and that of a wall running breadth-wise = breadth " " × "

∴ **area** of the 4 walls = 2 × length × height + 2 × breadth × height
= **2(length + breadth) × height.**

But 2 × (length + breadth) is the **perimeter** (or **circuit**) of the room, *i.e.*, the sum of the lengths of the walls.

∴ **area** of the 4 walls = **perimeter (or circuit) × height.**

N.B. The area of the walls obtained by the above RULE includes that of the doors, windows, fire-places, &c. In all questions on papering, plastering, painting, &c of rooms, deductions must be made for these. Where, however, no such mention is made, no deductions are required.

Note. The student should be careful to avoid the common mistake of multiplying the length, breadth and height together when they are required to find the area of the four walls.

275. The area of the **ceiling** of a room is the *same* as that of the floor; to find the area of a ceiling we have therefore to multiply the length of the room by its breadth.

N.B. Unless otherwise mentioned, **Papering** and **painting** refer to the *walls* only, **plastering** and **whitewashing** both to the *walls* and the *ceiling*, and **glazing** to the *windows*, only.

Ex. 1. Find the area of the 4 walls of a room 17 ft. 7 in. long, 13 ft. 5 in. broad and 16⅔ ft. high.

Area of the 4 walls = 2(17 ft. 7 in. + 13 ft. 5 in.) × 16⅔ ft.
$= 2 \times 31 \times \frac{50}{3}$ sq. ft. $= \frac{3100}{3}$ sq. ft. = 1033⅓ sq. ft.

Ex. 2. Find the length of paper ⅔ yd. wide, required to cover the walls of the above room.

The area of the 4 walls $= \frac{3100}{3}$ sq. ft.; width of paper = ⅔ yd. = 2 ft.
∴ length of paper $= (\frac{3100}{3} \div 2)$ ft. $= \frac{1550}{3}$ ft. = 172 yds 0 ft. 8 in.

Ex. 3. Find the cost of papering the above room at 2*a.* 3*p.* per yd.
2*a.* 3*p.* = 2¼*a.*; also length of paper $= \frac{1550}{3}$ ft. $= \frac{1550}{9}$ yds.
∴ reqd. cost of paper $= \frac{9}{4}a. \times \frac{1550}{9} = \frac{775}{2}a.$ = *Rs.* 24 3*a.* 6*p.* *Ans.*

Ex. 4. Find the cost of painting the walls and ceiling of a room 18 ft. 9 in. long, 11 ft. 3 in. broad and 15 ft. high, at 4*s.* per sq. yd., there being 3 doors 6 ft. by 3 ft. 6 in. each and 6 windows 5 ft. 6 in. by 3 ft. each.

Area of the 4 walls $= 2$ (18 ft. 9 in. + 11 ft. 3 in.) $\times$ 15 ft. $=$ 900 sq. ft.

Area of the ceiling $= 18\frac{3}{4} \times 11\frac{1}{4}$ sq. ft. $= \frac{3375}{16}$ sq. ft.

Area of the 3 doors $= 3 \times 6 \times \frac{7}{2}$ sq. ft. $= 63$ sq. ft., and that of the 6 windows $= (6 \times \frac{11}{2} \times 3)$ or 99 sq. ft.

$\therefore$ whole area to be painted $= (900 + \frac{3375}{16} - 63 - 99)$ sq. ft.

$= 948\frac{15}{16}$ sq. ft. $= 105\frac{7}{16}$ sq. yds.

$\therefore$ cost of painting $= 4s. \times 105\frac{7}{16} = 421\frac{3}{4}s. = £21.\ 1s.\ 9d.$ *Ans.*

276. *To find the height of a room, the area of the four walls and the length and breadth being given.*

RULE. Height $=$ (area of the 4 walls) $\div$ 2 (length + breadth).

Ex. 1. The breadth of a room is 14 ft. 8 in.; the cost of matting it at 5*a.* 4*p.* per sq yd. is *Rs.* 11, and that of papering it at 3*a.* per sq. ft. is *Rs.* 157. 2*a.* Find the height of the room.

Area of the floor in sq. yds $=$ *Rs.* $11 \div 5a.\ 4p = 11 \div \frac{1}{3} = 33$;

$\therefore$ length of the room $= 33$ sq. yds $\div 14\frac{2}{3}$ ft. $= (33 \times 9 \times \frac{3}{44})$ or $20\frac{1}{4}$ ft.

Area of the 4 walls in sq. ft. $=$ *Rs.* 157. $2a. \div 3a. = 838$;

2 (length + breadth) $= 2\ (20\frac{1}{4}$ ft. $+ 14\frac{2}{3}$ ft.$) = \frac{419}{6}$ ft.

$\therefore$ height of the room $= (838 \div \frac{419}{6})$ ft. $= 12$ ft. *Ans.*

Ex. 2. The height and breadth of a room are 15 ft. and 18 ft. 9 in. respectively, and the cost of papering the walls with paper $\frac{5}{6}$ yd. wide is *Rs.* 114. 10*a.* 8*p.*, at 10*a.* 8*p.* per yd.; find the length of the room.

Rs. 114. 10*a.* 8*p.* $=$ *Rs.* $114\frac{2}{3} =$ *Rs.* $\frac{344}{3}$; and 10*a.* 8*p.* $=$ *Re.* $\frac{2}{3}$;

$\therefore$ length of paper reqd. $= (\frac{344}{3} \div \frac{2}{3})$ of yds. $= 172$ yds.;

$\therefore$ area of the 4 walls $= (172 \times \frac{5}{6})$ sq. yds. $= \frac{430}{3}$ sq. yds. $= 1290$ sq. ft.;

$\therefore$ 2 (length + breadth) $= (1290 \div 15)$ ft. $= 86$ ft.;

$\therefore$ the reqd. length $= 43$ ft. $-$ 18 ft. 9 in. $= 24$ ft. 3 in. *Ans.*

Examples 116.

1. Find the area of the 4 walls of each of the following rooms:

(1) 24 ft. long, 15 ft. broad, and 16 ft. high.

(2) 20 ft. 6 in. long, 12 ft. 4 in. broad, and 14 ft. high.

(3) 27 ft. 6 in. „ 19 ft. 8 in. „ „ 15 ft. 6 in. high.

(4) 19 ft. 3 in. „ 16 ft. 6 in. „ „ 13 ft. 6 in. high.

2. Find the length of wall-paper required for each of the following rooms:

(1) 18 ft. 8 in. square and 15 ft. 9 in. high with paper $\frac{7}{8}$ yd. wide.

(2) 29 ft. 6 in. long, 13 ft. $10\frac{1}{2}$ in. broad, and 14 ft. 8 in. high, with paper $\frac{8}{11}$ yd. wide.

(3) 24 ft. 9 in. long, 17 ft. 6 in. wide, and 11 ft. 11 in. high, with paper 26 in. wide.

(4) 26 ft. 5 in. long, 16 ft. 3 in. broad, and 14 ft. 6 in. high, with paper 2 ft. 8 in. wide.

3. Find the cost of painting the walls and ceiling of each of the following rooms :

(1) 19 ft. 4 in. long, 15 ft. 8 in. broad, and 12 ft. high, at 10*a*. 8*p*. per sq. yd.

(2) 20 ft. 6 in. square and 14 ft. 4 in. high, at R2. 8*a*. per sq. yd.

(3) 7 yds. 2 ft. 8 in. long, 5 yds. 1 ft. 4 in. broad, and 4 yds. 2 ft. 8 in. high, at 12*s*. 6*d*. per 5 sq. ft.

(4) 47 ft. 3 in. long, 32 ft. 9 in. broad, and 18 ft. high, at 7*s*. 8*d*. per sq. yd.

4. Find the cost of papering each of the following rooms :

(1) 17 ft. 9 in. long, 14 ft. 3 in. broad and 12 ft. 6 in. high, with paper $22\frac{1}{2}$ in. wide at 6*a*. per yard.

(2) 23 ft. 8 in. long, 16 ft. 4 in. broad, and 15 ft. high, with paper 25 in. wide at 5*a*. 4*p*. per yard.

(3) 28 ft. 10 in. long, 19 ft. 2 in. broad, and 13 ft. 4 in. high, with paper 2 ft. 8 in. wide at 9*d*. per yard.

(4) 32 ft. 7 in. long, 21 ft. 5 in. broad, and 16 ft. high, with paper $\frac{2}{3}$ yd. wide at 15*s*. for 30 ft.

5. A room 24 ft. long, 16 ft. broad and 18 ft. high, contains 3 doors 10 ft. by 5 ft. each, 4 windows 8 ft. by 4 ft. each and a fireplace 6 ft. by 4 ft. Find the cost of painting the room at 5*s*. 6*d*. per sq. yd.

6. A hall 56 ft. long, 24 ft. broad and 27 ft. high, contains 5 archways 12 ft. by 6 ft. each : find the cost of papering the walls with paper $\frac{3}{4}$ yd. wide at R2. 12*a*. per yard.

7. A cistern 16 ft. long, 12 ft. wide and 8 ft. deep is to be lined with lead. How many square feet of lead will it require? What weight of lead will there be if a square foot weigh 5 seers? And what will the lead cost at R12. 8*a*. per maund?

8. Find the cost of whitewashing a room 32 ft. long, 20 ft. broad and 13 ft. 6 in. high at R1. 9*a*. per 100 sq. ft.

9. A room is 27 ft. 8 in long, 17 ft. 4 in. broad, and 12 ft. high. How many postage stamps, each $\frac{15}{16}$ in. by $\frac{3}{4}$ in., will be required for its walls?

10. The cost of painting the walls of a room 17 ft. long and 13 ft. broad at 12*a*. per sq. yd., is R100. Find the height.

11. The height of a room is 15 ft. and the cost of painting the walls at R2. 4*a*. per sq. yd. is R225. Find its length and breadth, the length being twice the breadth.

12. The length of a room is 28 ft. : the cost of papering the walls with paper $\frac{3}{4}$ yd. wide at 7*a*. per yd. is R49 ; and that of carpeting the room at R1. 3*a*. per sq. ft. is R465. 8*a*. Find the height of the room.

13. The height and breadth of a room are 11 ft. 6 in. and 15 ft. 8 in. respectively, and the cost of painting its walls at 6*s.* per sq. yd. is £27. 12*s.*; find the length of the room.

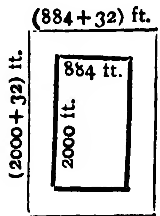
14. The length of a room is 56 ft.; the cost of matting the room at 5*a.* per sq. yd. is R29. 2*a.* 8*p.*, and that of papering the room with paper $\frac{2}{3}$ yd. wide at 10*a.* 8*p.* per yd. is R284. Find the breadth and the height of the room.

277. Paths, Moats, Foundations, &c.

(*a*) Where paths, moats, &c., go all round a field **outside** it, their areas may be found either (i) by **increasing** each dimension by *twice* the **uniform** breadth of the paths, &c., and subtracting the area of the given field from the increased area; or (ii) by **increasing** each dimension by the uniform breadth of the path, and then adopting the following RULE:

Area $= 2 \times$ (*increased* length + *increased* breadth) $\times$ uniform width of the path.

Ex. 1. A rectangular garden 2000 ft. long and 884 ft. broad, has a path of a uniform breadth of 16 ft. lying all round it. Find the cost of paving it at 6*a.* per sq. yd.



By the First Method.

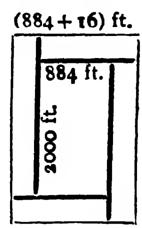
$2000 + 2 \times 16 = 2032$;

$884 + 2 \times 16 = 916$.

∴ the area of the path

$= (2032 \times 916)$ sq. ft. $- (2000 \times 884)$ sq. ft.

$= 93312$ sq. ft. $= 10368$ sq. yds.



By the Second Method.

$2000 + 16 = 2016$; $884 + 16 = 900$.

∴ the area of the path

$= 2(2016 + 900) \times 16$ sq. ft.

$= 93312$ sq. ft. $= 10368$ sq. yds.

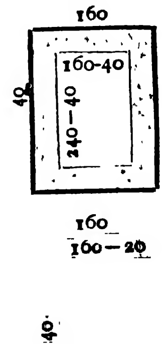
∴ reqd. cost $= 10368 \times$ *Re.* $\frac{3}{8} =$ *Rs.* 3888.

Note. The second method is usually the simpler of the two.

(*b*) When the path, moat, &c., go round the field **inside** it, each dimension may be (i) **diminished** by twice the **uniform** breadth of the path, &c., and the area found by subtracting the new from the old area: or (ii) **decreased** by the uniform breadth of the path, &c., and the following RULE then adopted:

Area $= 2$ (*diminished* length $+$ *diminished* breadth) $\times$ uniform breadth of the path.

Ex. 2. A rectangular field 240 yds. long and 160 yds. wide is enclosed ; there is a belt of plantation 20 yds wide all round between the outside fencing and tank in the centre. Find the area of the plantation.



By the First Method.

$240 - 2 \times 20 = 200$;

$160 - 2 \times 20 = 120$.

$\therefore$ area of the plantation in sq. yds.

$= (240 \times 160) - (200 \times 120)$

$= 14400$.

By the Second Method.

$240 - 20 = 220$; $160 - 20 = 140$.

$\therefore$ area of the plantation

$= 2(220 + 140) \times 20$ sq. yds.

$=$ 14400 sq. yds.

Ex. 3. A room 16 ft. long, 12 ft. broad, and 10 ft. high inside, is enclosed by walls 2 ft. thick ; find the area of the foundation : and also the surface of the outer walls which are 11 ft. high.

The thickness of the walls is equal to the uniform breadth.

$\therefore$ area of the foundation $= 2(18 + 14) \times 2$ sq. ft. $=$ 128 sq ft.

To find the area of the outside of the walls, we must first find the length and breadth of the outside of the walls.

Outer length $=$ length of the room $+ 2 \times$ thickness of the wall ; and outer breadth $=$ breadth of the room $+ 2 \times$ thickness of the wall.

$\therefore$ outer length $= (16 + 4)$ or 20 ft., and outer breadth $= (12 + 4)$ or 16 ft. ;

$\therefore$ area of the walls $= 2 \times (20 + 16) \times 11$ sq. ft. $=$ 792 sq. ft.

Ex. 4. In a rectangular court-yard, 120 yds. long and 80 yds broad, there are two paths crossing one another, each parallel to one side of the rectangle and 8 yds. broad. Find the cost of paving the court-yard with stones at 12*a.* per sq. yd. and gravelling the paths at 6*a.* per sq. yd.

Area of court-yard (including the paths) $= 120 \times 80$ or 9600 sq. yds.

" " (less the paths) $= 112 \times 72$ or 8064 " "

$\therefore$ area of paths $= (9600 - 8064)$ sq. yds $= 1536$ sq. yds. ;

$\therefore$ cost of paving the court-yard $= 8064 \times$ *Re.* $\frac{3}{4}$ $=$ *Rs.* 6048 ;

and " gravelling the paths $= 1536 \times$ *Re.* $\frac{3}{8}$ $=$ *Rs.* 576 ;

$\therefore$ the total cost reqd. $=$ *Rs.* 6624. *Ans.*

Ex. 5. Two rectangular roads each a furlong long cross each other. The one being 12 yds. wide and the other 10 yds., find the area covered by the two roads.

The area of the two roads.

$=220\times 12$ sq. yds. $+220\times 10$ sq. yds. -12×10 sq. yds.

$=2640$ sq. yds. $+2200$ sq. yds. -120 sq. yds.

$=4720$ sq. yds.

Examples 117.

1. A rectangular plot of ground 242 yds. long and 88 yds. wide is surrounded by a walk of a uniform breadth of 9 ft. Find the area of the walk.

2. What would be the cost of paving a path of a uniform breadth of 15 ft. running all round a rectangular garden 760 yds. long and 440 yds. broad, with pebbles at 2*a*. 9*p*. per sq. yd.?

3. How many paving stones each of which is 15 in. by 10 in., will be required to pave a foot-path of a uniform breadth of 18 ft., extending round a park 200 yds. long by 150 yds. broad.

4. A moat is to be dug out all round a fort 4500 yds. long and 1 mi. broad. How many acres of land will have to be taken for the purpose, supposing that the moat is to be 45 ft. wide everywhere?

5. A house 88 yds. long stands on an area of one acre, and is surrounded by a verandah 9 ft. wide extending all round. How many marble slabs 18 in. by 12 in. each, will be required to pave the verandah?

6. In a ship's hold which is 112 ft. long and 46 ft. broad, there is a gangway 4 ft. wide. Find the area of the gangway.

7. A hall 64 ft. long by 24 ft. broad, has a cornice 10 in. wide. Find the cost of gilding it at R2. 8*a*. per square foot.

8. In carpeting a room 36 ft. long and 16 ft. broad, a clear space of 3 ft. is left all round for matting. If the cost of the carpet be R1. 4*a*. per sq. ft. and that of the matting 5*a*. 6*p*. per sq. yd., find the total cost.

***9.** A rectangular piece of land 220 yds. long and 180 yds. broad is enclosed by walls. In the centre is a building which is surrounded by a granite path of a uniform breadth of 15 ft. Between this path and the outside walls there is a grassy walk 18 ft. wide. Find the area of the walk and the cost of the granite path at 10*s*. 8*d*. per square yard.

***10.** Find the cost of whitewashing the ceiling and the inside and outside walls of a room 24 ft. long, 18 ft. broad and 15 ft high, the walls being $1\frac{1}{2}$ ft. thick and 2 ft. higher at the outside, at 1*a*. 6*p*. per sq. yd.

11. A room 20 ft. 6 in. long and 12 ft. 6 in. wide was carpeted with a certain material. If the room had been 2 ft. longer and 1 ft. 6 in. wider, the additional cost would have been £2. 7*s*. How much did it cost to carpet the room?

***12.** In a rectangular building there are four equal rooms built side by side; each room is 22 ft. long by 13 ft. broad; the walls are 2 ft. thick, 10 ft. high at the inside and 12 ft. at the outside. Find the cost of whitewashing the ceiling and the inside and the outside walls, at 2*p*. a sq. ft.

13. In a rectangular village a mile long and half a mile broad, there are two main roads crossing one another. If they are 50 ft. and 40 ft. wide respectively, find the cost of paving them at 9*d.* per sq. yd.

14. The outer dimensions of a wooden box are 3 ft., 1 ft. 6 in. and 15 in. respectively, and the box is made of wood 2 in. thick. Find the cost of lining the inside with velvet at ₨4. 8*a.* per sq. yd.

CHAP. XXVIII. CUBIC MEASURE.

278. **A rectangular solid** or **parallelopiped** is a solid figure bounded by six rectangular surfaces, the opposite pairs being equal and parallel to each other.

279. A rectangular solid has three **dimensions**, *viz.*, length, breadth, and height, thickness or depth. A rectangular solid having all its three dimensions equal, is called a **cube.**

280. A rectangular solid measuring a yard each way is called a **cubic yard**; a rectangular solid measuring a foot each way is called a **cubic foot**; and so on.

281. Let the rectangular solid $oABC$ be 4 ft. long, 3 ft. broad, and 2 ft. thick.

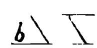


Fig. 3.

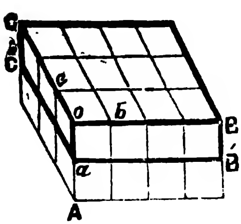


Fig. 1.

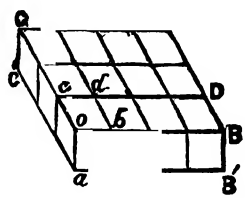


Fig. 2.

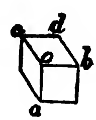


Fig. 4.

Cut the solid (*Fig. 1.*) into 2 equal slices along aB', aC'; each of these will be a rectangular solid of which the length is 4 ft., breadth 3 ft. and thickness 1 ft. (see *Fig. 2*). Again, cut each of these 2 slices into 3 smaller equal slices; each of these will be a rectangular solid, 4 ft. long, 1 ft. broad and 1 ft. thick (see *Fig. 3*). Lastly, cut each of the (3 × 2 or 6) smaller slices thus obtained into 4 equal parts (see *Fig. 4*). Each of

these parts is plainly a cubic foot. Now the whole figure has in the above way been divided into $4 \times 3 \times 2$ or 24 such slices. Therefore the solid contains $\underline{4 \times 3 \times 2}$ cubic feet.

Hence the **volume** or **cubic content** of a rectangular solid is obtained by multiplying its three dimensions ; or

Volume = Length × Breadth × Height.

$$\therefore \text{ length} = \frac{\text{volume}}{\text{breadth} \times \text{height}}; \quad \text{breadth} = \frac{\text{volume}}{\text{length} \times \text{height}};$$

and
$$\text{height} = \frac{\text{volume}}{\text{length} \times \text{breadth}}.$$

Ex. 1. Find the volume of a rectangular piece of timber, 30 ft. long, 3 ft. wide and 2 ft. thick.

The volume of the timber $= 30 \times 3 \times 2$ cub. ft. $= 180$ cub. ft. *Ans.*

Ex. 2. Find the breadth of a piece of timber containing $10\frac{1}{2}$ cub. ft. the length being 16 ft. and the height 9 in.

The breadth $= \dfrac{10\frac{1}{2} \text{ cub. ft.}}{16 \times \frac{3}{4} \text{ sq.ft.}} = \frac{7}{8}$ ft. $= 10\frac{1}{2}$ in. *Ans.*

Ex. 3. How many stones measuring 15 in. each way, will there be in a pile 25 ft. long, 15 ft. wide, and 10 ft. high ?

The volume of each stone $= (\frac{15}{12} \times \frac{15}{12} \times \frac{15}{12})$ cub. ft. $= \frac{125}{64}$ cub. ft.
The volume of the pile $= 25 \times 15 \times 10$ cub. ft. $= 3750$ cub. ft.
$\therefore$ no. of stones $= 3750 \div \frac{125}{64} = 3750 \times \frac{64}{125} = 1920$. *Ans.*

Ex. 4. A rectangular cistern is 40 ft. long by 25 ft. wide ; how much water must be drawn off to make the surface sink 4 ft. ?

As the surface sinks 4 ft. the depth of the quantity of water drawn off is also 4 ft.

$\therefore$ quantity of water drawn off $= 40 \times 25 \times 4$ cub. ft. $= \underline{4000 \text{ cub. ft.}}$

Ex. 5. A rectangular reservoir 80 ft. long, 50 ft. wide and 20 ft. deep is full of water ; 120 water-carts, each measuring 6 ft. long, 4 ft. wide and 15 in. high are employed to take away water from it. How much must the surface sink down, when each of the water-carts has been used ten times ?

Cubic content of each cart $= 6 \times 4 \times \frac{15}{12}$ cub. ft. $= 30$ cub. ft. ;
$\therefore$ quantity of water drawn off $= 30$ cub. ft. $\times 120 \times 10 = 36000$ cub. ft.
$\therefore$ the water will sink down 36000 cub. ft. $\div (80 \times 50)$ sq. ft. $= \underline{9 \text{ ft.}}$

Ex. 6. A rectangular piece of ground, 298 ft. long and 148 ft. broad, is enclosed by a wall 2 ft. thick and 10 ft. high. How many bricks, each 9 in. long, 4 in. wide and 3 in. thick, will be required to build the wall ?

Area of the foundation $= 2(300 + 150) \times 2$ sq. ft. $= 1800$ sq. ft. ;
$\therefore$ cubic content of the wall $= (1800 \times 10)$ cub. ft. $= 18000$ cub. ft.
Also " " of a brick $= \frac{3}{4} \times \frac{1}{3} \times \frac{1}{4}$ cub. ft. $= \frac{1}{16}$ cub. ft. ;
$\therefore$ no. of bricks $= 18000 \div \frac{1}{16} = 288000$. *Ans.*

Ex. 7. A box is to be made with wood one inch thick. Find the quantity of wood that would be required for the purpose, the external dimensions of the box being 32 in., 18 in. and 10 in., respectively.

The wood being an inch thick, the inner length, breadth and height are 30 in., 16 in. and 8 in., respectively.

The external volume $= 32 \times 18 \times 10$ cub. in. $= 5760$ cub. in.

" internal " $= 30 \times 16 \times 8$ cub. in. $= 3840$ "

$\therefore$ the quantity of wood $= (5760 - 3840)$ cub. in. $=$ 1920 cub. in.

Ex. 8. A hollow rectangular iron pillar 15 ft. long is made of iron half an inch thick. The hollow section is 11 in. by 8 in.; find the weight of the pillar, 1 cub. ft. of iron weighing 5 mds. 28 sr.; also find its cost at *Rs.* 5 a maund.

The external breadth $= (11 + 2 \times \frac{1}{2})$ in. $= 1$ ft.; and the external thickness $= (8 + 2 \times \frac{1}{2})$ in. $= \frac{3}{4}$ ft.

$\therefore$ the external volume $= (15 \times 1 \times \frac{3}{4})$ cub ft. $= \frac{45}{4}$ cub. ft., and the internal volume $= (15 \times \frac{11}{12} \times \frac{8}{12})$ cub. ft. $= \frac{55}{6}$ cub. ft.

$\therefore$ quantity of iron $= (\frac{45}{4} - \frac{55}{6})$ cub. ft. $= \frac{25}{12}$ cub. ft.

$\therefore$ weight of the pillar $= 5$ mds. 28 sr. $\times \frac{25}{12} = 11$ mds. 35 sr. }
and the required cost $=$ *Rs.* $5 \times 11\frac{7}{8}$ $=$ *Rs.* 59. 6*a.* } *Ans.*

Examples 118.

1. Find the cubic content of the rectangular solids having the following dimensions:

(1)	Length 15 ft.	breadth 12 ft. 8 in.	and height 8 ft.
(2)	12 ft. 6 in.	" 8 ft. 8 in.	" 3 ft.
(3)	7 ft. 6 in.	" 5 ft. 4 in.	" 1 ft. 6 in.
(4)	10 ft. 4 in.	" 6 ft. 6 in.	" 2 ft. 6 in.
(5)	27 yds. 2 ft.	" 22 yds. 1 ft.	" 5 yds. 1 ft.

2. Find the volume of a cube whose side is 7 ft. 6 in.

3. Find the height of a room 25 ft. long and 16 ft. broad, and containing 4800 cub. ft.

4. A block of stone 7 ft. 6 in. long and 2 ft. 8 in. wide, contains $26\frac{2}{3}$ cub. ft.; find the thickness of the block.

5. Find the price of a timber 30 ft. long, 3 ft. 4 in. wide and 2 ft. thick, at Re. 1. 5*a.* 4*p.* per cub. ft.

6. Find the weight of water in a cistern 32 ft. long, 25 ft. broad and 5 ft. deep, a cubic foot of water weighing 1000 oz. Avoir.

7. How many bricks will there be in a pile 22 ft. long, 16 ft. broad and 15 ft. high, a brick containing 108 cub. in.?

8. How many times can a bucket holding $2\frac{1}{4}$ cub. ft. of water, be filled from a cistern 7 ft. 6 in. long, 5 ft. broad and 4 ft. deep?

9. How many water-carts each 7 ft. long, 5 ft. 6 in. wide, and 18 in. thick, can be filled from a tank 28 ft. long, 22 ft. broad and $4\frac{1}{2}$ ft. deep?

10. The surface of a block of marble 5 in. thick, is 12 sq. ft.; find its weight, if a cub. ft. weigh 1 md. 20 sr.; also find its price at R15. 8*a.* per maund.

11. How many bricks each 9 in. by $4\frac{1}{2}$ in. and 3 in. thick, will be needed for a wall 63 ft. long, 36 ft. high and 3 ft. thick?

12. In what time can a tank 36 ft. long, 18 ft. wide, and 7 ft. 6 in. deep, be emptied by a pipe which discharges 12 cub. ft. of water per minute?

13. In a room 26 ft. 8 in. long and 11 ft 3 in. high, there are 5400 cub. ft. of space: how broad is the room?

14. A rectangular tank 5 ft. 6 in. deep, contains 616 cub. ft. of water; find the area of its surface.

15. Find the cost of painting the walls of a room, which is 20 ft. long and 15 ft. broad and contains 3300 cub. ft. of air, at 2*s.* 6*d.* per sq. ft.

16. Find the cost of carpeting a room which is $12\frac{1}{2}$ ft high and contains 2975 cub ft. of air at R1. 2*a.* per sq. ft

17. A barrack 50 ft. long and 20 ft. broad, contains 30 beds; how high must it be that each soldier may have 500 cub. ft of air.

***18.** A tank 64 ft 6 in. long, and 37 ft 6 in. wide and 16 ft. 6 in. deep, is emptied in 6 hrs. by a pipe whose aperture is 33 sq. in, at what rate per second does the water flow through the pipe?

***19.** A tank is 150 yds. long and 75 yds. broad; with what velocity per second must water flow into it through a drain 1 ft. broad by 9 in. deep, that the level may be raised 3 ft. in 36 hours?

20. A cistern 200 yds. long, 80 yds. broad and 15 yds. deep, has water in it 6 yds. deep; find how many oblong pieces of stones each 4 ft. long, 1 ft. broad and 9 in. thick, must be thrown into it that the water may rise just to the brim.

21. A tank is 36 ft. long by 15 ft. broad, and 12 ft. deep. How deep must another tank of equal capacity be, whose length and breadth are 27 ft. and 16 ft. respectively?

22. A cubic inch of gold is beaten into a leaf 16 in square; find how many such leaves will go to the inch.

23. From a tank 40 ft. long and 32 ft. broad, 2400 mds. of water are drawn of; by how much will the surface sink down, a cubic foot of water weighing 30 seers?

24. A gentleman wishes to raise a rectangular piece of ground (385 yds. long by 231 yds. broad) by 2 ft. 5 in., and digs out a moat all round it (11 yds. wide everywhere) for the soil; how deep must the moat be, supposing its depth to be uniform?

***25.** Two new roads both 66 yds. broad are to be constructed crossing one another; they are 2 mi. and $1\frac{1}{2}$ mi. long respectively, and are to be 18 ft high; to what depth must a rectangular piece of land 554 yds. long by 396 yds. wide be dug, to supply the earth for the purpose?

*26. A reservoir 55 yds. long, 25 yds. broad and 6 yds. deep, is to be filled with water by means of a pipe whose sectional area is 48 sq. in.; how fast does the water flow in the pipe per minute, supposing it to be filled in 2 hrs. 45 min.?

*27. An iron-chest 5 ft. long, 3 ft. broad and 2 ft. 6 in. high at the inside, is made of iron 1 in. thick. Find the weight of iron used (a cub. ft. of iron weighs 6 mds.); and also the cost at R15. 12*a.* per maund.

28. How many iron bars, each 6 ft. 8 in. long, 3 in. wide and 2 in. thick, can be made of 480 mds. of iron, a cub. ft. weighing 6 mds.?

*29. A cub. foot of gold weighs 19 times as much as water of the same bulk. Find the weight of the gold which a box made of wood an inch thick and with external dimensions 4 ft. 2 in., 3 ft. 2 in. and 2 ft. 2 in. respectively, can contain a cub. ft. of water weighing 1000 oz.

30. A rectangular garden 2 fur. long and $1\frac{1}{2}$ fur. broad is surrounded by a wall 24 in. thick and 12 ft. high; find the cost of constructing it at R20. 13*a.* 4*p.* per 100 cub. ft.

*31. In a barrack there are 60 beds placed in a single row with their lengths parallel to the breadth of the room, each bed being 7 ft. long and 4 ft. broad. There is a clear passage of 4 ft. all round between the beds and the walls, and a space of 3 ft. is left between every two beds. Find the height of the barrack, each soldier being allotted 10625 cub. ft.

*32. A hollow rectangular iron pillar 15 ft. high, is made out of a sheet of iron $1\frac{1}{2}$ in. thick and the hollow part is 10 in square at the end. Find the weight of the pillar, a cub. ft. weighing 6 mds. 5 sr.

33. A cistern is 36 ft. long, 20 ft. wide, and 16 ft. deep; what will be the depth of another cistern which is 80 ft. long by 72 ft. wide and contains 4 times as much water as the first?

*34. A tank is 66 yds. 2 ft. long, 40 yds. wide, and 16 ft. deep; how many times can 400 water-carts each carrying 24 mds. of water at a time, be filled before the tank falls empty, a cub. ft. of water weighing 30 sr.?

*35. How many bales each 6 ft. long, 3 ft. 3 in. broad, and 2 ft. 6 in. thick, can be stowed in the hold of a ship 104 ft. long, 60 ft. broad and 15 ft. high, supposing there to be a gangway 4 ft. wide?

36. A street is 3 miles long and 72 ft. wide; there are two foot-paths one on each side, covering an area $\frac{2}{9}$ of that of the street. If the foot-paths be raised a foot, find the cost at 1*a.* per cub. ft.

CHAP. XXIX. DUODECIMALS.

282. There is a method called that of **Cross Multiplication** or **Duodecimals**, for finding the areas of surfaces and volumes of solids, which for the sake of its simplicity is often used by painters, builders, and other practical men. It obviates the labour of reducing the given dimensions to the same denomination and multiplying them as fractions or otherwise, as has been done in the previous Chapter.

1 foot = 12 (linear) Primes
1 prime (1) = 12 Seconds

1 sq. ft. = 12 (superficial) primes
1 sup. prime (1′) = 12 „ seconds
1 sup. second (1″) = 12 „ thirds ;
and so on.

second (1″) = 12 Thirds
third (1‴) = 12 Fourths

cub. ft. = 12 (solid) primes
solid prime (1′) = 12 „ seconds
„ second (1″) = 12 „ thirds ;
and so on.

From the fact that each of the above units is **twelve** (*duodecimals*) times the next lower one, the method is called that of **Duodecimals.**

283. The following Examples will illustrate how quantities expressed in feet and inches are converted into Duodecimals, and *vice-versa.* In such reductions it must be remembered that a linear prime = a linear inch; a superficial second = 1 sq. in.; and a solid third = 1 cub. in.

Ex. 1. Express 15 ft. $7\frac{1}{3}$ in. in *duodecimals*, and 7 ft. 8′ 3″ in *ft.* and *in.*

15 ft. $7\frac{1}{3}$ in. = 15 ft. $7\frac{4}{12}$ in. = 15 ft. 7′ 4″.

7 ft. 8′ 3″ = 7 ft. $8\frac{3}{12}$ in. = 7 ft $8\frac{1}{4}$ in.

Ex. 2. Convert 216 sq. ft. 111 sq. in. to *duodecimals*, and 25 sq. ft. 11′ 9″ 8‴ to *sq. ft.* and *sq. in.*

216 sq. ft. 111 sq. in. = 216 sq. ft. 111″ = 215 sq. ft. 9′ 3″.

25 sq. ft. 11′ 9″ 8‴ = 25 sq. ft. 141″ 8‴ = 25 sq. ft. 141 $\frac{8}{12}$″
= 25 sq. ft. $141\frac{2}{3}$ sq. in.

Ex. 3. Express 26 cub ft. 857 cub. in. in *Duodecimals*, and 61 cub. ft. 7′ 8″ 9‴ in *cub. ft.* and *cub. in.*

26 cub. ft. 857 cub. in. = 26 cub. ft. 857‴ = 26 cub. ft. 71″ 5‴
= 26 cub. ft. 5′ 11″ 5‴.

61 cub. ft. 7′ 8″ 9‴ = 61 cub. ft. 1113‴ = 61 cub. ft. 1113 cub. in.

Examples 119.

1. Express as Duodecimals :

(1) 15 ft. 7 in.; 24 ft. $5\frac{3}{4}$ in.; 7 yds. 2 ft. $11\frac{3}{4}$ in.

(2) 15 sq. ft. 129 sq. in.; 45 sq. ft. $133\frac{1}{2}$ sq. in.; 26 sq. ft. $47\frac{1}{4}$ sq. in.

(3) 45 cub. ft. 461 cub. in.; 26 cub. ft. 947 cub. in.; 1525 cub. in.

2. Express in feet and inches :

(1) 29 ft. 8′ 9″ : 231 ft. 7′ 8″ ; 31 ft. 4′ 11″ 3‴.

(2) 61 sq. ft. 6′ 7″ 8‴ ; 46 sq. ft. 8′ 11″ 9‴ ; 42 sq. ft. 5′ 7″ 8‴ 10⁗.

(3) 58 cub. ft. 4′ 9″; 4 cub. ft. 4′ 7″ 8‴; 50 cub. ft. 8′ 9″ 10‴ 11⁗.

284. In finding areas and volumes by **Duodecimals**, the following measures will be required.

SQUARE MEASURE			CUBIC MEASURE.		
Feet × primes	= superficial	primes	Super. ft. × primes	= solid	primes
feet × seconds =	„	seconds	„ ft. × seconds	= „	seconds
feet × thirds =	„	thirds	„ ft. × thirds	= „	thirds
and so on.			and so on.		
Primes × primes =	„	seconds	„ primes × primes =	„	seconds
primes × seconds =	„	thirds	„ primes × seconds =	„	thirds
and so on.			and so on.		
Seconds × seconds =	„	fourths	„ seconds × seconds =	„	fourths
seconds × thirds =	„	fifths, &c.	„ seconds × thirds =	„	fifths, &c.

[1 ft. $\times 1' = 1 \times \frac{1}{12}$ sq. ft. $= \frac{1}{12}$ sq ft $= 1'$; 1 ft. $\times 1'' = 1 \times \frac{1}{144}$ sq. ft. $= \frac{1}{144}$ sq. ft. $= 1''$; &c.]

Ex 1. Find the area of a room 20 ft. 5′ 4″ long and 10 ft. 8′ 10″ broad.

20 ft.	5′	4″		
10		10		
204 sq. ft. 5′			10 ft. × 4″	= 40″ = 3′ 4″
13	7	8‴	10 ft. × 5′ + 3′	= 53′ = 4 ft 5′ &c.
1	5	5	8′ × 4″	= 32‴ = 2″ 8‴ &c.
			10″ × 4″	= 40⁗ = 3‴ 4⁗
			&c.	&c.

Ex. 2. Find the volume of a rectangular solid 7 ft. 6′ long, 5 ft. 7′ broad, and 3 ft. 10′ deep.

7 ft.	6′	
5	7	
37 sq. ft.	6	
4	4	6″
41 sq. ft.	10	6
3	10	
125 cub. ft. 7	6	
34	10	9
160 cub. ft. 6′	3″	

Thus the required volume
= 160 cub. ft. 6′ 3″
= 160 cub. ft. 900 cub. in.

N. B.—We first multiply all the terms of the multiplicand beginning with the number of feet in the multiplier ; then in a similar way by the primes ; then by the seconds ; and so on.

Examples 120.

1. Find by Duodecimals the following areas :

(1) 7 ft. 4′ by 6 ft. 8′ (2) 11 ft. 7′ by 9 ft. 8′

(3) 15 ft. 6′ by 11 ft. 9′ (4) 12 ft. 4′ 8″ by 7 ft. 4′

(5) 47 ft. 2′ 3″ by 4 ft. 5′ (6) 18 ft. 4′ 9″ by 8 ft. 2′ 1″

2. Find by Cross Multiplication the volumes of the following solids :

(1) 7 ft. 6′ by 5 ft. 5′ by 11 ft. 2′. (2) 22 ft. 8′ by 13 ft. 7′ by 10 ft. 4′.

(3) 16 ft. 5′ by 10 ft. 9′ by 6 ft. 4′. (4) 12 ft. 6′ 4″ by 8 ft. 6′ by 5 ft. 9′.

SQUARE AND CUBIC MEASURES OF BENGAL.

285. In Bengal, the areas of rectangular fields, &c., are found by a similar method which is called **Subhankar's Method.**

The following is Subhankar's RULE for finding areas :

bigha × bigha = bigha	bigha × chhatak = chhatak
bigha × katha = katha	katha × chhatak = ganda
katha × katha = dhul	chhatak × chhatak = kag.

[20 dhuls = 1 katha ; 16 gandas = 1 dhul ; 16 kags = 1 ganda.]

Ex. Find the area of a rectangular field 5 bi. 8 kat. long and 4 bi. 7 kat. broad.

5 bi	8 kat.	
4	7	
21	12	
1	17	16 dhul.
23 bi.	9 kat.	16 dhul.

4 bi. × 8 kat. = 32 kat. = 1 bi. 12 kat. ; *carry* 1 bi., 5 bi. × 4 bi. = 20 bi, 21 bi. ; 8 kat. × 7 kat. = 56 dhul = 2 kat. 16 dhul ; *carry* 2 kat., 5 bi. × 7 kat. = 35 kat., 37 kat. ; 1 bi. 17 kat.
16 dhul = 16 × 16 or 256 ga = 12 ch. 16 ga.

∴ area = 23 bi. 9 kat. 16 dhul = 23 bi. 9 kat. 12 ch. 16 ga.

Examples 121.

Find, by Subhankar's method, the area of :

1. 6 bi. by 5 bi.
2. 25 bi. 15 kat. by 11 bi.
3. 17 bi. 11 kat. by 15 bi. 10 kat.
4. 14 bi. 11 kat. by 9 bi. 15 kat.
5. 215 bi. 12 kat. by 145 bi. 10 kat.
6. 52 yds. by 40 yds.
7. 256 yds. by 120 yds.
8. 256 yds. by 220 yds.

286. The volume of a rectangular solid is found in the same way as in ART. 281.

Ex. Find the volume of a wall 60 cubits long, 10 cubits high, and 2 cubits thick.

Volume of the wall = 60 × 10 × 2 cub. cubits = 1200 cub. cubits.

Examples 122.

Find the volume from the following dimensions :

1. 25, 16, 8 cubits.
2. 250, 17, $2\frac{1}{2}$ cubits.
3. 260, 50, $7\frac{1}{2}$ cubits.
4 37 yds., 20 yds., 8 yds.
5. 24 yds., 15 yds., 9 yds.
6. 44 yds., 28 yds., 3 ft.

CHAP. XXX. MISCELLANEOUS PROPOSITIONS.

287. **Races** and **Games of skill.** If *B* can run only 80 yds. while *A* runs 100, *A* can **give** *B* (100 − 80) or 20 yds. in 100 ; so, if in a game like billiards, *B* can make only 90

points while A makes 100, A can **give** (100—90) or 10 points to B out of 100. B is said to **get** or **take** 20 yds. or 10 points in these two cases, respectively.

Ex. 1. A can beat B by 30 yds. in a race of 300 yds., and B can beat C by 30 yds. in the same race: by how much can A beat C in a race of 200 yards?

A can run 300 yds., when B can run (300−30) or 270.;

∴ „ „ $\frac{300}{270}$ yds. „ „ „ „ 1 yd.;

∴ „ „ $\frac{300}{270}\times 300$ or $\frac{1000}{3}$ yds when B can run 300 yds.

But when B can run 300 yds., C can run 270 yds.;

∴ A can run $\frac{1000}{3}$ yds, when „ „ „ 270 yds.;

or „ „ 1000 yds., „ „ „ „ 810 yds.;

or „ „ 200 yds., „ „ „ „ (810÷5) or 162 yds.

∴ A can beat C by (200−162) or 38 yds. in 200 yds.

otherwise thus:

While A can run 300 yds., B can run (300−30) or 270 yds.;

∴ while A runs 200 yds., the latter runs $200\times\frac{270}{300}$ or 180 yds.

Again, while B can run 180 yds., C can run $180\times\frac{270}{300}$ or 162 yds.;

∴ while A can run 200 yds., C can run 162 yds.;

∴ A can beat C by (200−162) or 38 yds. in 200 yds.

Ex. 2. At a game of skill, B can give A 10 points and C 18 points, out of 50; how many can A give C out of 150?

While A can make (50−10) or 40 points, C can make (50−18) or 32;

∴ A can give C (40−32) = 8 points out of 40;

∴ „ „ 1 „ „ 5;

∴ „ „ 30 „ „ 150. *Ans.*

Examples 123.

1. If A can beat B by 40 yds. in a race of 300 yds. and B can beat C by 50 yds. in a race of 500 yds.; by how much can A beat C in a race of 50 yds.?

2. In a mile race P gives Q 90 yds.' start and beats him by 20 yds. If P can run the mile in 5 minutes, how far can Q run in an hour?

3. In a mile race X can give Y 16 yds.' start and Z 80 yds.' start; what start can Y give Z in a race of 545 yds.?

4. A can run 25 yds. while B can run 20; B can run 8 yds. while C can run 12; if C can run a mile in 5 minutes, in what time can A run the same distance?

5. X can give Y 110 yds and Y can give Z $53\frac{1}{3}$ yds. in a race of 1 mile; how much should X give Z that they may run a dead heat?

6. In a game of skill A can give to B 15 points out of 70 and to C 18 points out of 98. Which is the better player, B or C? How many points can the one give the other in a game of 100 points?

7. In a game of skill P can give Q, and Q can give R, 10 points out of 50. How much should P give R out of 50 to make up an even match?

8. In a game of billiards A can give B 15 points out of 75, and C 20 points out of 80. Of B and C, which is the better player; and how many points can the one give to the other out of 16?

9 In a game of billiards X can give Y 20 points out of 120, and Y can give Z 4 points out of 80. How many can X give Z out of 24?

10. A can give B 220 yds. in a mile, and C can give B 330 yds in a mile; if A and C run over a mile, who should win and by how much?

*11. P and Q run a mile race, and P wins by 80 yds.; when P and R run over the same course, P wins by 20 sec.; Q and R run, and Q wins by 5 sec. In what time can P run the mile?

12 In a race M gained 30 yds. upon N in every 125 yds. and finally won by 72 yds.; find the length of the course.

13. In a game at fives, out of 15 points A can give B 3 points; A can also give C 7 points in the same; how many points can B give to C so as to make up an even match?

*14. M can give N a start of 60 yards in a race of 200 yards; and N can give P 60 yards' start in a race of 240 yards. What must be the length of a race-course on which M beats P by 19 yards?

15. A can give B 400 yds. and C 500 yds. in a mile race: if B can run the mile in 11 minutes, in what times can A and C run the same?

*16. A can give B 220 yds.' and C 330 yds.' start in a mile race: A can also give C a start of 2 min. 15 sec. in a race of $2\frac{1}{2}$ miles. In what time can each run a mile?

288. **Temperature.** The following **Thermometers** are commonly used.

The **Thermometer** is an instrument for measuring the **temperature** of bodies. It has been found that the atmospheric pressure remaining the same, water always boils and freezes at the *same* temperature. The two marks on the stem of the thermometer which correspond to these are respectively called the **boiling** and **freezing points.**

Fahrenheit's Thermometer is in common use in England. The **freezing point** is denoted in it by 32° and the **boiling point** by 212°. The space between these two points is marked off into 180 equal parts, each called a *degree* (1°F).

The **Centigrade** Thermometer is in use on the Continent. The *freezing* and *boiling points* are respectively marked 0° and 100°, the intervening space being divided into 100 equal parts, each called a *degree* (1°C).

Reaumur's Thermometer is used in Russia. The *freezing point* is called 0°R, and the *boiling point* 80°R.

289. The following facts have to be remembered in converting a temperature expressed in the Fahrenheit scale into the Centigrade or the Reaumur scale, and *vice-versa*.

(i) $32^\circ F = 0^\circ C = 0^\circ R$; (ii) $180^\circ F = 100^\circ C = 80^\circ R$.

$\therefore\ 1^\circ F = \frac{5}{9}{}^\circ C = \frac{4}{9}{}^\circ R$; $1^\circ C = \frac{9}{5}{}^\circ F = \frac{4}{5}{}^\circ R$; $1^\circ R = \frac{9}{4}{}^\circ F = \frac{5}{4}{}^\circ C$.

Ex. $124^\circ F = 32^\circ F + 92^\circ F = 0^\circ C + \frac{5}{9} \times 92^\circ C = \frac{460}{9}{}^\circ C = 51\frac{1}{9}{}^\circ C$.

To reduce C to F, multiply the number of C° by $\frac{9}{5}$ and add 32° to the product; and to reduce R to F, multiply the number of R° by $\frac{9}{4}$ and add 32° to the product.

The following formulæ will prove very useful in these reductions. :

$$\frac{F-32}{180} = \frac{C}{100} = \frac{R}{80} ;\ \text{or,}\ \frac{F-32}{9} = \frac{C}{5} = \frac{R}{4}.$$

Examples 124.

1. What degree C corresponds to 80°F ?

What degree F corresponds to 54°C ?

3. What temperature F is represented by 45°C ?

4. What temperature C is represented by 167°F ?

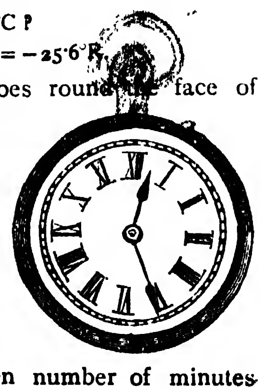
5. What temperature R is represented by 122°F ?

6. What temperature C is the same as 60°R ?

7. What temperature R is the same as 92°C ?

8. Shew that $-40^\circ F = -40^\circ C$ and $-25\cdot6^\circ F = -25\cdot6^\circ R$.

290. **Clocks.** The minute-hand goes round the face of the clock, *i. e.* moves 60 minutes, while the hour-hand moves 5 minutes round. The minute-hand therefore goes 12 minutes for every minute that the hour-hand goes: or, in other words, it gains 11 minutes for every 12 minutes it advances, or 1 minute for every $\frac{12}{11}$ minutes it advances. Hence to find the time in which the minute-hand will be **gaining** a certain number of minutes on the hour-hand, we have to multiply the given number of minutes by $\frac{12}{11}$.



It should be remembered that (1) when the minute-hand is either 15 min. *before* or 15 min. *behind* the hour-hand, the two

hands are *at right angles*; and (2) when the minute-hand is either 30 min. *before* or *behind*, they are *opposite to each other*.

Ex. 1. When will the hour and minute-hands be (1) together, (2) at right angles, and (3) opposite to each other, between 4 and 5 o'clock?

(1) The minute-hand starts at 4 o'clock 20 minutes behind the hour-hand. It has therefore to gain these 20 min., in order that the two hands may be together. But it takes $\frac{12}{11}$ min. to gain 1 min.; therefore the required time is $\frac{12}{11} \times 20$ min. or $21\frac{9}{11}$ min. after 4.

(2) The minute-hand was 20 minutes behind the hour-hand at 4 o'clock. Therefore it has to gain either $(20-15)$ min. or $(20+15)$ min. to be at right angles to the other. In the former case it will be *behind*, and in the latter case, *before* the hour-hand. Hence the required time will be either $\frac{12}{11} \times (20-15)$ min. or $\frac{12}{11} \times (20+15)$ min. after 4; *i.e.*, either $5\frac{5}{11}$ min. or $38\frac{2}{11}$ min. past 4.

(3) Since the minute-hand was only 20 min. behind at starting, it cannot be 30 min. *behind* at any time between 4 and 5 o'clock. Hence the two hands can be opposite to each other, only when the minute-hand is 30 min. *before* the other, *i.e.*, when it has gained $(30+20)$ min. or 50 min. over the hour-hand. Therefore the required time is $\frac{12}{11} \times 50$ min. or $54\frac{6}{11}$ min. past 4.

Ex. 2. When will the hands of a clock be at right angles to each other, (1) between 1 and 2 o'clock, (2) between 10 and 11 o'clock?

(1) At starting the minute-hand is 5 minutes behind the hour-hand; the two hands will therefore be at right angles only when the minute-hand has gained $(5+15)$ min. or $(5+45)$ min. on the other. In the first case it will be $\frac{12}{11} \times 20$ or $21\frac{9}{11}$ min. after 1; in the other, $\frac{12}{11} \times 50$ or $54\frac{6}{11}$ min., after 1.

(2) At starting the minute-hand was 10 min. before the other: so the two hands will be at right angles when the minute-hand has gained (1) 5 min. more, or (2) 35 min. more. In (1) it will be $5 \times \frac{12}{11}$ or $5\frac{5}{11}$ min., and in (2), $35 \times \frac{12}{11}$ or $38\frac{2}{11}$ min., after 10.

Examples 125.

1. When will the hour and minute hands of a clock be (i) at right angles to each other, (ii) together, and (iii) directly opposite to each other between the hours of ?

(1) 12 and 1. (2) 1 and 2. (3) 2 and 3. (4) 3 and 4.
(5) 4 " 5. (6) 5 " 6. (7) 6 " 7. (8) 7 " 8.
(9) 8 " 9. (10) 9 " 10. (11) 10 " 11. (12) 11 " 12.

2. At 12 o'clock the hour and minute hands of a clock are coincident. How often will they coincide again during the next 6 hours?

3. At 3 o'clock the hour and minute hands of a clock are at right angles. How often will they be at right angles to each other during the next 3 hours?

4. At 6 o'clock the two hands are opposite to each other. How often will they again be so during the next 5 hours?

291. A clock is said to be 15 min. **too slow**, when it indicates 15 minutes *behind* the true time; and 15 min. **too fast** when it indicates 15 min. *before* the true time.

Thus, if a clock indicates 9-50 when the correct time is 10, it is said to be 10 min. *too slow*; if, on the other hand, it indicates 10-5 at the same hour, it is called 5 min. *too fast.*

Ex. 1. A clock was 15 minutes too slow at noon 30 days before and to-day at the same hour it is 15 minutes too fast: when will it again shew correct time?

The clock has gained (15 + 15) min. or 30 min. in 30 days, or gains at the rate of a minute a day. Again when the time is really 12 o'clock, it indicates 12 hrs. 15 min. Therefore it will shew correct time when it will have gained (12 hrs − 15 min.) or 705 min. Now it gains 1 min. in 1 day; therefore it will gain these 705 min. in 705 days. *Ans.*

Ex. 2. Two clocks are set right at noon on Sunday: one gains 3 minutes and the other loses 2 minutes in 24 hrs.: (i) when will there be a difference of one hour between the times indicated by them; (ii) what time will the first indicate when the second indicates noon, a week afterwards; (iii) what time will the second indicate, when the first indicates 6 P. M. on the following Thursday; (iv) what will be the true times, when the first clock indicates 3 P. M. on the following Wednesday and the second indicates 10 A. M. on the following Friday?

(i) The first gaining 3′ and second losing 2′ per day, they will differ by 5 in 1 day. Therefore they will differ by 1 hour or 60 min. in 12 days. Therefore the required time is Saturday noon, 12 days later *Ans.*

(ii) When the second indicates 23 hrs. 58 min. or $\frac{719}{30}$ hrs, the first indicates 24 hrs. 3 min. or 5′ *more.*

∴ When the second indicates 1 hr., the first indicates $5 \times \frac{30}{719}$ min. more;
∴ „ „ 7 × 24 hrs. „ „ $\frac{150}{719} \times 7 \times 24$ min. „
or $35\frac{35}{719}$ „ „

∴ the first will indicate $12\text{-}35\frac{35}{719}$ P. M., when the second indicates noon a week afterwards. *Ans.*

(iii) From Sunday noon to Thursday 6 P. M. there are 102 hours.

When the first indicates $24\frac{1}{20}$ hrs., the second indicates $23\frac{29}{30}$ or 5′ *less*;
∴ „ „ 1 hr. „ „ $5 \times \frac{20}{481}$ min. „
∴ „ „ 102 hrs. „ „ $\frac{100}{481} \times 102$ „ „
or $21\frac{99}{481}$ „ „

∴ the second will indicate $21\frac{99}{481}$ min. less or $5\text{-}38\frac{382}{481}$ P. M., when the first indicates 6 P. M., on the following Thursday. *Ans.*

(iv) *Firstly*: From Sunday noon to Tuesday 3 P. M. there are 51 hrs.

When the first indicates $24\frac{1}{20}$ hrs. the true time is 3′ *less*;
∴ „ „ 1 hr. „ „ $3 \times \frac{20}{481}$ min. less;
∴ „ „ 51 hrs. „ „ $\frac{60}{481} \times 51$ „ „
or $6\frac{174}{481}$ „ „

∴ the true time is $6\frac{174}{481}$ min. behind or $2\text{-}53\frac{307}{481}$ P. M. when 3 P. M. is indicated by the first on the following Tuesday. *Ans.*

Secondly : From Sunday noon to Friday 10 A. M. there are 118 hrs. When the second indicates $23\frac{29}{30}$ hrs., the true time is 2 min. *more* :

$\therefore$ " " " 1 hr. " " " $2 \times \frac{30}{719}$ min. more;
$\therefore$ " " " 118 hrs. " " " $\frac{60}{719} \times 118$ " "
or $9\frac{609}{719}$ " "

$\therefore$ the true time is $9\frac{609}{719}$ min. before or 10-$9\frac{609}{719}$ A. M., when 10 A. M. is indicated by the second on the following Friday. *Ans.*

Examples 126.

1. A clock was 12 min. too slow at 4 P. M 20 days ago, and today at the same hour it is 12 min. too fast : when will it shew correct time ?

2. A clock was 10 min. too fast at noon 24 days ago, and today at the same hour it is 10 min. too slow : when will it shew correct time ?

3. A watch is 15 minutes too fast at noon on Tuesday, and gains 2′ 30″ per day ; what will be the time by the watch at half past 4 P. M. on the following Friday ?

4. A watch is 10 min. too slow at noon on Monday, and gains 3 min. a day ; what time will the watch shew at 4-30 P. M. on Sunday next ?

5. Two clocks indicate 12 at the same time : one gains 6 seconds and the other loses 9 seconds, in 12 hours ; in what time will one have gained and hour on the other, and what time will then be indicated by each. ?

6. A clock being regulated at 4 P. M. on Monday indicated 35 min. past 12 at noon on Sunday next. At what rate per hour did it gain ?

7. A clock which uniformly gains 15 seconds per hour, is set right at 8 P. M. on March 16 ; when will it again indicate true time ?

8. A clock is set right at 10 P. M. on Wednesday : at 10 A. M. on the following Saturday it is 5 min. too fast : supposing it gains uniformly, what will be the correct time when the clock strikes 10 P. M. on Monday next.

9. Two clocks, which gains 3 min. and 2 min. respectively in 12 hours, are set right on Monday at noon : when will they again strike together ?

10. Of two clocks, one gains 1′ 30″ and the other gains 2′ 30″ a day ; the first is set right at 4 P. M. on Monday, and the second at 10 A. M. on the following Wednesday : when will they indicate the same time ?

11. Two clocks are set right at noon on the 10th January 1892 ; one gains 3 min. and the other loses 2 min. per day ; when will they both indicate the same time, and what hour will then be indicated by each ?

***12.** Two clocks strike 11 together on Friday morning. On Monday morning, one of them wants 15 min. to 7 when the other strikes 7. How much must the faster clock be put back or the slower clock put forward, that they may strike 4 P. M. together in the afternoon ?

13. A clock, which is 15 min. too fast on Friday at noon, is 2 min. too fast at midnight on the following Sunday : what does it lose per day ?

14. A chronometer which gains 6 min. a day is 20 min. too fast at noon on Monday. What time will it indicate at 3 P. M. on the following Thursday ?

15. One clock gains 6 min. in 8 days and another 2 min. in 3 days: if they be set right at noon on Monday, when will they differ by 2 min.?

***16.** Two clocks strike 11 together to-day morning; one gains 3 min. and the other loses 2 min. in 24 hours: when will one be 1 hr. 30 min. before the other and what times will they then respectively indicate?

17. A clock, which has 16 min. too slow 24 days ago, is 16 min. too fast to-day at the same hour. When did it last shew correct time, and when will it shew correct time again?

***18.** A clock gains $2\frac{8}{11}$ min. an hour: how much must it be put back at noon, that it may indicate true time at 6 P. M., and what will be the position of the hands when it is thus put back?

19. A clock is 12 min. too fast at noon; it loses $2\frac{1}{2}$ minutes an hour: what is the true time when the hands are (i) at right angles, (ii) coincident, and (iii) directly opposite, between 3 and 4 o'clock?

***20.** A clock is 10 min. too slow at noon, and gains 2 min. an hour; what will be the true time when the hands are directly opposite for the third time after noon?

***21.** A clock indicates true time when its hands are at right angles for the first time after noon: if it gains $1\frac{1}{2}$ min. per hour, what time will it shew at noon on the following day?

292. **Time and Distance.** If a man walks at the rate of 3 mi. an hour, he will walk in 4 hours 3×4 or 12 miles; *conversely*, if a man walks at the rate of 3 miles an hour, he takes $(10 \div 3)$ hrs. to walk 10 miles, and if he takes $2\frac{1}{2}$ hrs. to walk 12 mi., he walks at the rate of $(12 \div 2\frac{1}{2})$ mi. an hour.

Ex. 1. One man starts from Calcutta for Raneegunge, which is 120 mi. off, at the rate of $4\frac{1}{3}$ mi. an hour; another man starts at the same time from Raneegunge for Calcutta at the rate of $3\frac{2}{3}$ mi. an hour. When and at what distance from Calcutta will they meet?

Two persons approaching each other from opposite directions, meet when they have walked the whole distance between them.

They together walk $(4\frac{1}{3}+3\frac{2}{3})$ or 8 miles in 1 hour;

$$\left.\begin{array}{l}\therefore \text{ " \quad " \quad " } \quad 120 \text{ miles in } (120 \div 8) \text{ or } 15 \text{ hours;} \\ \text{and the required distance from Calcutta} = (4\frac{1}{3} \times 15) \text{ or } 65 \text{ mi.}\end{array}\right\} \textit{Ans}$$

Ex. 2. *A* starts from *P* for *Q* a distance of 147 miles, walking at the rate of 4 mi. an hour. Three hours and a half later, *B* starts from *Q* for *P* at 3 mi. an hour. When will they meet?

When *B* starts, *A* has already travelled $(3\frac{1}{2} \times 4)$ or 14 miles: therefore the distance between them is then only $(147-14)$ or 133 miles.

Now *A* and *B* together walk $(4+3)$ or 7 miles in 1 hour.

$\therefore$ " " " " 133 miles in $(133 \div 7)$ or 19 hrs;

i.e., they will meet 19 hrs. after *B* started. *Ans.*

Ex. 3. A train leaves Howrah at 5 P. M. and travels at the rate of 25 mi per hour; another train leaves Howrah on a paralell line of rails at 8 P. M. and tarvels at the rate of 35 mi. an hour. When and where will the second train overtake the fiirst?

The first starts 3 hrs. before the other, and is therefore 25×3 or 75 mi. ahead of it. But the second train *approaches* the first at the rate of $(35-25)$ or 10 mi. per hour. Therefore the one will overtake the other in $(75 \div 10)$ or $7\frac{1}{2}$ hrs., *i e*, at 3-30 A. M. Also the place where they come together is $7\frac{1}{2} \times 35$ mi. or $262\frac{1}{2}$ mi. from Howrah. *Ans.*

Ex. 4. Two men start from the same place at the same time in opposite directions, and are $49\frac{1}{2}$ mi. apart in $5\frac{1}{2}$ hrs.: if they had walked in the same directions, they would have been $37\frac{1}{8}$ mi. apart in $12\frac{3}{8}$ hours. Find their speeds per hour.

When they go in opposite directions, the distance between them increases every hour by the *sum* of their rates: and when they walk in the same direction the distance increases by the *difference.*

∴ the sum of their rates per hour $=(49\frac{1}{2} \div 5\frac{1}{2})$ or 9 miles;

and the difference ,, ,, ,, ,, ,, $=(37\frac{1}{8} \div 12\frac{3}{8})$ or 3 ,,

$$\left.\begin{array}{l} \therefore \text{the rate of the faster} = \frac{1}{2}(9+3) \text{ or 6 miles per hour;} \\ \text{and ,, ,, ,, slower} = \frac{1}{2}(9-3) \text{ or 3 ,, ,, .} \end{array}\right\} \textit{Ans.}$$

Ex. 5. *B* starts from Calcutta for Burdwan at 6 A. M, walking $3\frac{1}{3}$ mi. an hour. Two hours later, *A* starts from Burdwan and reaches Calcutta at the same time that *B* reaches Burdwan: the distance between the two places is 60 miles. Find the speed of *A*.

A takes 2 hrs. less than *B* to travel 60 miles.

Now, *B* takes $(60 \div 3\frac{1}{3})$ or 18 hrs.; ∴ *A* takes 16 hrs.

∴ *A*'s speed is $(60 \div 16)$ or $3\frac{3}{4}$ miles per hour. *Ans.*

Ex. 6. *A* to *B* is 16 mi., 2 mi. of which are uphill, and 4 mi downhill. Which is the shorter Journey, *A* to *B* or *B* to *A*; and by how much?

The speed uphill is 3 mi., downhill 5 mi., and on the level 4 mi. an hour.

A to *B* is 2 mi. uphill, 10 mi. level, and 4 mi. downhill;

∴ the whole time taken $=(\frac{2}{3}+\frac{10}{4}+\frac{4}{5})$ hrs. $=\frac{238}{60}$ hrs. $=3$ hrs. 58 min.

B to *A* is 4 mi. uphill, 10 mi. level, and 2 mi. downhill;

∴ the whole time taken $=(\frac{4}{3}+\frac{10}{4}+\frac{2}{5})$ hrs. $=\frac{254}{60}$ hrs $=4$ hrs. 14 min.

∴ the journey from *A* to *B* will be performed 16 min. *sooner*. *Ans.*

Ex. 7. A man can row 36 mi. down or 12 mi. up a stream in 6 hrs.; find the rate of his rowing and of the stream per hour.

When going down the stream, his speed is $(36 \div 6)$ or 6 mi. per hour;

,, ,, against ,, ,, ,, ,, ,, $(12 \div 6)$ or 2 ,, ,, ,, .

Now, in the former case he is helped, and in the latter case opposed, by the stream; therefore the sum of his rate of rowing and the velocity of the stream is 6, and the difference 2.

$$\left.\begin{array}{l} \therefore \text{his rate of rowing} = \frac{1}{2}(6+2) \text{ or 4 miles an hour;} \\ \text{and the ,, ,, the stream} = \frac{1}{2}(6-2) \text{ or 2 ,, ,, ,, .} \end{array}\right\} \textit{Ans.}$$

Ex. 8. A hare is 100 of her own leaps before a greyhound; she takes 4 leaps for every 3 that the other takes, but can only cover as much ground in 3 leaps that the other does in 2; how many leaps will the greyhound have taken before she is caught?

2 leaps of the greyhound $= 3$ leaps of the hare;
$\therefore$ 1 leap " " $= \frac{3}{2}$ " " "
or 3 leaps " " $= \frac{9}{2}$ " " "

But when the greyhound takes 3 leaps, the hare takes 4; $\therefore$ the greyhound gains $(\frac{9}{2}-4)$ or $\frac{1}{2}$ a leap of the hare on 3 of his own leaps;

$\therefore$ he gains 1 leap of the hare on 6 of his own leaps;

$\therefore$ " " 100 " " " on 600 " " " *Ans.*

Ex. 9. Two guns are fired from the same place after an interval of 21 min.; but a person approaching the place hears the reports after an interval of only 20 min 15 sec.: find his rate of progress, sound travelling at the rate of 1125 ft. per second.

In 20 min. 15 sec., or 1215 sec., the man travels a distance which sound would take (21 min. $-$ 20 min. 15 sec) or 45 sec. to travel.

But sound travels, in 45 sec., 45×1125 ft.;

$\therefore$ the man travels 45×1125 ft in 1215 sec., or $\frac{125}{3}$ ft. in 1 sec., or $\frac{125}{3} \times 60 \times 60$ ft. in 1 hour, or $28\frac{9}{22}$ miles in an hour. *Ans.*

Ex. 10. I have to be at a certain place in a certain time; If I walk at the rate of 3 mi. an hour I shall be 30 min. too late; but if I walk at the rate of 4 mi. an hour, I shall be 30 min. too soon; how far have I to walk?

3 mi. an hour $=$ 1 mi. in 20 min.; 4 mi. an hour $=$ 1 mi. in 15 min.

Therefore, 5 min. are *gained* in each mile, or 1 min. in each $\frac{1}{5}$ mile by increasing the speed from 3 to 4 miles an hour.

Now the whole time gained in the required distance $=$ 60 min.;

$\therefore$ the required distance $= 60 \times \frac{1}{5}$ mi. $=$ 12 mi. *Ans.*

Ex. 11. A monkey, in climbing up a greased pole 33 ft high, ascends 7 ft. and slips down 4 ft. in alternate minutes: how long will it take him to get to the top?

In 2 min. the monkey gains $(7-4)$ or 3 ft., *i.e.* in one min. $\frac{3}{2}$ ft., an *even* number of minutes being taken

Now the **first time** that, after his slipping down, there will remain a length *not more than 7 ft*, the monkey will climb it during the next minute and get to the top; hence we should first find what *least* number, not less than $(33-7)$ or 26, is a multiple of $\frac{3}{2}$ or 3 Now this number is evidently 27;

also to ascend the first 27 ft the monkey takes $(27 \div \frac{3}{2}$ or 18 min.);
and " " " remaining 6 ft. " " $\frac{6}{7}$ "

$\therefore$ the whole time taken is $18\frac{6}{7}$ min. *Ans.*

Ex. 12. Two trains 110 yds. and 66 yds. long respectively, run at the rates of 25 and 20 miles an hour on parallel rails: find how long a person in the first train would take to pass the other train, and how long the two trains would take to pass each other; supposing the trains were running, (i) in opposite directions, (ii) in the same direction.

When running in opposite directions, the two trains separate at the rate of $(25+20)$ or 45 mi. an hour; when running in the same direction the one separates from the other at the rate of $(25-20)$ or 5 mi. an hour.

The person in the first train would pass the second train when the distance which he gains over the train passed is equal to the length of the train, which is 66 yds. or $\frac{3}{80}$ mi.

$$\left.\begin{array}{l}\therefore \text{ for opposite directions, the time} = (\tfrac{3}{80} \div 45) \text{ hr.} = 3 \text{ sec.} \\ \text{and ,, the same ,, } \qquad\qquad = (\tfrac{3}{80} \div 5) \text{ hr.} = 27 \text{ ,,}\end{array}\right\} Ans.$$

Again, one train passes another when it has gained over the other a distance equal to the sum of the lengths of the trains which is here $(110+66)$ or 176 yds. or $\frac{1}{10}$ mi.

$$\left.\begin{array}{l}\therefore \text{ for opposite directions, the time} = (\tfrac{1}{10} \div 45) \text{ hr.} = 8 \text{ sec.} \\ \text{and ,, the same ,, ,, ,, } = (\tfrac{1}{10} \div 5) \text{ hr.} = 72 \text{ ,,}\end{array}\right\} Ans.$$

Examples 127.

1. Two persons who are 169 miles apart are travelling towards each other, one at the rate of 16 miles an hour and the other at 10 miles an hour: when will they meet?

2. P sets out from Howrah to go to Burdwan, a distance of 63 miles, and travels at the rate of $3\frac{1}{2}$ miles an hour; $3\frac{1}{7}$ hours afterwards, Q sets out from Burdwan to go to Howrah, and travels at the rate of 3 miles an hour; at what distance from Howrah will they meet?

3. A train leaves Calcutta at 5-30 A.M. and travels 30 miles an hour; another train leaves Calcutta at 2 P.M. and travels 40 miles an hour; when and where will the first train be overtaken by the second?

4. Two runners pass through Hooghly 7 hours one after the other. They travel the same way at the rates of 8 and 12 miles an hour respectively: how long and how far will the first have travelled before he is overtaken by the second?

5. A hare which is 66 yds. before a greyhound is not observed till she has run for 2 min. 30 sec., when the hound gives chase at 10 mi. an hour. If she runs at the rate of 8 mi. an hour, how long will the chase continue, and how far will the hound have run over before he catches the hare?

***6.** A train leaves Calcutta for Benares, a distance of 500 miles After having travelled 50 miles at the rate of 20 miles an hour, it meets with an accident which detains it for 30 minutes, and has to reduce its speed by $\frac{2}{5}$ths of its former rate; how much will the train be behind time?

7. A man rides at the rate of 10 miles an hour and stops 5 minutes to change horses at the end of every eighth mile; how long will it take him to perform a journey of 106 miles?

8. A man rides at the rate of 12 miles an hour and stops 8 minutes to change horses at the end of every tenth mile; how long will it take him to perform a journey of 160 miles?

***9.** Two stations A and B are connected by a railway 280 miles long and a trunk road 250 miles long. A coach starts from A for B, and is

followed after 5 hours by a train running at the rate of 28 miles an hour. A person teaching B by train returns immediately, walking at the rate of 5 miles an hour and meets the coach after 5 hours. Find the speed of the coach per hour.

10. A train leaves Howrah at 6-30 A.M., and reaches Raneegunge at 12-30 P. M.; another train leaves Raneegunge at 8-15 A.M. and reaches Howrah at 1-30 P. M.: when do they meet?

11. Two trains start from Q at the same time on parallel lines for P which is 900 miles off; the first, travelling 15 miles an hour, reaches P 12 hrs. before the other: at what rate does the second run per hour?

12. A train starts from Howrah towards Mirjapur at the rate of 20 miles an hour; three hours afterwards, another train starts from Howrah, and travelling 25 miles an hour, reaches Mirjapur 2 hours before the first Find the distance between Howrah and Mirjapur.

13. Two trains starting from the same station and travelling in opposite directions, are 252 miles apart in 7 hrs. 12 min.; had they been travelling in the same direction, they would have been 77 miles apart in 15 hrs. 24 min. Find the speed of each per hour.

14. Two trains start at the same time, one from Howrah and the other from Ferozabad, and proceed towards each other at the rates of 15, and 12 miles per hour respectively. When they meet, the faster train has run 90 miles more than the other. Find the distance between Howrah and Ferozabad.

15. An ordinary train leaves Calcutta for Goalundo (a distance of 150 miles) running at the rate of 15 miles an hour; 2 hrs. 30 min. after, an express train leaves Calcutta and reaches Goalundo at the same time as the ordinary. Find the rate of the express train per hour.

16. The distance from A to B is 30 miles, of which 8 miles are uphill and 7 miles downhill. In what time can a person walk from A to B and back again, if his pace be uphill 3 miles, downhill 5 miles, and on level ground 10 miles per hour?

17. A ship 40 miles from the shore springs a leak which admits $3\frac{3}{4}$ tons of water in 12 minutes. 60 tons would suffice to sink her; but her pumps can throw out 12 tons of water in an hour. Find the average rate of sailing, that she may reach the shore just as she begins to sink.

18. The distance from P to Q is 25 miles, of which 6 miles are uphill and 4 miles downhill; if a person walk from P to Q and back again, which of the journeys will take a longer time, and by how much; supposing his pace uphill to be 3 miles, downhill 4 miles, and on level ground $7\frac{1}{2}$ miles per hour?

19. A man can row 20 miles down a stream in 3 hrs. 20 min., and row back again in 6 hrs. 40 min.; at what rate can he row, and how fast does the stream flow per hour?

20. A man can row 25 miles and back again in 10 hours, when there is no current: find in what time he can do the same when the stream is flowing at the rate of $2\frac{1}{2}$ miles an hour.

21. A man who can row 5 mi. an hour, rows 30 mi. down the stream in 5 hrs.; how long will he be in rowing 40 mi. against the stream?

22. A man who can row 3 miles an hour, rows 15 miles against the stream in 10 hrs.; how long will it take him to row down 18 miles?

23. When a stream is flowing at the rate of 3 miles an hour, a man can row 16 miles against it in 8 hours; how long will he be in rowing 40 miles with the stream?

24. A man can row 15 miles in 4 hours down a stream flowing at the rate of $1\frac{1}{4}$ miles an hour; find in what time he can row 10 miles against it.

25. Two persons *A* and *B* are 30 miles apart in 3 hrs., when they row from the same place in opposite direction; but they are 12 miles apart in 6 hours, when they both row down the stream. Find the rate at which each can row per hour.

26 A hare which is 120 of her own leaps before a greyhound, takes 3 leaps for every 2 that he takes; but he covers as much ground in 3 leaps as she does in 5; how many leaps will the greyhound have taken before she is overtaken?

27. A hare is 91 of her own leaps before a greyhound. She takes 7 leaps for every 4 that he takes, but 5 leaps of the greyhound cover as much ground as 12 leaps of the hare; how many leaps will the hare have taken before she is overtaken?

***28.** A man starting from Benares at 8 A. M. overtakes another who left Benares at 6 A. M. in 6 hours. Had this man been 3 miles further off when the other started, he would have been overtaken in 7 hrs. 30 min. Find their rates.

29. *P* and *Q* start at the same time from Howrah and Hooghly respectively, each walking towards the other at the rate of 4 mi. an hour. After they meet, *P* increases his speed by half a mile per hour and *Q* reduces his by a mile. If *P* reaches Hooghly in 2 hrs. 40 mi., in what time will *Q* reach Howrah? Also find the distance between Howrah and Hooghly.

***30.** An express train leaving Howrah at 10 A M., overtakes in 10 hrs. an ordinary train which left Howrah at 5 P. M. If the ordinary train had been 15 miles further on when the express train started, it would have been overtaken in 13 hrs. Find their rates of travelling per hour.

***31.** Two guns are fired from the same place at an iaterval of 16 minutes, but a person going towards the place observes that 15 min. 10 sec. elapse between the reports; if sound travels 1122 ft. per second, find his rate of travelling per hour.

***32.** Two guns are fired at an interval of 28 minutes; if a person riding away from them hears the report of the first gun 10 seconds after it was fired and that of the second 40 seconds after, with what speed does sound travel per second, supposing that the man rides $13\frac{7}{11}$ miles an hour?

***33.** A gun is fired at a certain place at regular intervals; a person riding towards it at 13 mi. an hour, hears the reports 30 sec. and 12 sec. respectively after the flashes were seen. At what intervals must the gun have been fired, sound travelling 1144 feet per second?

***34.** A gun is fired twice in a ship sailing towards the shore. A man hears the first report 20 seconds after seeing the flash, and begins to walk towards the ship at the rate of 6 mi. an hour. Observing the second flash $5\frac{1}{3}$ minutes after the first, he stops and hears the report 16 seconds after : with what speed does the ship sail, supposing that sound travels 1155 feet per second ?

***35.** Two guns are fired in a ship sailing towards the shore. A man hearing the report of the first $26\frac{2}{5}$ seconds after seein the flash, walks towards the ship at the rate of 6 miles an hour ; on seeing the second flash 20 min. $26\frac{2}{5}$ sec. after the first, he stops and hears the report in $4\frac{33}{40}$ seconds. Find the speed of the ship per hour, the velocity of sound being 400 yards per second.

***36.** A monkey, climbing up a greased pole, ascends 12 feet and slips down 8 feet, in alternate minutes. If the pole be 56 feet high, how long will it take him to reach the top ?

***37.** A snail creeps 25 in. up a pole during the 12 hours in the night and slips down 18 in. during the 12 hours in the day. If the pole is 21 ft. high, in how many hours will it get to the top ?

38. What time will two trains, 88 yds. and 66 yds. long, running with uniform velocities at the rates of 25 and 20 miles respectively per hour on paralell rails in opposite directions, take to pass each other ?

39. In what time will two trains 110 yds. and 88 yds. long, moving with uniform velocities at the rates of 25 and 20 miles respectively per hour in the same direction, pass each other ?

40. Two trains 100 yds. and 76 yds. long, move with uniform velocities in opposite directions, and pass each other in 9 sec. ? when moving in the same direction with the same velocities as before, the faster train passes the other in 45 sec. ; find the rate of each per hour.

41. Two trains 99 yds. and 77 yds. long, move with uniform velocities in opposite directions and pass each other in 6 sec. ; when moving in the same directions, with the same velocities as before, the faster train passes the other in 36 sec. ; find the rate at which each moves.

42. Two trains 110 yds. and 66 yds. long run wrth uniform velocities at the rates of 18 and 12 miles respectively per hour. How long will a person sitting in the faster train take to pass the other, when going in (1) an opposite direction, (2) the same direction ?

***43.** A train 88 yds. long, overtook a person walking the same way along the line at the rate of 4 mi. an hour, and passed him in 10 sec. ; later on, it overtook another person walking in the same direction and passed him in 9 sec. At what rate per hour was this second person walking ?

***44.** A man is to be at a certain place at a certain hour, and finds that if he walks at the rate of 4 miles an hour, he will reach the place 2 hrs too late and if at the rate of 6 miles an hour, he will be there 1 hr. 20 min. too early : what distance has he to travel ?

***45.** Two trains start from *A* to arrive at a fixed time at *B*, travelling at the rates of 20 and 25 miles an hour respectively. The first reaches *B* 45 min. behind time, and the second 30 min. before. Find the distance between the two places ?

*46. Two horsemen start together to go to a certain place at an appointed time; but riding at the rates of 8 and 10 mi. per hour respectively, the first reaches the place 30 minutes after the time fixed and the second one our before. What distance did they travel?

Problems worked out.

Ex. 1. A horse is sold for *Rs.*210 at a loss; had it been sold for *Rs* 290, the gain would have been four times the loss. What did it cost?

The difference between *Rs.* 210 and *Rs.*290 is $(4+1)$ or 5 times the loss; therefore the loss $=\frac{1}{5}$ of *Rs.* $(290-210)=\frac{1}{5}$ of *Rs.*80 = *Rs.*16.

$\therefore$ the cost price = *Rs.*210 + *Rs.*16 = *Rs.*226. *Ans.*

Ex. 2. A is twice as old as B, and 5 years older than C; the sum of their ages is 95 years: find A's age.

If C were 5 years older, than A's age would be equal to C's and the sum of the ages of A, B, C would be $(95+5)$ or 100 years. Also putting B's age as 1, A's age would be 2, and C's age 2.
Now $2+1+2=5$; $\therefore$ A's age $=\frac{2}{5}$ of 100 years = 40 years. *Ans.*

Ex. 3. The product of two numbers is 750, and the quotient when one is divided by the other is $\frac{6}{5}$ Find the numbers.

The product × the quotient = the square of the dividend.
Now $750\times\frac{6}{5}=900$: and $\sqrt{(900)}=30$:
$\therefore$ one number = 30, and other = $(750\div30)$ or 25 *Ans.*

Ex. 4. In a division, a motion was carried by a majority equal to $\frac{1}{17}$ of the votes of the winning side; had 5 votes been transferred to the losing side, it would have been lost by a majority of 1. Find the total number of votes.

Since a transfer of 5 votes would convert the majority into a minority of 1, the majority $=2\times5-1=9$.

$\therefore\frac{1}{17}$ of the votes of the winning side = 9;
$\therefore$ no. " " " " " $=9\div\frac{1}{17}=153$;
$\therefore$ no " " " losing " $=153-9=144$;
$\therefore$ the total number of votes $=153+144=297$. *Ans.*

Ex. 5. The cost of carpeting a room 30 ft. long is £48; if the breadth had been 4 ft. more, the cost would have been £60: find the breadth.

£60 − £48 or £12 = the cost of carpeting a strip 4 ft. wide;
$\therefore$ £12 × 4 or £48 = " " " " " 16 ft. " ;
$\therefore$ the breadth of the room = 16 ft. *Ans.*

Ex. 6. I divided *Rs.* 57 amongst 200 children; each girl had 8*a.*, and each boy 4*a.*; how many boys were there?

Each girl may be paid 4*a.* along with the boys, and 4*a.* ***afterwards separately***, so that she may get 8*a.* altogether.

The 200 children would in this way first get 4*a.* × 200 or *Rs.* 50. The remaining *Rs.* $(57-50)$ or *Rs* 7 would have to be distributed amongst the girls at the rate of 4*a.* each; $\therefore$ the no. of girls = *Rs.*7 ÷ 4*a.* = 28; and the no. of boys = $200-28=172$. *Ans.*

Ex. 7. In rifle-shooting, a bull's eye counts 4, a centre 3, an outer 2. Eleven men fire five shoots each at target, and score 113; 8 misses are made and 3 bull's eyes: find the number of centres and outers.

The 11 men fire 5×11 or 55 shots; 8 being misses, 47 shots score 113. The 3 bull's eyes score $4 \times 3 = 12$. Therefore the remaining $(47-3)$ or 44 shots, which are either centres or outers, score $(113-12)$ or 101. But if the 44 shots had been all outers, the score would have been 44×2 or 88.

$\therefore$ $(101-88)$ or 13 points are due to centres only;

$\therefore$ the no. of centres $= 13$, and that of outers $= (44-13)$ or 31. *Ans.*

Ex. 8. A man when distributing some pice amongst a certain number of beggars finds that if he gives 3 pice to each there will be 9 pice left; but if he gives 5 pice to each he will require 11 pice more. How much money has he and how many beggars are there?

A rise in the rate from 3 pice to 5 pice or by 2 pice, increases the money required by $(11+9)$ or 20 pice.

$\therefore$ the no. of beggars $= 20 \div 2 = 10$;
and the man has $(3 \times 10 + 9)$ price or 39 pice } *Ans.*

MISCELLANEOUS EXAMPLES V.

1. A rectangular piece of ground is 64 ft. long and 46 ft. broad. Find the cost of enclosing it with a path 8 ft. broad, at 2*s.* a sq. yd.; (i) when the path is outside the ground, (ii) when the path is part of the ground.

2. In a rectangular court-yard measuring 200 ft. by 80 ft., there are (one at each corner) 4 grass plots each 37 ft. 6 in. by 20 ft. Find the cost of paving the remaining area at 5*d* per square foot.

3. The length, breadth and height of a room are 32 ft., 20 ft. and $17\frac{1}{2}$ ft. respectively. It has 4 windows each 8 ft. by $4\frac{1}{2}$ ft, 3 doors each $7\frac{1}{2}$ ft. by 4 ft., and 2 fire-places each 6 ft. by $4\frac{1}{2}$ ft. Find the expense of papering the room, with paper 2 ft. wide, at $5\frac{1}{2}d.$ a yard.

***4.** A room is 54 ft. long by 34 ft wide; how many people can be seated in it on chairs which are $1\frac{1}{2}$ ft. wide and placed 2 ft. apart from back to back, allowance being made for a passage 4 ft. wide down the middle of the room, and for a clear space 12 ft. deep at the end?

5. If in the formation of a square tank 45 ft. in length, 1225 cub. yds. of earth were dug out, shew that the depth of the tank was 16 ft. 4 in.

6. Supposing the average rainfall in Calcutta in a given year to have been 11 ft. 3 in.; find how deep this quantity of water would fill a tank 2 miles square, Calcutta being 6·4 miles long and 2·5 miles broad.

7. A Turkey carpet 11 ft. by 12 ft. 6 in. is laid down in a room 20 ft. by 13 ft. 9 in.: find the cost of staining the borders at $6\frac{3}{4}d.$ the sq. yd.

8. What length must be cut off from a beam $13\frac{1}{2}$ in. broad and 8 in. thick, so as to sell for R162. at 1*p.* the cub. inch.

9. Round a square field containing 90 acres, three boys *A*, *B*, *C*, begin to walk continually, starting from the same place and walking 54, 45, and 36 yards respectively per minute: when will they be again at the starting point, and how many times will each have walked round it?

***10.** A piece of work was done by a certain number of men and boys in 4 days: but when only half the men were present, it required 7 days to do a similar piece of work. Shew that the men can do six times as much work as the boys.

11. A room 18 ft. by 24 ft. and 12 ft. high, is to be carpeted with a carpet 32 in. wide at 2*s*. 8*d*. a yard, and papered with paper $2\frac{2}{3}$ ft. wide at 3*d*. a yard: the windows, taking up a seventh of the walls, are to be glazed at 1*s*. 3*d*. a sq. ft., and the doors, taking up a fourth of the walls, are to be painted at 2*s*. 3*d*. a sq. yard; find the whole cost.

12. A path 3 ft. wide has to be made 5 ft. from the walls round a court measuring 100 yards by 92 yards: find the cost of gravelling it at 3*d*. per square yard.

13 *A* and *B* run a 100 yds.' race and *A* wins by 2 yds.; *C* and *D* run over the same course, and *C* wins by 6 yds.; *B* and *D* also run over it and *B* wins by 4 yds. If *A* and *C* run, which will win and by how much?

14. *A* barters some tea with *B* for flour which is worth 2*s*. $3\frac{1}{2}$*d*. per stone, but uses a false pound-weight of $15\frac{5}{7}$ oz.; what value should *B* set upon his flour that the exchange may be fair?

***15.** A square field is surrounded by a wall. The part immediately within the wall and round the field is covered with gravel, 13 ft. in width: and two straight gravel walks 13 ft. in width run across the field, joining the middle parts of the opposite sides. The ungravelled parts of the field contain $2\frac{1}{4}$ acres. What is the length of the bounding wall?

16. A rectangular room 20 ft. 6 in. long and 12 ft. 6 in. wide, was carpeted with a certain material. If the room had been 2 ft. longer and 1 ft. 6 in. wider, the additional cost would have been £2. 7*s*.; how much did it cost to carpet the room?

***17.** The length, breadth and height of a covered box (outside measure) are as 5, 4, 3. If their sum be 72 in., and the wood of which it is made be $\frac{1}{4}$ in. thick, what is the cubical content of the box?

***18.** In a boat race 7 mi. long, *A* beats *B* by 10 yds., rowing down stream, and *C* by 60 yds., also rowing down stream If it takes $1\frac{3}{4}$ times longer to row up stream than to row down stream, by how much would *B* beat *C* if the race were rowed up stream?

***19** A man has to be at a certain place at a certain time. If he walks 4 miles an hour, he will be 5 min. too late; if he walks 5 miles an hour he will be 10 min. too soon. How far has he to go?

20 In a lake of 18 acres, with an average depth of 8 ft., there is an island 9 roods in extent. If the lake were drained, how much soil would be required to fill it up?

21. The distance from *A* to *B* is 15 mi., the road going over a hill whose top is 3 mi. from *A*. Two men start from *A* and *B* at the same time. The first man walks 4 mi. an hour up-hill and $5\frac{1}{4}$ mi. down; the second man walks $3\frac{1}{2}$ mi. an hour up-hill and $4\frac{1}{2}$ down; where will they meet?

22 *A*, *B*, *C* divide a quantity of goods which cost £100, and mutually agree that *B* should have a third part more than *A*, and *C* a fourth part more than *B*. What must each pay?

***23.** A clerk was engaged on the condition that he should receive a *cowrie* for every 5 words he wrote. On finishing the work, he received R16. 8*a.* 1 *ga.* 1*co.* for his labour; find his hourly earnings, supposing that he wrote as many words in an hour as the number of hours he was at work.

***24.** A train leaves London at 6 A. M., with directions to reach York, by 11 A. M.: at some distance from London it is detained by an accident for 20 min., after which the speed is increased for the remainder of the distance, 5 mi. an hour, and thus York is reached by the appointed time. Where did the accident take place, York being 200 mi. from London?

25. A body of troops consisting of 14600 men is formed into 4 battalions, so that $\frac{1}{2}$ of the first, $\frac{2}{3}$ of the second, $\frac{3}{4}$ of the third, and $\frac{5}{6}$ of the fourth are all composed of the same number of men: how many are there in each battalion?

26. A set of boatmen can row 6 mi. an hour on still water: they row 10 mi. and back again on a stream which has a velocity of 2 mi. an hour; in what time will they accomplish the distance?

27. Find the square root of 6 to 4 places of decimals, and hence find the value of $\sqrt{\frac{2}{3}}$.

28. Find the weight of a rectangular vessel of iron without a top, the vessel being 10 ft. 8 in. long, 8 ft. 4 in broad, and 5 ft. deep: supposing the iron of such a thickness that 4 inches of surface weigh 1 lb.

29. Two persons start at the same time from places 120 mi. apart and meet after travelling 6 hrs.: the one travelled 2 mi. an hour more than the other; at what rate did each travel?

30. If a lecture-room 93 ft. long and 72 ft. broad accommodate 756 persons; how many persons can be accommodated after its length has been increased by one-third and its breadth by one sixth?

31. Find the least number of ounces of gold that can be coined (i) into an exact number of half-sovereigns; (ii) into an exact number of guineas: standard gold being £3. 17*s.* $10\frac{1}{2}$*d.* an ounce.

32. From a vessel containing 50 sr. of milk, 10 sr. are drawn off, and the vessel filled with water; 10 sr. of the mixture are drawn off, and the vessel again filled with water; shew that there are 32 sr. of milk in it now.

***33.** Two persons run a distance of 605 yds.; the first runs at the rate of 11 miles an hour and the second at the rate of $8\frac{4}{5}$ mi.; what start may be allowed to the second that the first may win the race by one yard?

34. In what time would a column of horse extending 1100 ft. march through a defile a mile in length at the rate of 99 paces a minute, the average pace of a horse being $2\frac{1}{2}$ ft.?

***35.** An express train starts from station (*A*) at 10 o'clock for a station (*B*) at 30 mi. an hour and in a quarter of an hour is followed by an ordinary train at 20 mi. an hour. A train from *B* to *A*, at 25 mi. an hour, after travelling one hour meets the express, and in 20 min. more meets the ordinary train. At what time did the train leave *B*?

36. Divide £510 among *A*, *B*, and *C* in such a manner that $\frac{1}{6}$ of *A*'s share may be equal to $\frac{1}{4}$ of *B*'s share, and *C*'s share may be equal to $\frac{5}{12}$ of the sum of the shares of *A* and *B*.

37. Two clocks are together at 12. When the first again comes to 12 it has gained a minute; and when the second comes to 12, it has lost a minute. When will the clocks be quarter of an hour asunder?

38. A gentleman, being 5 min. late for his train, after a minute's consideration determined to take a special train, which was got ready in $16\frac{1}{2}$ min. If the rate of the ordinary train was 32 mi. an hour, and that of the special train 48 mi. find how far the ordinary train had travelled before it was caught by the special train.

39. *A* can give *B* 5 yds. in every 100: how many second's start can he give in a mile race, if *B* can run the 100 yds. in 11 sec.?

40. A yard of silk sells for R2. 8*a*. at a loss; if it had sold for R4, the gain would have been twice the loss. What did it cost?

41. I sell my stock for R4000 at a loss; had it sold for R4400 the gain would have been thrice this loss. Find the proper value.

42. A father is thrice as old as his second son, and 25 yrs. older than his eldest son. The sum of their ages is 80 yrs.; find the father's age.

43. Two trains moving with uniform velocities at the rates of 25 and 20 miles respectively per hour in opposite directions, pass each other in 9 sec.; when they are moving in the same direction with the same velocities as before, a person sitting in the faster train observes that he passes the other in 36 sec. Find the lengths of the trains.

***44.** In going to his office, a man finds that if he walks at the rate of 5 miles an hour he will be 20 min. too late, and if at the rate of 8 miles an hour, an hour too soon. Find the distance he has to travel.

45. The product of two numbers is 864, and the quotient when the greater is divided by the smaller is $\frac{3}{2}$. Find the numbers.

46. The area of a room is 80 sq. ft., and the length is greater than the breadth by a fourth. Find the length and breadth of the room.

47. The cost of matting a room is R8. 5*a*. 4*p*. Had it been 3 ft. wider, it would have cost R9. 14*a*. 4*p*.; find the width of the room.

48. The cost of painting a room 37 ft. long and 23 ft. broad is £24; that of painting another 3 ft. higher and having an equal perimeter is £30. Find the height of each of the rooms.

***49.** In a school there are 400 boys. 1785 oranges are distributed amongst them, each boy of the first four classes getting 5 oranges and each boy of the next five classes getting 4. Find the number of boys in the five lower classes.

***50.** I divided £152 among 300 children so that each boy got half a guinea and each girl two crowns. Find the number of girls

***51.** A squad of 11 boys fired 10 shots each at a target, and scored 286; 20 bull's eyes were made and 11 misses. How many centres and outers were there? (A bull's eye scores 4, a centre 3, and an outer 2.)

52. A man had a certain number of eggs: he sells half the number and one more to one person, half the remainder and one more to a second, and half the remainder and one more to a third, by which time he has disposed of all that he had. How many had he?

53. A man had a certain number of apples; he sells one-third of the number and 6 more to one person, half the remainder and 4 more to a second, one-fourth of the remainder and 3 more to a third, and two-thirds of the remainder and 5 more to a fourth, by which time he has only 3 left. How many had he at first?

54. Which will be more advantageous to employ on a piece of work,—6 men who work 10 hours a day for R36, or 9 boys who work 8 hours a day for R30, it being given that a man can do as much work in 40 min. as a boy in an hour?

55. There is leak in the bottom of a cistern. When the cistern was in thorough repair, it could be filled in 5 hrs.; but now it takes 1 hr. longer. If the cistern is full, how long would it be in leaking itself empty?

56. There are two casks *A* and *B*, each containing 8 gal. of two fluids; a gal. is taken out of each and put into the other; this is done 4 times: how much of the original fluid of *A* is now remaining in *A*?

***57.** A trader bought 120 yds of silk and velvet for R360; for the silk he paid R1 8*a.* per yd. and for the velvet R6 per yd; find the quantity of each bought.

***58.** I went to a book-shop to purchase a few copies of a book with a certain sum of money in my pocket, and found two different editions of the same, the one at R2. 8*a.* a copy and the other at R3. Had I bought the former edition I would have R7. 8*a.* left but choosing to buy the latter edition, I had to borrow R7 8*a.* How many copies did I buy, and how much money had I with me?

***59.** If mangoes are a dozen for the rupee, I have to pay 8*a.* more than what I have, for a basket; if they sell at a rupee for a score, I shall have R3. to spare: find the number of mangoes, and the money I have.

***60.** A float is dropped at Hugli at 9 A. M. just as the tide begins to ebb. It is carried down 10 miles during low tide and pushed up 8 miles during high tide: when will it reach Calcutta which is 24½ mi. from Hugli, it being known that there are 2 low and 2 high tides in 24 hrs. 48 min.?

CHAP. XXXI. RULE OF THREE.

293. **Ratio** is the relation which one quantity bears to another of the same kind in respect of magnitude, the relation being ascertained by comparing what multiple, part, or parts the one is of the other.

Thus the ratio of 3 to 4 is ascertained by the fraction $\frac{3}{4}$; that of R12 to R3. by $\frac{12}{3}$ or 4.

Of the two quantities forming a ratio, the *first* is called the **antecedent**, and the *second* the **consequent**; and the two quantities which form the ratio are called its **terms**.

Thus, in the ratio 22*s.* to 10*s.*, 22*s.* is called the *antecedent* and 10*s.* the *consequent*; also 22*s.* and 10*s.* are its *terms*

294. The ratio 22*s.* to 10*s.* is usually written, 22*s.* : 10*s.*

Note. The sign (:) is an abbreviation of the sign of division ($\div$).

295. When the terms of a ratio are interchanged, the resulting ratio is called the **inverse** of the given ratio.

Thus 10*s.* : 22*s.* is the *inverse* or the **reciprocal** of 22*s.* : 10*s.*

296. The ratio 5*s.* : 7*s.* $= \frac{5}{7}$; the ratio 5 yds. : 7 yds. $= \frac{5}{7}$; the ratio 5 lbs. : 7 lbs. $= \frac{5}{7}$. The ratios 5*s.* : 7*s.*, 5 yds. : 7 yds., 5 lbs. : 7 lbs. are therefore all equal. Thus the value of a ratio does not depend upon the nature of the quantities, and is an *abstract* number.

297. The equality of two ratios is **Proportion** ; and when the ratio of the first of four numbers or quantities to the second is equal to that of the third number or quantity to the fourth, the four numbers are said to be **in proportion** or **proportionals.**

Thus 15 : 20 is equal to 21 : 28 ; for the ratio 15 : 20 $= \frac{15}{20}$ or $\frac{3}{4}$, and the ratio 21 : 28 $= \frac{21}{28}$ or also $\frac{3}{4}$. Therefore the ratios being equal the four numbers 15, 20, 21, 28 are *in proportion* or *proportionals.*

298. When four quantities are proportionals, they are not necessarily *all* of the same kind ; but the first two of them must be of the same kind, also the other two.

Thus, we have 15*s.* : 20*s.* = 21 lbs. : 28 lbs. ; but we cannot have 15*s.* : 21 lbs. = 20*s.* : 28 lbs.

The proportion among four quantities or numbers is usually denoted by placing the sign (::) between the second and the third. The sign (::) is read **equals** or **as.**

Thus, the proportion among the four numbers 15, 20, 21, 28 is written 15 : 20 :: 21 : 28. It may also be written 15 : 20 = 21 : 28.

In the above, 15 and 28 are called the **extremes,** and 20 and 21 are called the **means** ; and 28 is called a **fourth proportional** to 15, 20 and 21.

299. When four numbers are proportionals, the product of the *extremes* is equal to the product of the *means.*

Thus, in the proportion 15 : 20 :: 21 : 28, it is easy to see that $15 \times 28 = 20 \times 21$; for $15 \times 28 = 420$, and $20 \times 21 = 420$.

Hence, an extreme = product of the means $\div$ the other extreme ; and a mean = product of the extremes $\div$ the other mean.

When, therefore, three of the numbers in a proportion are given, we can easily find the fourth. The RULE by which this is done is a very important one in the solution of arithmetical problems, and is called **The Rule of Three.**

300. The **Rule of Three** is the method by which we can find out the fourth term of a proportion, when the first three are known ; *i. e.*, find out a number or quantity whose ratio to the third is the same as that of the second to the first.

RULE. Of the three given quantities, that which is of the same kind as the *fourth* or *the required quantity* (or which is different from the other two quantities, by the nature of the question) should be placed as the *third* term of the proportion. Of the two remaining quantities, one should be placed as the *first* and the other as the *second* term of the proportion ; care being taken, however, *to place the larger or the smaller of the two quantities as the second term, according as the fourth term or required quantity will be greater or less than the third term.*

Before proceeding to find out the fourth term, the first and second terms must both be reduced to the *same* denomination. It should also be remembered that the answer should always be of the *same* denomination as that to which the third term is reduced.

Ex. 1. If the price of 19 yds. of cloth be *Rs.* 76, find that of 24 yds.

Here the three given quantities are 19 yds., 24 yds. and *Rs.* 76 ; and we have to find out the price of 24 yds. Therefore the required term must be of the same denomination as *Rs.* 76 ; hence *Rs.* 76 should be placed as the *third term* of the proportion. Again, because of the two remaining quantities 19 yds. and 24 yds., the price of 24 yds. ought to be *greater* than the price of 19 yds. : therefore 24 yds. should be placed as the *second term* and 19 yds. as the *first.*

The terms should therefore be placed thus :

19 yds. : 24 yds. :: *Rs.* 76 :

and the entire proportion should be written as follows :

19 yds. : 24 yds :: *Rs.* 76 : the required price :

$\therefore$ the required price $= Rs. \frac{24 \times 76}{19} = Rs.\ 96.$ *Ans.* (ART. 299.)

Ex. 2. If 15 men can do a piece of work in 12 days, in what time can 18 men do the same ?

As the fourth term here will be *less* than the third, the proportion should stand thus :

18 men : 15 men :: 12 days : reqd. time :

$\therefore$ reqd. time $= \frac{15 \times 12}{18}$ days $= 10$ days. *Ans.*

301. The RULE OF THREE may thus be classified under two different heads, the first being called the **Rule of Three Direct** and the second the **Rule of Three Inverse.**

The Rule of Three is said to be direct when a greater number or quantity requires a greater answer, and a less number or quantity a less answer. The *first* of the foregoing two Examples is an instance of the **Rule of Three Direct**: for 24 yds. being greater than 19 yds., the price of 24 yds. is necessarily *greater* than that of 19 yds.

The Rule of Three is called inverse when a greater number or quantity requires a less answer, and a less number or quantity requires a greater answer. The *second* of the foregoing two Examples is an instance of the **Rule of Three Inverse**: for 18 men being more than 15 men, it is plain that 18 men will perform the work in a *less* time than 15 men.

302. The following Examples are instances of the Rule of Three Direct.

Ex. 1. Find the value of $8\frac{1}{2}$ tons at the rate of £9. 15*s.* for 3 tons 5 cwt.

3 tons 5 cwt : $8\frac{1}{2}$ tons :: £9. 15*s.* : reqd. cost :

or $\frac{13}{4}$ tons : $\frac{17}{2}$ tons :: £$\frac{39}{4}$: reqd. cost ;

$\therefore$ the reqd. cost $=£\dfrac{\frac{17}{2}\times\frac{39}{4}}{\frac{13}{4}}=£\dfrac{17\times39\times4}{2\times4\times13}=£25.\ 10s.$ *Ans.*

Ex 2. If a man earns *Rs.*6. 14*a.* $7\frac{1}{2}p.$ in 15 days, in what time should he earn *Rs.* 34. 9*a.* $1\frac{1}{2}p.$?

Rs. 6. 14*a.* $7\frac{1}{2}p.$: *Rs.* 34. 9*a.* $1\frac{1}{2}p.$ :: 15 days. : the reqd. time ;

or *Rs.* $\frac{885}{128}$: *Rs.* $\frac{4425}{128}$ :: 15 days : the reqd. time.

$\therefore$ the reqd. time $=\dfrac{4425\times15\times128}{885\times121}$ days $=75$ days. *Ans.*

Ex. 3. If the tax on £250. 17*s.* 6*d.* amounts to £12. 10*s.* $10\frac{1}{2}d.$, what ought to be paid on £100. 10*s.*?

£250. 17*s.* 6*d.* : £100. 10*s.* :: £12. 10*s.* $10\frac{1}{2}d.$: reqd. tax ;

or £$\frac{2007}{8}$: £$\frac{201}{2}$ :: £$\frac{2007}{160}$: reqd. tax.

$\therefore$ the reqd. tax $=£\dfrac{201\times2007\times8}{2007\times2\times160}=£5.\ 0s.\ 6d.$ *Ans.*

Ex. 4. A bankrupt's debts amount to *Rs.* 5600 and his assets are *Rs.* 2100 : how much can he pay in the Rupee ?

*Rs.*5600 : *Re.* 1 :: *Rs.* 2100 : what he can pay in the rupee :

$\therefore$ what he can pay per *Re.* $=$ *Re.* $\dfrac{2100\times1}{5600}=$ *Re.* $\frac{3}{8}=6a.$ *Ans.*

Ex. 5. If after paying an income tax of 15*d.* in the £, a man has £675 left ; what is his income ?

After paying 15*d.* in the £, there is left 225*d.* or £$\frac{15}{16}$ out of £1 income.

£$\frac{15}{16}$: £675 :: £1 : reqd. income ;

$\therefore$ reqd. income $=£\dfrac{675\times1\times16}{15}=£720.$ *Ans.*

Ex. 6. If 3 horses or 25 sheep can eat 24 tons of hay in a certain time, what quantity is required for 6 horses and 75 sheep in the same time?

3 horses = 25 sheep; ∴ 6 horses = 50 sheep; ∴ 6 horses and 75 sheep = (50 + 75) or 125 sheep.

25 sheep : 125 sheep :: 24 tons : no. of tons reqd.;

$$\therefore \text{ no. of tons reqd.} = \frac{125 \times 24}{25} = 120. \quad \textit{Ans.}$$

Ex. 7. I can travel 250 miles by railway for *Rs.* 7. 13*a.*; how far can I travel for *Rs.* 15?

$$Rs.\ 7.\ 13a = Rs. 7\tfrac{13}{16} = Rs.\ \tfrac{125}{16}.$$

$$Rs.\ \tfrac{125}{16} : Rs.\ 15 :: 250 \text{ mi.} : \text{reqd. distance};$$

$$\therefore \quad \text{reqd. distance} = \frac{15 \times 250 \times 16}{125} \text{mi.} = 480 \text{ miles.} \quad \textit{Ans.}$$

303. The following are Examples of the RULE OF THREE INVERSE.

Ex. 1. If 24 men can do a piece of work in 18 days, how many men can do it in 9 days?

9 days : 18 days :: 24 men : reqd. no. of men;

$$\therefore \quad \text{reqd. no. of men} = \frac{18 \times 24}{9} = 48. \quad \textit{Ans.}$$

Ex. 2. If a piece of cloth 66 yds. long be $\frac{7}{8}$ yd. wide, how long should another piece be which is $\frac{3}{4}$ yd. wide, in order to serve the same purpose?

$\frac{3}{4}$ yd. : $\frac{7}{8}$ yd. :: 66 yds. : reqd. length;

$$\therefore \quad \text{reqd. length} = \frac{7 \times 66 \times 4}{3 \times 8} \text{yds.} = 77 \text{ yds.} \quad \textit{Ans.}$$

Ex. 3. If the four-penny loaf weighs $3\frac{1}{2}$ lbs. when wheat is £4. 16*s.* per quarter, what should it weigh when wheat is £5. 12*s.* per quarter?

£5. 12*s.* : £4. 16*s.* :: $3\frac{1}{2}$ lbs. : reqd. weight;

$$\therefore \quad \text{reqd. weight} = \frac{96 \times 7}{2 \times 112} \text{lbs.} = 3 \text{ lbs.} \quad \textit{Ans.}$$

Ex. 4. If 66 cwt. be carried 120 miles for 16, how far must 55 cwt. be carried for the same sum?

55 cwt. : 66 cwt. :: 120 miles : reqd. distance;

$$\therefore \quad \text{reqd. distance} = \frac{66 \times 120}{55} \text{miles} = 144 \text{ miles.} \quad \textit{Ans.}$$

Ex. 5. If a quantity of rice serve 2560 men for 60 days allowing each man 18 ch. a day, to what must the daily allowance be reduced that it may last the same number of men for 72 days?

The number of men may be neglected, as it is the same in both cases.

72 days : 60 days :: 18 ch. : reduced daily allowance;

$$\therefore \quad \text{the reduced daily allowance} = \frac{60 \times 18}{72} \text{ch.} = 15 \text{ ch.} \quad \textit{Ans.}$$

Ex. 6. If in a town 1200 men have provisions for 80 days, and if after 10 days 200 men go away, how long will the remaining provisions last the number left ?

After 10 days there will be provisions left of 1200 men for 70 days, while the number of men will be reduced to 1000.

1000 men : 1200 men :: 70 days : no. of days reqd. ;

$\therefore$ no. of days reqd. $= \dfrac{1200 \times 70}{1000} = 84.$ *Ans.*

Ex. 7. How many yards of velvet at *Rs.* 10. 10*a.* 8*p.* per yard may be exchanged for 252 yards of silk at *Rs.* 8 per yard ?

The price of velvet being greater than the price of silk, the quantity of velvet must be less than the quantity of silk exchanged.

Rs. 10. 10*a.* 8*p.* = *Rs.* $10\frac{2}{3}$ = *Rs.* $\frac{32}{3}$.

Rs $\frac{32}{3}$: *Rs.* 8 :: 252 yds. : quantity of velvet ;

$\therefore$ quantity of velvet $= \dfrac{8 \times 252 \times 3}{32}$ yds. = 189 yds. *Ans.*

Examples 128.

[All the Examples set on the Unitary Method may, if necessary, be worked out by the Rule of Three. *See* pp. 86, 136.]

1. If $103\frac{1}{2}$ yards of cloth cost £37. 3*s.* $10\frac{3}{8}$*d.*, how many yards can be bought for £35. 11*s.* $6\frac{3}{4}$*d.* ?

2. If 54 men can finish a piece of work in 80 days, in what time can 72 men perform double the work ?

3. The assets of a bankrupt whose debts amount to £2625 are £1274. 4*s.* $4\frac{1}{2}$*d* : what can he pay in the £, and what will a creditor lose on a debt of £133 ?

4. By selling 32 copies of a book at 5*s.* 6*d.* per copy, a bookseller clears $\frac{3}{8}$ of the prime cost of each. He then raises the price to 6*s.* 0*d.*, and sells 44 copies more ; how much does he gain on the whole ?

5. *A* borrowed of *B* R5700 for 7 months ; what sum should *A* lend *B* for $9\frac{1}{2}$ months in return ?

6. If a piece of land be 375 ft. 6 in. long and 75 ft. 6 in. broad, how broad must a piece of equal size be whose length is 250 ft. 4 in. ?

7. The circumference of a circle is to its diameter as 22 : 7 ; find the number of revolutions that a wheel, whose diameter is 3 ft. 5 in., will make in 3 mi. 6 fur. 30 po.

8. A servant entered on a situation at noon on Nov. 1, 1891 at an annual salary of R73, and left it at noon on the 9th April 1892 : what did he receive for his service ?

9. If I lend a friend R15000 for 16 months, for what time should he lend me R10000 to repay the obligation ?

10. If 5 men and 7 children can mow 120 acres of grass in 15 days, in what time can 10 men and 3 children mow the same field, supposing a child can do $\frac{1}{3}$ as much as a man in the same time?

11. A family of 12 persons can be maintained for 18 mo. with R1250 when rice is R4. 4*a.* a maund; what family can be maintained for the same time with the same sum, when rice is R3. 6*a.* $4\frac{4}{5}$*p.* a maund?

12. 15 seers of flour may be had for R1. 14*a.*, when wheat is at R4 per maund: what ought to be the price of a maund of flour, when wheat is sold at R6 a maund?

13. A railway train travels 30 miles an hour when it does not stop, and 24 miles per hour with stoppages; in what distance will the train lose $2\frac{1}{2}$ hours by stoppages?

14. If 56 horses and 35 sheep are sold for R3710, and 2 horses are worth as much as 65 sheep, find the price of 10 sheep.

15. A garrison of 2000 men have provisions for 42 days; if after 14 days 400 men be sent away, how long will the remaining provisions last the number left?

16. A ship having a crew of 36 persons has provisions for 25 days: after having been at sea for 13 days, she picks up a party from a wreck, and it is then found that the provisions will last only 8 days more; find the number of persons taken from the wreck.

17. One man walks 5 mi. in 48 min., and another can walk $7\frac{1}{2}$ mi. in 1 hr. 15 min.; how much start must the slower walker have, that in a 10-mile race they may walk a dead heat?

18. A person gave away R140 to an equal number of men and women: each man received R1. 9*a.* 4*p.*, and each woman 12*a.* How many persons were relieved?

19. After deducting 6*p.* in the rupee for income tax $\frac{3}{25}$ of the income of the whole estate for collecting rents, the net income of a gentleman is R20370. What is the annual income of the estate?

20. Two clocks, one gaining 4 min. and the other losing 3 min per day, are set right at noon; what time is it by the second clock when the first indicates noon a fortnight after? Also find the correct time.

21. A man starting from Calcutta takes 1 hr. 15 min. to reach Jadubpore by road, and 1 hr. 7 min. 30 sec. to return across fields walking at the same rate. If the distance by road be 5 mi., what is it across fields?

22. A contractor undertook to construct a road in 35 days, and engaged 30 men to do the work. After 12 days he thought it necessary to employ 15 men more, and in consequeuce the work was done 3 days before the time. How many days would he have been behind-hand, had he tried to finish the work with the original number of men?

23. If *A* can beat *B* by $\frac{4}{7}$ of a mile in a race of 6 mi., what start must he give that they may reach the fourteenth mile-stone together?

24. A contractor undertakes to finish a piece of work in 300 days and immediately employs 80 men to work on it; at the end of $\frac{4}{5}$ of the time the work is only half done. How many men must he add to the number to have the work finished in the appointed time.

***25.** A train without stopping can perform a distance of 66 mi. in $2\frac{3}{4}$ hrs. If it stops at 4 stations and loses 3 min. at each; if also it goes at half speed for the first two miles after each start (not including the first); how long is it in performing the distance?

26. It is agreed that one-half of a fixed rent, which is £900, should be paid in wheat at 45*s.* per quarter and the other half in barley at 30*s.* per quarter. What will the rent amount to, when the price of wheat has advanced to 50*s.* and that of barley to 36*s.* per quarter?

27. *A* alone can do a piece of work in 8 days and *B* in 12 days, when they work 8 hours a day: in what time can they jointly do it when they work 12 hours a day?

28. A person bought 1260 gallons of wine for £756; 60 gallons leaked out; at what price per gallon must he sell the remainder so that he may gain £144 by his bargain?

29. 3 gal. of brandy cost as much as 4 gal. of rum, and 5 gal. of rum as much as 9 gal. of gin; the cost of 3 gal. of a mixture formed by taking them in equal quantities is £2. 12*s.*; find the value of each sort per gallon?

30. A hare is 96 yds. before a greyhound, and is not perceived by him till she has been up 72 seconds; the hare makes at the rate of 10 miles an hour, and the greyhound makes after her at the rate of 14 miles an hour. What distance will the greyhound run over before the hare is caught?

31. A hare is pursued by a greyhound, is 54 yds. in advance of him at starting; to every 4 leaps of the hare the greyhound takes 3, but one leap of the hare covers only $1\frac{7}{8}$ yds. while one of the hound covers $2\frac{3}{4}$ yds.; how far must the greyhound run before he overtakes the hare?

32. A hare is 40 of her own leaps before a greyhound; she takes 5 leaps for every 3 that he takes, but he covers as much ground in one leap as she does in 2; how many leaps will each have taken before she is overtaken?

33. If 7 gal. of brandy cost as much as 9 gal. of rum and 9 gal. of rum as much as 12 gal. of gin, and the cost of 3 gal. of these, taken one of each kind, was £2. 2*s.* 6*d.*: what was the value of each per gal.?

***34** A clock is 10 minutes too fast at 12 o'clock noon, on Monday; at what rate per day does it gain, if at a quarter past 10 o'clock A. M. on the following Saturday, it indicates 10 hrs. $40'$-$36\frac{7}{48}''$?

35. A ship sailed from a certain port for another with provisions for 64 days, the daily allowance of each man being 18 oz. After she had been at sea for 16 days, a storm drove her out of her course, which delayed her 6 days. Find to what the daily allowance should be reduced so that the provisions might last until she reached the port.

36. A hare starts with 50 of her own leaps in advance of a greyhound and takes 4 leaps to the greyhound's 3; but two of the greyhound's leaps cover as much ground as 3 of the hare's; find the number of leaps the greyhound must take to overtake the hare.

37 What is the value of $\cdot 09$ of $\cdot 08\dot{3}$ of an estate, if a person who owns $\cdot 2\dot{7}$ of it, sells $\cdot 1$ of $\cdot 875$ of his share for R7000 ?

38. A passenger and a mail train started at the same time from a station A to go to B, a distance of 300 mi., their rates of travelling being 20 and 30 miles per hour, rerpectively. The mail train on reaching B returned immediately. How far from A did the two trains meet ?

CHAP. XXXII. COMPOUND RULE OF THREE.

304. It has been seen that in simple proportion (corresponding to **Simple Rule of Three),** there are two pairs of quantities, each of the same kind, whose ratios are equal. In Compound Proportion (corresponding to **Compound Rule of Three),** there will be several pairs of such quantities, which are so related that the ratio between one pair will be the **product** of the ratios between the other pairs.

Thus in **Compound Rule of Three,** there will be given 5, 7, 9,... quantities, one of which is of the same kind as the *quantity required to be found*, while the others form pairs, each of the same kind.

305. Examples on Compound Rule of Three are worked out in almost the same way as those on Simple Rule of Three.

RULE. Arrange the pairs of numbers in two lines, in such a way that the corresponding quantities fall under one another, and the unknown quantity is placed *last* in the *second* line ; and draw an arrow *upwards* by the side of the unknown quantity.

Take each of the pairs, and ascertain whether the ratio is to be direct or inverse ; *i. e.*, whether the effect of *increasing* the number in the lower line is to *increase* or *diminish* the unknown quantity, and *vice-versa.* If the ratio is direct, draw an arrow downwards, if not, draw it upwards.

Multiply the number corresponding to the unknown quantity by the product of the number lying at the *heads* of the arrows and divide by the product of the others. The quotient will give the quantity required.

Ex. 1. If 17 men can earn £24 in 40 days ; how much would 51 men earn in 25 days at the same rate ?

men	days	£	
17 ↓	40 ↓	24 ↑	More men, *more* earnings ;
51	25	no. of £	more days, *more* ,,

$\therefore$ required no. of £ $= \dfrac{51 \times 25 \times 24}{17 \times 40} = 45.$ *Ans.*

[The following solution by the Unitary Method is instructive.

17 men in 40 days earn £24 ;
$\therefore$ 17 ,, ,, 1 day ,, £$\frac{24}{40}$;
$\therefore$ 1 man ,, 1 day ,, £$\frac{24}{40} \times \frac{1}{17}$;
$\therefore$ 1 ,, ,, 25 days ,, £$\frac{24}{40} \times \frac{1}{17} \times 25$;
$\therefore$ 51 men ,, 25 ,, ,, £$\frac{24}{40} \times \frac{1}{17} \times 25 \times 51 =$ £45]

Ex. 2. If 13 men can reap a field of 10 acres in 9 days of 7 hrs. each, how many hours a day must 91 men work to reap 15 acres in 2 days?

men	acres	days	hours	
13 ↑	10 ↓	9 ↑	7 ↑	More men, *less* hours;
91	15	2	no. of hrs.	more acres, *more* „ ; more days, *less* „

$\therefore$ no. of hrs. reqd. $= \dfrac{13 \times 15 \times 9 \times 7}{91 \times 10 \times 2} = 6\frac{3}{4}$. *Ans.*

Ex. 3. A six-penny loaf weighs $3\frac{1}{2}$ lbs. when wheat is 6*s.* 3*d* per bushel; what ought to be the weight of the four-penny loaf when wheat is 8*s.* a bushel?

d.	*s.*	lbs.	
6 ↓	$6\frac{1}{4}$ ↑	$3\frac{1}{2}$ ↑	More price of loaf, *more* weight;
4	8	reqd. wt.	more price of wheat, *less* „ .

$\therefore$ reqd. weight $= \dfrac{4 \times 6\frac{1}{4} \times 3\frac{1}{2}}{6 \times 8}$ lbs. $= \dfrac{175}{96}$ lbs. $=$ 1 lb $13\frac{1}{3}$ oz. *Ans.*

Ex 4. A garrison of 10000 men have provisions for 4 mo. at 2 lbs. daily for each man; how many must be sent away, that by increasing the daily allowance to each man by $\frac{1}{2}$ lb. the provisions may last 16 mo.?

The new allowance $= 2\frac{1}{2}$ lbs. Let x be the no. of men that remain

mo.	lbs	men	
4 ↑	2 ↑	10000 ↑	More months, *less* men;
16	$2\frac{1}{2}$	x	more allowance, *less* „ .

$$\therefore x = \frac{4 \times 2 \times 10000}{16 \times 2\frac{1}{2}} = 2000.$$

$\therefore$ no. of men to be sent away $= 10000 - 2000 = 8000$ *Ans.*

Ex. 5. If 60 guns firing 5 rounds in 8 min kill 350 men in 15 min., how many guns firing 7 rounds in 9 min. will kill 980 men in 25 min, at the same rate?

Let x be the no of guns required.

rounds	interval	time	men	no. of guns	
5 ↑	8 ↓	15 ↑	350 ↓	60 ↑	More rounds, *less* guns; more interval, *more* „ ;
7	2	25	980	x	more time, *less* „ ; more men, *more* „ ;

$$\therefore x = \frac{5 \times 9 \times 15 \times 980 \times 60}{7 \times 8 \times 25 \times 350} = 81. \quad \textit{Ans.}$$

Examples 129.

[Students are recommended to work out the following Examples both by the Compound Rule of Three and by the Unitary Method.]

1. If 15 men earn R555 12*a.* in 13 weeks, how many men can earn R114 in 5 weeks?

2. If 8 men can mow a field of 10 acres in 15 hrs, in how many hours will 36 men mow 16 acres?

3. If 56 horses eat 31 mds. of oat in 6 days, in what time will 280 horses eat $516\frac{2}{3}$ mds. ?

4 If the expenses of a family of 7 persons for 13 days be R58. 8*a.*, what will be the expenses of a family of 12 persons for 21 days ?

5. If 630 mds. be carried 45 mi. for R91. 14*a.*, for what sum of money will 660 mds. be carried 36 miles ?

6. If by placing R4480 in business for $4\frac{1}{2}$ months R360 be gained, what sum must be placed in the business so that in $10\frac{1}{2}$ months R682. 10*a*. 8*p*. may be gained. ?

7. If 16 fires burning 9 hours a day, consume 160 maunds of coal in 20 days, what quantity of coal will be consumed by 30 fires burning 10 hours a day in 30 days ?

8. If 30 men can do a piece of work in 16 days of 10 hours, how many hours a day would 75 men have to work in order to do a piece of work 6 times as great in 48 days ?

9. If 20 masons build a wall 50 ft. long, 2 ft. thick, and 14 ft. high, in 12 days of 7 hours each, how many hours a day, will 60 masons have to work in order to build in 64 days a wall 500 ft. long, 4 ft. thick and 16 ft. high ?

10. If 25 men can dig a trench 600 ft. long, 10 ft. wide and 8 ft. deep in 48 days, working 10 hours a day, for how many days of 12 hours each, must 400 men be employed to dig a trench 1500 yds. long, $10\frac{2}{3}$ ft. wide, and 12 ft. deep ?

11. If 8 men can pave a rectangular court-yard 300 yards long and 80 yds. wide in 15 days of 11 hours each, how many men, working 12 hrs. a day for 33 days, will be required to pave a rectangular area measuring 540 yds. by 120 yds. ?

12. If 44 men can do a piece of work in 18 days of 7 hours each, how many women will do the same work in 22 days of 14 hrs. each, supposing that a man can do as much work as two women ?

13. If 72 men or 108 boys can earn £1148. 11*s*. in 31 days, how many boys and 10 men will earn £308. 15*s*. in 20 days ?

14. If the carriage of 55 mds. 11 sr. for 200 miles cost R383. 13*a*. 8*p*., what distance will 162 mds. be carried for R618. 12*a*. ?

15. If with a capital of R315 a trader gain R20. 4*a*. in 5 months, what sum must he put in, in order to gain R90 in 7 months ?

16. A garrison of 20,000 men having provisions for 27 weeks, at 12 oz. to each man per day, is reinforced by 4000 men : what must be the reduced daily allowance that the provisions may last 30 weeks ?

17. A garrison of 12000 men have provisions for 30 days, allowing 15 oz. to each man every day ; if, after 12 days, 2000 men go away, how long will the remaining provisions serve the number left, the daily allowance being reduced by 3 oz. ?

***18**. If 12 compositors set up 14 sheets of 16 pages, each page containing 48 lines of 65 letters, in 20 days of 8 hours each ; how many compositors will set up 26 sheets of 24 pages, each page containing 56 lines of 55 letters, in 12 days of 11 hours each ?

19. If 26 men in 22 days working 10 hours a day can pave a rectangular field 2002 yards long by 540 yards wide; how long must another field whose breadth is 360 yards be, which 64 men can pave in 20 days working $7\frac{1}{2}$ hours a day?

20. If in Calcutta a family of 10 persons can live for 16 months on R4000, what will it cost a family of 7 persons to live in Midnapur for 8 months, prices in Midnapur being $\frac{4}{7}$ of what they are in Calcutta?

21. If 15 horses and 148 sheep can be kept for 9 days for £75. 15*s*., what sum will keep 10 horses and 132 sheep for 8 days, supposing 5 horses eat as much as 84 sheep?

22. If 1200 copies of a book of 15 sheets require 36 reams of paper, how much paper will be required for 2000 copies of a book of 28 sheets, both the books being of the same size?

23. If 756,000 bricks, each 12 in. long, $4\frac{1}{2}$ in. broad and 4 in. thick, are required to build a house, how many bricks which are each one-third less in every dimension, will be required to build a house three times as large?

24. If the wages of 60 men amount to R480 in 24 days, how many women must work 36 days to receive R1008, the daily wages of a woman being one-half of those of a man?

25. If the cost of printing a book of 720 leaves, with 32 lines on each page and 15 words in each line, be R256; find the cost of printing a book of 2100 pages with 27 lines on each page, and 12 words in each line.

***26**. A ship having a crew of 51 men and provisions for 100 days at 20 oz. a day, encountered a storm after 20 days and lost 6 of her crew; and on the same day took on board 15 persons from a wreck; what must the daily allowance of each man be reduced to, that the provisions might just last for 85 days more?

27. If in an asylum 100 beggars be fed 16 days for R200 when rice is at R2. 8*a*. a maund, how many beggars may be fed there 20 days for, R360 when rice is at R3 a maund?

28. If the four-penny loaf weigh 2 lbs. when wheat is 60*s*. the quarter what should the three-penny loaf weigh when wheat is 54*s*. the quarter?

***29**. If 16 cannon, firing 5 rounds in 6 minutes, kill 2500 men in an hour, how many cannon, firing 4 rounds in 5 minutes, will kill 3000 men in 1 hour and 20 minutes?

***30**. If 15 cannon, which fire 3 rounds in 5 minutes, kill 450 men in 1 hour 40 minutes, how many men would be killed by 24 cannon, which fire 5 rounds in 6 minutes, in 1 hour 20 minutes?

31. If the expenses of a family of 9 persons for 16 months be R1500 when rice is R4. 8*a*. per maund, what will be the expenses of another family of 15 persons for 18 months, when rice is R5 per maund?

32. How far must 4 tons 7 cwt. 2 qr. be carried for £3. 12*s*. 11*d*., when 17 tons 10 cwt. are carried 180 miles for £13. 2*s*. 6*d*.?

33. If 7 horses be kept 40 days with R280, when oats are R2. 8*a*. a maund, how many horses can be kept 9 days with R810, when oats are R3 a maund ?

34. If the wages of 50 men amount to R1533 5*a*. 4*p* in 16 days, how many men must work 48 days to earn R2070, supposing 3 men of the former set can do as much work as 2 men of the latter ?

35. If 18 men can do a piece of work in 10 days, how many boys will do another piece of work 4 times as great in one-fifth of the time, 3 men doing the same amount of work as 7 boys in the same time ?

36. A contractor agrees to construct a road in a fixed time. He employs 2500 men who work 9 hours a day. After $\frac{2}{7}$ of the time has expired, he finds that only $\frac{5}{16}$ of the work has been done. How many more men should he employ to finish the work within the appointed time, now that the daily work has been increased to 10 hours a day ?

37. If 3000 copies of a book of 17 sheets require 102 reams of paper, how much paper will be required for 10000 copies of a book of 24 sheets, half as large again as the former both in length and breadth ?

38 If 225 men can dig a trench 600 yds. long, 10 ft. wide and $6\frac{3}{4}$ ft. deep, in 48 days of 9 hours each, how many men must be employed for 20 days of 10 hours each, to dig a trench 1200 yds. long, 15 ft. wide and 10 ft. deep, each man of the latter set doing $\frac{1}{11}$ part more work in an hour, than each of the former ?

39. If 30 men dig a tank 510 yards long, 216 yards broad and 14 ft. deep in 153 days working 10 hrs. a day, how many days of 12 hrs. must 120 men take to dig another which is 720 yds. long, 420 yds. broad and 18 yds. deep, the hardness of the soil in the two cases being as 5 : 6 ?

***40**. If 35 men can dig out 340 tons of coal in 44 days working 9 hrs. a day for R577. 8*a*., how many boys must assist 14 men to dig 910 tons of coal for R850 in 39 days of 7 hrs. each, the work of a man and that of a boy being as 3 : 1 ?

41. A town defended by 3000 troops has provisions for $17\frac{1}{2}$ months, each man being allowed a ration of 2 lbs. a day : how many must be sent away so that by increasing the daily allowance by half a lb., the provisions may last $3\frac{1}{2}$ years ?

42. If 128 horses consume a stack of hay 32 ft. long, 19 ft. broad and 27 ft 6 in. high in 10 days, how high must another stack 55 ft. long and 16 ft. 6 in broad be, which will supply 200 mules for 12 days, the voracity of a horse being $\frac{2}{17}$ths more than that of a mule ?

***43**. If 15 cannon, firing 6 rounds in 10 minutes, kill 2000 men in 1 hour 40 minutes, how many men would be killed by 20 cannon, firing 7 rounds in 12 minutes, in 1 hour 48 minutes ?

44 If the four-penny loaf weighs 2·9 lbs. when wheat is 7·25*s*. the bushel, what weight of bread can be purchased for 16*s*. 8*d*., when wheat is 14·5*s* the bushel ?

45. If, when rice is selling 16 sr. for the rupee, R700 can meet the expenses of a family of 11 persons for 7 months, what must the expenses amount to, if a family of 16 persons have to be maintained for 11 months when rice is selling 12 sr., for the rupee ?

46. If 16 pumps, each making 30 strokes 3 ft. in length in 7 min., empty a tank in 12 days of $10\frac{1}{2}$ hrs. each, how many pumps, making 32 strokes $2\frac{1}{2}$ ft. in length in 5 min, will empty another tank of double the size in 9 days of 15 hrs. each ?

47. A farmer contracts to reap a field of 57 bighas in 8 days, and employs 12 men on the work; at the end of 5 days only 30 bighas are reaped; how many more men should he employ to fulfil the contract ?

***48.** If 10 men dig a moat of a uniform breadth and depth of 6 ft. and 9 ft. respectively and surrounding a rectangular field, 300 yds. long by 160 yds. broad, in 150 days working 12 hrs. a day; in how many days of 10 hrs. each will 200 men dig a trench 250 yds. long, 58 yds. broad and 10 ft. deep, 5 men of the former set digging as much as 4 men of the latter in the same time ?

49. In a boat race one crew rows 40 strokes to the others' 36; but the strokes of the latter are $\frac{1}{15}$th stronger than those of the former. If it takes the first crew 30 min. to row a race of $3\frac{1}{2}$ miles, by how many yards and by how many seconds will they win ?

50. A contractor engages 88 men to do a piece of work in 42 days. They work at the rate of 8 hrs. a day, but after 20 days only a quarter of the work is done. The contractor then engages some women to help to finish it in time. If a man can do half as much again as a woman in the same time and if a woman works only 4 hrs. a day, how many women must be engaged to finish the work in the given time ?

CHAP. XXXIII MISCELLANEOUS PROPOSITIONS.

306. **Incomes, taxes, rates, &c.** The **gross income** is the total income on which taxes, rates, &c, have to be paid; the **net income** is that which remains after the taxes, rates, &c. have been paid out of this gross income.

Ex. 1. A man paying an income-tax of 6*p*. in the rupee, has *Rs*. 1240 left; find his gross income.

After paying 6*p*. in the rupee, he has $15\frac{1}{2}a$ or *Re*.$\frac{31}{32}$ left out of *Re*. 1.

Re.$\frac{31}{32}$: *Rs*.1240 :: *Re*.1 : gross income;

$\therefore$ gross income $= 1240 \times \frac{32}{31} \times$ *Re*.1 $=$ *Rs*.1280. *Ans.*

Ex. 2. A man whose net income is £6786 had to pay an income tax of 6*d*. in the pound, and 8*d*. in the pound on the remainder for rates; find his gross income.

After paying 6*d*. in the £ for income-tax, he has £$\frac{234}{240}$ or £$\frac{39}{40}$ left out of £1. For rates, he pays on the remainder $\frac{39}{40} \times 8d. = 7\frac{4}{5}d.$ Therefore he altogether pays $(6+7\frac{4}{5})d.$ or $13\frac{4}{5}d.$ in the pound.

$\therefore$ he has $(240-13\frac{4}{5})d.$ or $226\frac{1}{5}d.$ or £$\frac{1131}{1200}$ left out of £1.

£$\frac{1131}{1200}$: £6786 :: £1 : gross income;

$\therefore$ gross income $=$ £7200. *Ans.*

Ex. 3. The income-tax having been raised to $7d.$ in the £, a person has to pay £35 more than when it was $5d.$ in the £; what is his income?

The tax being raised to $7d.$ in the £, he has to pay $2d.$ or £$\frac{1}{120}$ more on £1.

£$\frac{1}{120}$: £35 :: £1 : income ; ∴ income = £4200. *Ans.*

Ex. 4. A man hires a house for £240 a year, which is assessed in the rate-book at $\frac{7}{8}$ of its rent : he agrees to pay the following rates upon it :— 3 poor's rates of $6d.$, $8d.$ and $10d.$ respectively, a church-rate of $14d.$, and a paying rate of $16d.$ in the £. What is the annual cost of the house?

Assessed value = $\frac{7}{8}$ of £240 = £210.

Rates on £1 amount to $(6+8+10+14+16)d. = 54d. =$ £$\frac{9}{40}$;

∴ rates on £210 = £$\frac{9}{40} \times 210$ = £47. 5s.;

∴ the annual cost of the house = £240 + £47. 5s. = £287. 5s. *Ans.*

Ex. 5. An occupier has to pay the following rates upon the assessment of the house he occupies; *viz.*, police rates of $8p.$, water rate of $16p.$, house rate of $18p.$, and lighting rate of $14p.$ in the rupee per annum. If the whole annual cost of the house be *Rs.* 1170, find its assessed value, which is $\frac{3}{4}$ of the actual rent.

The rates amount to $(8+16+18+14)$ or $56p = Re.\frac{7}{24}$ in the rupee. If the assessed value = *Re.* 1, the actual rent = *Rs.* $\frac{4}{3}$;

∴ the annual cost = *Rs.* $(\frac{4}{3}+\frac{7}{24})$ or *Rs.* $\frac{13}{8}$.

Rs. $\frac{13}{8}$: *Rs.* 1170 :: *Re.* 1 : assessed value; ∴ assessed value = *Rs.* 720.

Examples 130.

1. After paying an income-tax of $4p.$ in the rupee, a person has R940 left : find his income.

2. A gentleman pays R147. $14a.$ for income-tax at the rate of $7p.$ in the rupee; what is his income?

3. A talukdar after paying half an anna in the rupee for road-cess, has R15500 remaining. Find the annual rental of his taluk.

4. A zemindar after paying $5a.$ $4p$ in the rupee for land revenue and $6p.$ in the rupee on the remainder for road-cess, has R35743 left. How much does his estate produce annually?

5. A gentleman can spend £4830 for personal expenses after paying $5s.$ $4d.$ on every pound of his income for the education of his children and $15d.$ in the pound on charity. Find his income.

6. A person after spending $10a.$ $8p.$ in the rupee for family expenses and $5a.$ $4p.$ in the rupee on the remainder for other expenses, saves R200. Find his income.

7. An income-tax of $6p.$ in the rupee being paid on the whole annual cost of an estate, and $10p.$ in the rupee on the remainder for collecting rents, a net income of R14105 is obtained. What is the income-tax?

8. A person after paying $16d.$ in the pound for income-tax and $\frac{1}{16}$ of the remainder for rates, has £7036. $15s.$ left : what is his income?

9. A gentleman after spending 2*a.* 8*p.* on every rupee of his income for charitable purposes and saving $\frac{1}{8}$ of the whole, can spend R8874 per annum. What does he save annually ?

10. Before the introduction of the income-tax, the yearly income of a gentleman was £6900. How much must it be now with an income-tax of 10*d.* in the pound, that he may still have £6900 a year net ?

11. I hire a house for £360 a year, the assessed value of which is $\frac{8}{9}$ of its rent ; I have also to pay the following rates upon it ; *viz.*, poor's rate of 9*d.*, church-rate of 6*d.*, lighting rate of 8*d.*, water-rate of 10*d.* and police-rate of 3*d* in the pound ; what is the annual cost of the house ?

***12.** An occupier pays house-rate of 2*a.*, police-rate of $\frac{1}{2}$*a.*, water-rate of 1*a*, and lighting rate of $\frac{1}{3}$*a.* in the rupee. If the rent (which is $1\frac{1}{8}$ times the assessed value) and rates amount to R550, what is the assessed annual value of the house ?

13. A person whose income is £750 a year pays an income-tax of £20. 6*s.* 3*d.* ; what is the rate per £, and what is the income of another person who pays annually a tax of £30. 9*s.* 11*d.* ?

14. The income-tax having been raised to 10*d.* in the pound, I have to pay £53. 10*s.* 8*d.* more than when it was 7*d.* Find my income.

307. **Percentage.** The term **per cent** or **per centum** means 'for a hundred.'

The sign °/₀ or *p. c.* is often used for the words *per cent.*

"A man gains or loses 5°/₀ or 5 per cent.," means that he gains or loses R5 for every R100 invested ; or, in other words, that he gains or loses $\frac{5}{100}$ or ·05 of the outlay. Also 5 is called the rate per cent.

The following Examples will clearly illustrate the meaning of the term 'per cent.'

Ex. 1. How much per cent. is 5 of 12, *i.e*, what is the number which will bear the same ratio to 100 that 5 bears to 12 ?

12 : 100 : : 5 : the reqd rate ; ∴ the reqd. rate per cent.$=41\frac{2}{3}$. *Ans.*

Ex. 2. In a school of 300 boys, 250 are present. Find the percentage of attendance.

300 : 100 : : 250 : the required percentage ;

∴ the reqd. percentage of attendance$=83\frac{1}{3}$. *Ans.*

Ex. 3. In 1891, the number of boys in a school was 400 ; in 1892 it rose to 500. Find the increase per cent.

500 − 400 = 100 ; thus on 400 there is an increase of 100.

400 : 100 : : 100 ; the increase per cent. ;

∴ the increase per cent. = 25. *Ans.*

Ex. 4. Deaths are 15 per cent. of the population in a town containing 160000 people ; find the number of deaths.

100 : 160000 : : 15 : the number of deaths ;

∴ the number of deaths = 24000. *Ans.*

Ex. 5. In 1883, a person started business with a certain sum of money which, increasing by 20 per cent, amounted to *Rs.* 28800 in 1890. What sum did he start with?

Every *Rs.* 100 invested in 1883, *increased* to *Rs.* 120 in 1890. Therefore, *Rs.* 120 in 1890 corresponds to *Rs.* 100 in 1883.

∴ *Rs.* 120 : *Rs.* 28800 :: *Rs.* 100 : the sum required;

∴ the sum required $=$ *Rs.* 24000 *Ans.*

Ex. 6. In a mercantile concern, 42 per cent. of the whole profits were spent in paying off salaries and other expenses, and 50 per cent. were paid to the shareholders. The rest, which £8840, was reserved: find the total profits.

$100-(42+50)=8$; ∴ 8 per cent. was reserved.

£8 : £8840 :: £100 : the total profits;

∴ the total profits $=$ £110500. *Ans.*

Ex. 7. The death-rate in a town is 1·5 per cent. in the year, and the birth-rate is 2·4 per cent. The town contains 12000 inhabitants: find the population at the end of the year.

For every 100 persons at the beginning of the year, there will be at the end of the year $(100-1\cdot5+2\cdot4)$ or 100·9 persons.

100 : 12000 :: 100·9 : the reqd. population ;

∴ the reqd. population $=120\times100\cdot9=12108$. *Ans.*

Ex. 8. Standard gold contains 22 parts of pure gold and 2 parts of alloy. Find the percentage of pure gold in it.

$22+2=24$; there are 22 parts of pure in 24 parts of standard gold.

∴ 24 : 100 :: 22 : percentage of pure gold ;

or percentage of pure gold $=91\frac{2}{3}$. *Ans.*

Ex. 9. A person lays out £2400 and £1600 respectively in two different speculations; in the first he loses 4 per cent. and in the second he gains 10 per cent. Find his gain or loss per cent., on the sum invested.

100 : 2400 :: £4 : the loss on the first speculation ; ∴ loss $=$ £96 ;

100 : 1600 :: £10 : the gain „ second „ ∴ gain $=$ „ 160 ;

∴ on £$(2400+1600)$ or £4000, he gains £$(160-96)$ or £64.

∴ 4000 : 100 :: 64 : gain% ; ∴ gain% $=1\cdot6$. *Ans.*

Examples 131.

1. Find the value of :

(1) 4 per cent. of 250. (2) $7\frac{1}{2}$% of £420. (3) $3\frac{3}{4}$% of 120 mds.

(4) 22·5% of R1000. (5) $12\frac{1}{2}$% of $33\frac{1}{3}$ cwt. (6) $\frac{1}{8}$% of 225 days.

2. What percentage is :

(1) 12 of 25? (2) $6\frac{1}{4}$ of 80? (3) 15 of 93?

(4) $13\frac{1}{2}$ lbs. of 50 lbs. ? (5) £66. of £8 ? (6) 57 men of 600 men ?

(7) R7·8 of R520 ? (8) 235·5 bighas of 750 bighas ?

(9) 15 cwt. 3 qr. 21 lbs. 8 oz. of 3 tons 19 cwt. 2 qr. 23 lbs. 8 oz. ?

3. The income of a gentleman is R1220 a year; if he spends 85 per cent. of it, how much does he save ?

4. How many persons are engaged in agriculture, when they are 35 per cent. of a population of 4,620,520 ?

5. The population of a town was 856000 ten years ago, but now it is 860280; what is the increase per cent. ?

6. A father gave R5000 to each of his sons. After three years the eldest lost 42 per cent. and the second gained as much: what per cent. was the eldest son's money then of that of the second ?

7. Out of a pipe of wine, 21 gallons leaked out; how much per cent. was the leakage ?

8. A gas company reduces the price of its gas from R4. 11*a*. to R3. 12*a*. per 1000 cub. ft.; what is the reduction per cent. ?

9. *A* owed *B* R1235, and has paid R1100; what per cent. of the debt remains unpaid ?

10 A man owed R660, which is 75 per cent. of what he is worth; how much is he worth ?

11. The population of a certain town is 24350, and is 25 per cent. more than what it was five years ago; what was it five years ago ?

12. After paying 10 per cent. income-tax on his income, a man had R15000 left; what was his income ?

13. A man started business with a certain sum of money, which increasing by 25 per cent. every year, amounted to £100000 in three years. What capital did he start with ?

14. Brass is composed of copper and zinc in the ratio of 63 : 34. Find the percentage of copper and zinc in the compound.

15. Out of the population of a town, 45 per cent. are Hindus, 30 per cent. Mahomedans, 15 per cent. Christians, and the rest, amounting to 12000, of other creeds; find the entire population of the town.

16. Two schools have 400 and 500 pupils respectively. The average daily attendance in the first is 87·5 per cent. and in the second. 90 per cent.; find the percentage of average daily attendance in both the schools together.

***17**. The number of male and female criminals in a certain country are 12350 and 9880 respectively. While the former decrease by 4·6 per cent., the latter increase by 9·8 per cent., at the end of the year: find the annual increase or decrease per cent. in the whole number of criminals.

***18**. If the increase in the number of male and female criminals together is 1 8 per cent., while the decrease in the number of males alone is 4·6 per cent. and the increase in the number of females is 9·8; find the number of female criminals, the number of male criminals being 1235.

*19. Of the candidates at an examination, 85 per cent. passed in Arithmetic and 80 per cent. in English; 169 passed in both subjects and no candidate failed in both. How many candidates were there?

308. **Profit** and **Loss.** We have already given the student some idea of *Gain* or *Loss* (see ART. 121). The questions on Gain and Loss all come under the head of **Profit** and **Loss.** By 'gain or loss per cent.,' it is meant that so much per cent. is gained or lost on the **cost price** or **prime cost.**

When a trader is said to gain 10 per cent., it is meant that on every R100 worth of goods he gains R10, *i.e.*, that he sells the goods for R(100+10) or R110.

When a trader is said to lose 10 per cent., it is meant that on every £100 worth of goods he loses £10, *i.e.*, that he sells the goods for £90

Ex. 1. I buy some articles for R40; what must I sell them for, so as to gain 20 per cent.

20 per cent. having to be gained, *Rs.* 100 worth must be sold for *Rs.* 120.

Rs. 100 : *Rs.* 40 :: *Rs.* 120 : selling price;

$\therefore$ selling price $= \frac{40}{100} \times$ *Rs.* 120 = *Rs.* 48. *Ans.*

Ex. 2. If I buy a horse for £125, what do I sell it at, to lose 10%?

10 per cent. being lost, £100 worth of goods are sold for £90

£100 : £125 :: £90 : selling price;

$\therefore$ selling price $= \frac{125}{100} \times$ £90 = £112. 10*s.* *Ans.*

Ex. 3. An article, which costs £20, is sold for £22; what is the gain per cent.?

Gain on £20 = £22 − £20 = £2.

20 : 100 :: 2 : gain per cent.; $\therefore$ gain = 10 per cent. *Ans.*

Ex. 4. If by selling an article for *Rs.* 57. 8*a.*, 15 per cent. be gained, what is the cost price?

Rs. 100 worth of goods are sold for *Rs.* 115.

Rs. 115 : *Rs.* $57\frac{1}{2}$ :: *Rs.* 100 : cost price; $\therefore$ cost price = *Rs.* 50. *Ans.*

Ex. 5. If, by selling a horse for £60, I lose 20 per cent.; what must I sell it for, to gain 10 per cent.?

£(100 − 20) : £60 :: £100 : prime cost; $\therefore$ prime cost = £75

Again, £100 : £75 :: £110 : selling price; $\therefore$ selling price = £82 10*s.*

Examples 132.

1. I buy slates at 3*a.* 9*p.* each; what must I sell each for, to gain 20 per cent.?

2. I buy a book at 1*s.*; what must I sell it for, to gain $12\frac{1}{2}$ per cent.?

3. If tea be bought at 5*s.* 5*d.* per lb. and sold at 6*s.* 6*d.* per lb., what is the gain per cent.?

4 A trader, by selling an article for R51. 4*a.*, loses 18 per cent.; what is the prime cost?

5 Bought sugar at $6\frac{1}{2}a.$ per seer, and sold it at R18. 12*a.* per maund; what is the gain or loss per cent.?

6. Tea is bought at £30. 16*s.* per cwt., and sold at 5*s.* $11\frac{1}{2}d.$ per lb.; find the gain per cent.

7. If 4 lbs. of tea be sold for the sum of money that is paid for 5 lbs.; what per cent. do I make?

8. If 6 articles be sold for the same sum of money that was paid for 5, what is the loss per cent.?

9. A shop-keeper sells a book for £2. 1*s.*, and loses 18 per cent.; what was the cost price of the book?

10 By selling an article for R2. 8*a.*, a person loses 5 per cent. What must he sell it for, to gain $4\frac{1}{2}$ per cent.?

11. If I buy 2048 yards of linen at 3*s.* $2\frac{1}{2}d.$ per yard, and sell the whole for £358. 8*s.*; find the gain per cent.

12 A chair was bought for R33. 5*a.* 4*p*; the expenses were 5 per cent. upon the cost price, and the profit was 25 per cent. upon the whole outlay; find the price for which it was sold.

13. If by selling a horse for R1260, I lose 16 per cent.; how much per cent should I have gained or lost, had it been sold for R1680?

14. If I sell $\frac{2}{3}$ of a certain quantity of rice at $\frac{7}{8}$ of its cost, what is the profit per cent.?

15. I sold a farm for £5000 gaining thereby £500; what per cent. would have been lost, had it been sold for £4000?

16. A wine merchant buys 100 bottles of brandy for R318. 12*a.* and sells a dozen for R40. $12\frac{1}{8}a$: find his gain or loss per cent.

17 I sell a horse for R230 at a profit of 15 per cent.: what per cent. should I have gained had I sold it for R210?

18. If 8 per cent. be gained by selling 218 yds. of cloth for £92. 13*s.*: at what price per yard must it be sold, so as to gain 17 per cent.?

19. Bought 50 yds. of muslin at 3*s.* 4*d.* per yd., of which 15 yds. were sold at 4*s.* a yd., 20 yds. at 3*s* 10*d* a yd., and the rest at 3*s.* 8*d.* a yd.; how much per cent. was gained on the whole?

20. By selling an article for R45. 8*a.*, a trader loses 9 per cent.; at what price must he sell it, that he may gain 7 per cent.?

21. A person mixes 9 gallons of water with 63 gallons of gin which cost £23. 12*s.* 6*d.*: what does he gain per cent. by selling the mixture at 7*s.* 9*d.* per gallon?

22 If I buy 28 pieces of stuff at R5 a piece, and sell 12 of them at R6 a piece, and 8 at R5: at what rate per piece must I sell the rest to gain 20 per cent. on the outlay?

23. If 5 per cent. more be gained by selling a house for £5250 than by selling it for £5042, find cost price of the house.

24. A person having bought goods for R60 sells one-third of them at a loss of 10 per cent. : for how much must he sell the remainder so as to gain 15 per cent. on the whole?

25. A man bought a shawl and sold it at a loss of 10 per cent.; had he received R90 more, he would have gained $12\frac{1}{2}$ per cent. : what did the shawl cost him?

26. A shop-keeper bought a bale of silk at R200 a piece, and found that it contained 8 pieces of linen, which fetched only R12 8*a.* a piece. By selling the remaining pieces at R250 per piece, he, however, gained Rs.500 on the whole. Find the number of pieces in the bale.

***27.** By means of false scales, a shop-keeper defrauds both the man from whom he buys and the man to whom he sells, to the extent of 20 per cent. : find his whole percentage of gain.

28. A speculator invested his capital successively in four different speculations : in the first, he had his capital doubled, but in each of the others, he lost 20%: did he gain or lose, and how much per cent?

29. A gentleman buys an estate, and sells it when the price has risen 30 per cent. : the expenses of sale amounting to 5 per cent., he receives £6483. 15*s.* : what did the estate cost him?

30. If mangoes be bought at the rate of 5 a rupee, how many should be sold for R7 to gain 40 per cent.?

31. A trader bought 240 yds. of silk at R3. 15*a.* per yd., and sold one-fourth at R5. 2*a.*, one-third at R4. 4*a.*, and the rest at R3. 8*a.* per yd.: what was the gain or loss per cent. upon the whole outlay?

32. A man sells on credit R2400 worth of goods at a profit of 50 per cent. ; but he can realise only 10*a.* in the rupee. How much per cent. does he gain or lose by the transaction?

33. A farmer sells to *A* the produce of a farm which has cost him R300 at a profit of $12\frac{1}{2}$ per cent. : *A* sold it to *B* at a profit of 5 per cent. on what it cost him : what would the farmer have gained per cent., had he sold it directly to *B* for the price which *B* paid to *A* for it?

34. The prime cost of a 50-gal. cask of wine is R150, and 10 gallons are lost by leakage ; at what price per gal. must the remainder be sold so as to gain 10 per cent. on the original cost?

35. A person bought tea at R2. 8*a.* a sr. which he was obliged to sell at R2. 2*a.* a sr. : find his loss per cent. What would have been his gain per cent., if he had bought at the latter price and sold at the former?

36. A tea-merchant sells tea to a grocer at 19 per cent. profit ; the grocer sells to his customers at 25 per cent. profit. How much per cent. more than the prime cost do the customers pay?

37. A farmer bought a flock of sheep consisting of 560 heads at £2. 10*s.* a head. He had to rent a pasture at £15. 0*s.* 10*d.* a month, and engage a shepherd at 10*s.* a week. At what price per head should he sell them at the end of the year so as to realize all his expenses, as well as to make a profit of 10 per cent. on the cost price of the sheep?

38. A man bought 4 qr. of wheat at £1. 19*s.* per qr. and 16 bushels of a better quality at £2. 7*s.* per qr.; he mixed them, and sold the mixture at 6*s.* 3*d.* per bushel: how much per cent. did he gain?

39. A wine-merchant mixes 2 pipes of wine at R6. 4*a.* a gallon with 1 pipe at R9. a gallon; at what price must he sell the mixture per gallon that he may gain 25 per cent. on his outlay?

40 A wine-merchant buys two sorts of wine at R8. 4*a.* and R5. 4*a.* per gal. He mixes them in the proportion of 5 parts of the cheaper sort to 7 of the dearer sort. At what rate per gallon must he sell the mixture to make a profit of 25 per cent.?

41 If 80 gal. of spirit at R12. 8*a.* per gal. be mixed with 55 gal. at R13. 12*a.* per gal.; at what price per gal. must the mixture be sold to yield 35 per cent. on the outlay?

42. A wine-merchant mixes 8 gal. of gin. at R7. 8*a.* per gal. with 4 gal. of water and 2 gal. of base spirit worth R10: What per cent. will he gain on the money invested, if he sell the mixture at R1. 6*a.* 8*p.* per bottle, the bottles being six to the gallon.

309. **Division into Proportional Parts.** When a given quantity is divided into parts which are *proportional* to certain given numbers, it is said to be divided into **proportional parts.**

310. *To divide a given quantity into parts which shall be proportional* to certain given numbers.

RULE. (i) Divide the given quantity by the sum of the given numbers, and multiply the quotient separately by each number. The products so obtained are the respective parts.

(ii) The sum of the given numbers : any one of the given numbers :: the given quantity : the corresponding part.

Ex. 1. Divide *Rs.* 30 between *A* and *B*, so that their shares may be proportional to 6 and 4.

By the **Unitary Method.** $6+4=10$. *Rs.* $30 \div 10 =$ *Rs.* 3.

∴ *A*'s share = *Rs.* $3 \times 6 =$ *Rs.* 18, and *B*'s share = *Rs.* $3 \times 4 =$ *Rs.* 12.

By Proportion. $6+4=10$.

$$\left.\begin{array}{l} \therefore 10 : 6 :: Rs.\ 30 : A\text{'s share}; \ \therefore A\text{'s share} = Rs.\ 18; \\ \text{and } 10 : 4 :: \ ,,\ 30 : B\text{'s} \ ,, \ ; \ \therefore B\text{'s} \ ,, \ = \ ,, \ 12. \end{array}\right\}$$

Ex. 2. Divide 1064 oranges among *A*, *B*, and *C* in such a manner that their shares may be proportional to 2, $2\frac{1}{4}$, $2\frac{2}{5}$.

$$2 + 2\tfrac{1}{4} + 2\tfrac{2}{5} = \tfrac{133}{20}; \text{ also } 1064 \div \tfrac{133}{20} = 160.$$

$$\left.\begin{array}{l} \therefore A\text{'s share} = 160 \times 2 \quad = 320 \text{ oranges}; \\ \quad B\text{'s} \ \ ,, \ = 160 \times 2\tfrac{1}{4} = 360 \quad ,, \quad ; \\ \text{and } C\text{'s} \ ,, \ = 160 \times 2\tfrac{2}{5} = 384 \quad ,, \quad ; \end{array}\right\} \ Ans.$$

Ex. 3. Divide £1400 among *A*, *B*, *C*, in such a manner that as often as *A* gets £5, *B* shall get £4, and as often as *B* gets £3, *C* shall get £2.

When *B* gets £3, *C* gets £2 ; ∴ when *B* gets £1, *C* gets £$\frac{2}{3}$; ∴ when *B* gets £4, *C* gets £$\frac{8}{3}$; ∴ the shares are proportional to 5, 4, and $2\frac{2}{3}$.

$$5+4+2\tfrac{2}{3}=\tfrac{35}{3}\text{ ; also }1400\div\tfrac{35}{3}=120$$

$$\left.\begin{array}{lll} \therefore\ A\text{'s share} & = £120\times 5 & = £600\ ; \\ B\text{'s} \quad ,, & = ,,\ 120\times 4 & = ,,\ 480\ ; \\ \text{and } C\text{'s} \quad ,, & = ,,\ 120\times \tfrac{8}{3} & = ,,\ 320. \end{array}\right\}\ Ans$$

Ex. 4. Gunpowder is composed of 19 parts of nitre, $3\frac{1}{2}$ of charcoal, and $2\frac{1}{2}$ of sulphur ; find the quantity of nitre in 20 mds. of powder.

$$19+3\tfrac{1}{2}+2\tfrac{1}{2}=25.$$

∴ quantity of nitre $=\frac{20}{25}$ mnd. × 19 = 15 mds. 8 sr *Ans*

Ex. 5. There are 400 coins in guineas, half-sovereigns and half-crowns ; the values of the guineas, the half-sovereigns and the half-crowns are as 14 : 8 : 3 ; find the number of guineas.

Suppose the values to be all expressed in shillings.

The numbers of guineas, half-sovereigns, and half-crowns are as

$14\div 21 : 8\div 10 : 3\div 2\frac{1}{2}$, or as $\frac{2}{3} : \frac{4}{5} : \frac{6}{5}$.

Now $\frac{2}{3}+\frac{4}{5}+\frac{6}{5}=\frac{8}{3}$; also $400\div\frac{8}{3}=150$;

∴ no of guineas $=\frac{2}{3}\times 150=100$. *Ans.*

Ex. 6. The sum of Rs. 51 is made up of half-rupees, quarter-rupees and two-anna pieces, and the numbers of coins are in the proportion of 3, 7, 8 : find the respective number of half-rupees and quarter-rupees

The values of the groups of coins (half-rupees, quarter-rupees, and two-anna pieces) are as $3\times 8 : 7\times 4 : 8\times 2$, *i e*, as 6 : 7 : 4.

Now $6+7+4=17$; and *Rs.* $51\div 17=$ *Rs.* 3.

∴ the groups of coins are respectively worth *Rs.* 3×6, *Rs* 3×7 and *Rs.* 3×4, *i.e*, *Rs.* 18, *Rs.* 21, *Rs.* 12.

∴ there are 18×2 or 36 half-rupees and 21×4 or 84 quarter-rupees. *Ans.*

Examples 133.

1. Divide :

(1) R156 so that the parts may be proportional to 3, 4, 5

(2) £442. 15s. ,, ,, ,, 5, 7, 11.

(3) 7 mds. 6 sr. $10\frac{3}{4}$ ch. ,, ,, 10, $2\frac{1}{2}$, 1, $\frac{1}{3}$, $\frac{1}{3}$.

(4) 90 lbs. into parts proportional to 3·3, ·7, ·5.

2. In England, gunpowder is made of 75 parts of nitre, 10 of sulphur and 15 of charcoal ; find the weight of sulphur in 10 sr. of powder.

3. In France gunpowder is made of 77 parts of nitre, 9 of sulphur and 14 of charcoal ; how much powder can be formed with 36 lbs. of sulphur, and how much nitre will be required ?

4. English gunpowder is made of 75 parts nitre, 10 parts sulphur, and 15 parts charcoal, and French gunpowder is made of 77 parts nitre, 9 sulphur and 14 charcoal. If two tons of each be mixed together, find the weight of each ingredient in the compound.

5. In 50 parts of wheat-ash there are potash 7·5 parts, phosphate of lime 16, chloride of potassium 0·08, earthy phosphates 22·25, silica 0·25, and metallic oxides 0·125. Find the weight of each in 10 mds.

6. Brass is composed of 31·5 parts of copper and 15·5 of zinc: what weight of each metal ought to be taken so that 23 mds. 20 sr. of brass may be obtained?

7. *A*, *B*, *C*, and *D* rent a pasture for R520: *A* puts in 25, *B* 30, *C* 35, and *D* 40, cattle. What should each pay?

8 Divide R28. 10*a.* 4*p.* into two parts, so that one of them may be three-sevenths of the other.

9. A father left a farm to be divided among his three sons who were 20, 17, and 13 years of age, respectively. He directed that their shares should be proportional to their ages. If the eldest son's share be 277 bi. 17 kat. 8 ch., find the other shares, and the size of the whole farm.

10. Divide 103 tons 4 cwt. of coal among *A*, *B*, and *C*, so that for every lb. given to *A*, *B* may get 10 lbs., and *C* 10 lbs. 8 oz.

11. Divide R173. 5*a.* 4*p.* among *A*, *B*, and *C*, so that *A* gets three times as much as *B*, and *B* three times as much as *C*.

12. Divide 2800 boxes of tea among *A*, *B* and *C*, so that *A* may have half as many as *B*, and *C* as many as *A* and *B* together.

13. Find the area of a rectangular piece of ground whose perimeter is 880 yds., and whose length is $\frac{7}{3}$ of its breadth.

14. Brass is composed of 63 parts of copper and 31 of zinc: what quantity of brass contains 20 sr. more of copper than of zinc?

15. Pure water contains 88·9 parts oxygen and 11·1 hydrogen; find the weight of each in 10 cub. ft. of water, a cub. ft. of water weighing 31 sr. 4ch.?

16. Divide 8970 mds. 36 sr. of rice among 6 men, 5 women and 6 children, so that each man may get thrice and each woman twice the share of each child.

17. A certain sum is divided among *A*, *B*, *C*, so that *A* gets three times as much as *B*, and *B* three times as much as *C*; *A* received R40 more than *B*. Find the entire sum.

18. Divide 80000 tons into parts which should have the same ratio to one another as the cubes of the first four natural numbers.

19. The sides of a triangle are proportional to 5, 7, and 9; the smallest side is less than the greatest by 10 cubits: find the perimeter.

20. Divide 78400 mds. among *A*, *B*, *C*, and *D* so that *B*'s share : *A*'s share :: 1 : 2, *B*'s : *C*'s :: 3 : 4, and *C*'s : *D*'s :: 6 : 5.

***21.** 314 coins consist of guineas, sovereigns and crowns, their values being as 14 : 11 : 7. Find the number of each.

***22.** 213 coins consist of rupees, half-rupees and quarter-rupees, whose values are as 15 : 12 : 8. Find the number of each.

***23.** £2099 is made up of moidores, guineas and sovereigns; and the numbers of coins are proportional to the numbers 24, 31, 40. Find the number of coins of each kind.

311. When two or more people enter into partnership, they each subscribe a certain amount of money called the **Capital,** to work the business.

Fellowship or **Partnership** is the method of finding out the share of profit or loss belonging to each partner of a company, in proportion to his share of the joint capital.

312. Fellowship is either **Simple** or **Compound.**

In *Simple Fellowship,* the respective partners employ their capitals *for the same time; in Compound Fellowship,* for unequal times.

313. **Simple Fellowship.** In simple Fellowship, the profits arising at the end of any given time are usually divided among the partners in *proportion* to the *amount* of capital employed by each.

Ex. *A*, *B* and *C* start a business, *A* puts in *Rs.* 3000, *B* *Rs.* 5000, and *C* *Rs.* 7000; they gain *Rs.* 750: how should the profit be divided?

We have to divide *Rs.* 750 in the proportion of *Rs.* 3000, *Rs.* 5000, and *Rs.* 7000: or in the proportion of the numbers 3, 5, 7.

Now, $3+5+7=15$.

$$\left.\begin{array}{lll} \therefore & A\text{'s share} = \frac{3}{15} \text{ of } Rs.\ 750 = Rs.\ 150; \\ & B\text{'s} \quad ,, \ = \frac{5}{15} \text{ of } ,, \ 750 = \ ,, \ 250; \\ \text{and} & C\text{'s} \quad ,, \ = \frac{7}{15} \text{ of } ,, \ 750 = \ ,, \ 350. \end{array}\right\} \quad Ans.$$

314. **Compound Fellowship.** In Compound Fellowship, the profits are divided in *Proportion* to the *product* of the amount of capital subscribed by each and the period of time for which this capital has been employed.

Ex. 1. *A*, *B*, and *C* trade together; *A* puts in *Rs.* 3000, for 9 months, *B* *Rs.* 2400 for 6 months, and *C* *Rs.* 4000 for 5 months. They gain *Rs.*3070: how should the profit be divided?

$3000 \times 9 = 27000$; $2400 \times 6 = 14400$; $4000 \times 5 = 20000$. Also $(27000+14400+20000)=61400$.

$$\left.\begin{array}{lllll} 61400 : 27000 : & Rs.\ 3070 : A\text{'s share}; & \therefore A\text{'s share} = Rs.\ 1350; \\ 61400 : 14400 : & 3070 : B\text{'s} \quad ,, \ ; & ,, \ B\text{'s} \quad ,, \ = \ ,, \ 720; \\ 61400 : 20000 : & 3070 : C\text{'s} \quad ,, \ ; & ,, \ C\text{'s} \quad ,, \ = \ ,, \ 1000. \end{array}\right\} Ans.$$

Ex. 2. A gentleman distributed *Rs.* 820 amongst 20 men, 30 women and 15 children; for every 10*a.* given to a man, a woman was given 6*a.* and a child 2*a.*; at what rate were they paid?

$10 \times 20 = 200$; $6 \times 30 = 180$; and $2 \times 15 = 30$.

$200 + 180 + 30 = 410$.

410 : 200 :: *Rs.* 820 : 20 men's share ; ∴ 20 men's share = *Rs.* 400 ;
410 : 180 :: „ 820 : 30 women's „ ; ∴ 30 women's „ = „ 360 ;
410 : 30 :: „ 820 : 15 children's „ ; ∴ 15 children's „ = „ 60.

∴ a man gets *Rs.* 20 ; a woman, *Rs.* 12 ; and a child *Rs.* 4. *Ans.*

Ex. 3. A man with a capital of £2261, after $2\frac{1}{2}$ month's trading, takes in a partner with £1680. At the end of 6 months from the beginning, the profits are £762. 0s. 5*d.* How should they be divided ?

The first man uses £2261, for 6 months and the second £1680 for $(6-2\frac{1}{2})$ or $3\frac{1}{2}$ months. Therefore their profits should be as 2261×6 and $1680 \times \frac{7}{2}$, *i.e.*, as 323 and 140. Now $323 + 140 = 463$.

∴ 1st man's profits $= \frac{323}{463} \times$ £762 0s. 5*d.* = £531. 12s. 1*d.* ;
and 2nd man's „ $= \frac{140}{463} \times$ „ 762. 0s. 5*d.* = „ 230. 8s. 4*d.* } *Ans.*

Examples 134.

1. There are two partners in a business ; the one puts in R5460, and the other R4290 , they gain R1950. Divide the profit between them.

2. *A* and *B* enter into partnership ; *A* supplied R8400 and *B*, R10800 ; what is *A*'s share in a loss of R4322 ?

3. *A*, *B*, and *C* start a business ; *A* furnishes £5000, *B* £2500, and *C* £7500 ; the gain is £600. Divide the profit fairly amongst them.

4. *A*, *B*, *C* and *D* are four partners in a mercantile business ; their shares are as 3*s.*, 4*s.*, 6*s.*, 7*s.*, respectively. How should they divide a profit of £2610 amongst them ?

5. *A*, *B*, and *C* freighted a ship with 5620 maunds of rice, of which *A* supplied 1260 mds., *B* 2700 mds. and *C* the rest. In a storm 702 mds. 20 sr. were thrown overboard. How should the loss be divided ?

6. *A*, *B*, and *C* hire a pasture for £160 : *A* puts in 96, *B* 104, and *C* 120 sheep ; what part of the rent should each pay ?

7. *A*, *B*, and *C* are three partners in a mercantile business ; *A*'s share of the profit is $\frac{6}{13}$, *B*'s $\frac{3}{10}$, and *C*'s the rest. If *A* puts in R455 more than *C*, find the capital invested by each.

***8.** *A*, *B*, and *C* enter into partnership, *A*'s share of the profits being $\frac{1}{13}$, and *B* and *C* sharing the remainder equally between them. If the profits rise from $\frac{1}{15}$ of the whole capital to $\frac{1}{14}$, and *A* receives in consequence R80 more, what are the respectable capitals ?

9. In a joint-stock company *A* puts in R7600 for 5 months, *B* R4200 for 9 months and *C* R5200 for 11 months. Divide a profit of R3990 amongst them.

10. *A*, *B*, *C* and *D* enter into partnership. On Jan. 1st *A* puts in R1200, on April 1st *B* puts in R1500, on July 1st *C* puts in R1800, and on October 1st *D* puts in R2100. How should a profit of R900 be divided among them at the end of the year ?

11. A, B, and C rent a pasture for R660. A puts in 8 oxen for 10 months, B 7 oxen for 12 months, and C 20 oxen for 5 months. How should the rent be divided ?

12. A puts in R6000 in a mercantile business. B three months later R3000. After three months more, they again put in R3000 each. If they gain R3750 in a year and a half, how must they share it ?

13. A, B and C undertook to do a piece of work for R132. 4a.; C did $\frac{1}{2}$ of the work, and the work done by A is to that done by B as 10 : 13 ; what should A and B each receive ?

14. Two persons A and B agree to work a business for a year. They respectively contribute R5000 and R3000, after 4 mo A again contributes R3000, and after 9 mo. B withdraws R2500. At the end of the year, the profits are R4600. What part of the profits should each get ?

15. A, B, and C are three merchants trading together ; A gets a third of the whole profit, B $\frac{1}{4}$ and C the rest, which is R5900 less than what A and B together receive ; find the total gain.

16. A and B start a mercantile concern ; A at first puts in £5000, then at the end of a year another £5000, and again at the end of the second year £45000 ; B at first contributes £8000, at the end of a year and a half adds £5000, but after another six months withdraws £5000. Six months after the commencement of the business, C enters with a capital of £10000. After 3 years from the commencement, they divide a profit of £40216. 10s. between them ; find the profit obtained by each.

***17.** £76. 8s. is paid for a piece of work in which a certain number of men, women, and boys are engaged ; for every 10s. a man gets, a woman gets 2s. 6d. ; and a boy 3d. ; and the numbers of men, women, and boys are as 4, 6, 11. Find the number of each.

18. A, B, C rent a pasture for R16 8a. ; A puts 40 sheep for 14 mo., B 25 oxen for 8 mo., and C 12 horses for 10 mo. If 2 horses can eat as much as 3 oxen, and 2 oxen as much as 7 sheep, how should the rent be divided among them ?

315. **Bankruptcy.** When a tradesman becomes **insolvent,** *i. e.*, cannot pay his debts, he becomes a **Bankrupt.** The amount of his debts is called his **Liabilities,** and the amount of his property is called his **Assets.** The persons to whom he owes his debts are called his **Creditors.**

When the affairs of the bankrupt are settled, *i. e.*, during **Liquidation,** each creditor receives out of his assets a sum which is proportional to the amount owed to him.

The moneys which other people owe to the bankrupt are called **Book-debts** ; they are, of course, a part of the assets.

Ex. 1. A bankrupt owes *Rs.* 2760, and his assets amount to *Rs.* 1840 ; what can he pay in the rupee ?

On 2760 rupees he can pay 1840 rupees ;

$\therefore$ on one rupee he can pay *Rs.* $\frac{1840}{2760}$ = 10a. 8p. *Ans.*

Ex. 2. A bankrupt owes £5764 and pays $17\frac{1}{2}s.$ per £; find his assets.

He can pay $17\frac{1}{2}s.$ or £$\frac{7}{8}$ on £1;

∴ on £5764 he pays £$5764\times\frac{7}{8}$ = £5043. 10*s.* *Ans.*

Ex. 3. The assets of a bankrupt are *Rs.* 1540 and he can pay 11*a.* in the rupee; find the amount of his debts.

He can pay *Rs.* $\frac{11}{16}$ on every rupee of the debts

$\frac{11}{16}$: 1540 :: *Re.* 1. : reqd. debts; ∴ reqd. debts = *Rs.* 2240. *Ans.*

Ex. 4. A bankrupt can pay 5*a.* 4*p.* in the rupee, had he *Rs.* 3500 more he could have paid 8*a.* in the rupee. Find his debts and assets.

8*a.* − 5*a.* 4*p.* = 2*a.* 8*p* = *Re* $\frac{1}{6}$;

∴ he could have paid *Re.* $\frac{1}{6}$ more on each rupee of his debts.

∴ $\frac{1}{6}$: 3500 :: *Re.* 1 : his debts; ∴ his debt = *Rs.* 21000; } *Ans.*
also his assets = 5*a.* 4*p.* × 21000 = „ 7000. }

Ex. 5. A bankrupt pays £ 570 on his whole liabilities, at the rate of 13*s.* in the pound on half his debts and 17*s.* in the pound on the other half; find the amount of his debts.

He pays $\frac{13}{2}s.$ in 10*s.* and $\frac{17}{2}s.$ in 10*s.*; *i.e.* $(\frac{13}{2}+\frac{17}{2})s.$ or £$\frac{3}{4}$ in £1.

∴ $\frac{3}{4}$: 570 :: £1 : his debts; ∴ his debts = £760. *Ans.*

Ex. 6. A bankrupt owes £10000 to three creditors *A*, *B*, and *C*; to *A* he owes £2000, to *B* £3000 and to *C* the rest. If his assets amount to £2500, how much can he pay in the pound, and what loss does each creditor sustain?

He owes to *C* £10000 − £(2000 + 3000) = £5000.

He can pay £$\frac{2500}{10000}$ or £$\frac{1}{4}$ in the pound; ∴ his creditors lose £$\frac{3}{4}$.

∴ *A* loses $\frac{3}{4}$ × £2000 = £1500; }
B „ $\frac{3}{4}$ × „ 3000 = „ 2250; } *Ans.*
and *C* „ $\frac{3}{4}$ × „ 5000 = „ 3750. }

Ex. 7. A bankrupt has book-debts equal in amount to his liabilities; but on £6000 of them he can recover only 6*s.* 8*d.* in the £ and the expenses of bankruptcy are £5 for every £100 of the book-debts; if he pays 11*s.* in the £, what is the amount of his liabilities?

As he can recover 6*s.* 8*d.* or £$\frac{1}{3}$ in the £, he recovers £$\frac{1}{3}$ × 6000 or £2000 out of the £6000; therefore his *loss* amounts to £4000.

Again, he pays £5 for £100, or 1*s.* in the £ for expenses. Therefore he recovers $(11+1)s.$ in the £, and his loss per £ = 8*s.* or £$\frac{2}{5}$.

∴ $\frac{2}{5}$: 4000 :: £1 : *x* (amount of liabilities); ∴ *x* = £10000.

Examples 135.

1. A bankrupt's estates amount to R10593 and his debts to R14124; how much can he pay in the rupee?

2. The assets of a bankrupt are £2464. 16*s.*, and he can pay 13*s.* 4*d.* in the pound; find the amount of his debts.

3. A bankrupt's debts are R7621. 8*a*., and he can pay 10*a* in the rupee; what do his assets amount to?

4. A bankrupt's effects amount to £4560 and his debts to £5700; what loss does a creditor sustain to whom he owes £2640?

5. A bankrupt owes his creditors £7649. and pays them 14*s*. 6¾*d*. in the pound. How much do they receive altogether?

6. A bankrupt owes R6400 to *A*, R7200 to *B*, and R8000 to *C*; and his assets are R18900: how much can he pay in the rupee, and what loss will each of his creditors sustain?

7. A bankrupt failed to the amount of R16400 and paid 12*a*. 6*p*. in the rupee; find his assets, and the loss of a creditor to whom he owed R7500.

8. A bankrupt owes R45600 to three creditors, and his whole property amounts to R30000; two of the creditors who claim R20000 and R16000 respectively accept 10*a*. in the rupee; how much in the rupee can the third creditor realise?

9. The assets of a bankrupt are £3035, and he makes a composition of 16*s*. 8*d*. in the pound. Find the amount of his liabilities.

10. A bankrupt's assets amount to R32760 and his debts to R46800. What can he pay in the rupee, and what will a creditor lose on a debt of R17615?

***11** A bankrupt has book-debts equal in amount to his liabilities; but on £5000, £7500 and £6000 of them he can recover only 15*s*., 9*s* and 6*s*. in the pound respectively; the expenses of bankruptcy are 5 per cent. on the book-debts. If he pays 13*s*. in the pound, find the amount of his liabilities.

12. A creditor first gets a dividend of 6*s*. 8*d*. in the pound, and then a further dividend upon the deficiency of 13*s*. 4*d*. in the pound. What does he get in the pound altogether?

13. A bankrupt can pay 10*a*. in the rupee; had he R3150 more, he could have paid 13*a*. in the rupee. Find his debts and assets.

14. A bankrupt can pay 13*s*. 4*d*. in the £: if his assets were £1200 more, he could pay 16*s*. in the £ Find his debts and assets.

15. A bankrupt's assets are R616. Out of this he pays 12*a*. in the rupee on half his debts and 10*a* in the rupee on the other half What is the amount of his debts?

***16**. A bankrupt's assets are equal to his liabilities, but on £7560 of his book-debts he can recover only 14*s*. in the £; the expenses of bankruptcy are 5 per cent of the liabilities: if he pay 16*s*. 6*d*. in the £, what is the amount of his liabilities?

17. A bankrupt owes R36740 of which R8460 are due to *A*, R7280 to *B*, R1000 to *C*, and the rest to *D*. How much does he pay in the rupee when *D*'s receipts amount to what *C*'s should be; and how much must each of the other creditors lose?

316. **Average.** The **average** or **mean** of two or more numbers is found by dividing the *sum* of the given numbers by the *number* of those given numbers.

To find the average of 12, 20, 33, 45, 57 and 60, we first find the sum of the numbers, which is 227; and as the number of the given numbers is 6, we find the average to be $227 \div 6$ or $37\frac{5}{6}$.

Examples 136.

1. Find the average value of the following numbers :

(1) 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.

(2) 14, 18, 24, 50, 68, 100. (3) $3\frac{3}{4}$, $3\frac{1}{8}$, $4\frac{3}{8}$, $4\frac{2}{3}$, $1\frac{1}{10}$.

(4) 15·5, 36·75, 17·625, 10·375, 74·2, 28·25, 33.

2. The populations of 3 towns are 119503, 56781, and 9129 : what is the average population ?

3 A man runs 51 yds in the first minute, 54 yds. in the next 60 yds. in the third, 64 yds. in the fourth and 76 yds. in the fifth minute : find the average distance that he runs over per minute.

4. A train travels $\frac{1}{2}$ mi a min. in the first 5 min, $\frac{3}{4}$ mi. a min. in the next 5 min., 1 mi a min. in the next, $1\frac{1}{4}$ mi. a min. in the next, $1\frac{1}{2}$ mi. a min. in the next, and 2 mi a min. in the last 5 min. ; find the average speed per hour.

5 The weights of a boat's crew are respectively 8 st. 12 lbs., 9 st. 7 lbs., 9 st., 10 st 2 lbs, 11 st, 11 st. 5 lbs., 12 st and 8 st. 2 lbs. : what is the average weight of the crew ?

6. The average weight of the crew in the above Example is increased by $2\frac{1}{3}$ lbs., by including the weight of the cox-swain ; find his weight.

***7.** The average weight of a crew of 8 men is increased by 3 lbs. when one of the crew weighing 10 st. 4 lbs. is replaced by a new man ; what is the weight of this new man ?

8. A zamindar lets 500 bighas of land at R3 per bigha, 400 bighas at R4 per bigha and 300 bighas at R5 per bigha. Find the average rent per bigha.

9. A cricketer plays 18 innings in 1890, 23 in 1891, and 19 in 1892 ; his batting averages in the three years are 22·7, 21·4 and 32·1, respectively. Find his average for the three years.

10. A bowler during 4 seasons takes 101, 84, 93, and 122 wickets at an average cost of 8·7, 11·4, 13·5 and 12·8 wickets respectively ; find his average for the four seasons.

11 The population of five villages are 2406, 4554, 8262, 9018, and 5238 respectively ; what is the population of a sixth village, if the average population of the sixth is 6215 ?

12. The average dividend paid during 8 years by a railway company is 4·75 per cent. The dividends paid in the first 7 years were respectively 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, and 6 per cent. What was the dividend in the eighth year ?

13. The populations of 3 towns in 1874 were 356850, 328625, and 728140 respectively. By 1884, the first two had increased by 6 per cent. and 8 per cent. respectively, and the third diminished by 10 per cent.; find the average population in 1884.

14. The average age of A, B and C is 25 years; that of B, C and D is 28 years: if the age of D is 21 years, find that of A.

***15.** The average age of 14 boys is diminished 3 months when one of them who is 16 years old is replaced by a new boy: find the age of the new boy.

16. The average temperature for Monday, Tuesday and Wednesday was $59°$: that for Tuesday, Wednesday and Thursday was $66°$; if the temperature on Thursday was $68°$, what was it on Monday?

17. A wine-merchant buys 50 doz. of wine at R21 per doz., 40 doz. at R10. 8*a*. per doz., and 36 doz. at R15. 12*a*. per doz.; find the average cost per doz. If the 50 doz. of wine be sold at a profit of 10 per cent., the 40 doz. at a profit of 20 per cent., and the 36 doz. at a profit of 25 per cent.; find the average selling price per doz.

317. **Equation of payments.** When a person owes another several sums of money payable at different times, we may determine the time, called the **equated time,** in which all these debts may be cleared off at once by a single payment, neither the creditor nor the debtor losing thereby. The RULE by which we determine this *equated time*, is called the **Equation of payments.**

318. *To find the equated time of different payments.*

RULE. Multiply each debt by the time after which it becomes due, and divide the sum of the products by the sum of the debts: the quotient will give the required time.

Ex. 1. A owes B £3000 payable after 3 mo., £4000 after 4 mo., and £5000 after 6 mo. When may the three debts be paid together?

$$\text{Reqd. time in months} = \frac{3000 \times 3 + 4000 \times 4 + 5000 \times 6}{3000 + 4000 + 5000} = 4\tfrac{7}{12}. \quad \textit{Ans.}$$

Ex. 2. *Rs.* 10000 is due from A to B payable at the end of 9 mo. A pays however *Rs.* 2000 at the end of 3 mo., and *Rs.* 3000 at the end of 8 mo.; when will the remainder be due?

$$\text{The remainder} = Rs.\ (10000 - 2000 - 3000) = Rs.\ 5000.$$

$$\therefore \text{ reqd. time in mo.} = \frac{10000 \times 9 - (2000 \times 3 + 3000 \times 8)}{5000} = 12. \quad \textit{Ans.}$$

Examples 137.

1. If £1750 be due 37 days hence; £2100, 40 days hence; and £2700, 65 days hence; find the equated time of payment.

2. R7000 is due in 3 mo., R8000 in 5 mo., R5000 in 10 mo.; when may the three debts be paid off by a single payment?

3. A person owes to another two different sums of money payable at different times; one being £865 to be paid after 120 days, and the other £423. 6*s*. 8*d*. to be paid after 288 days; find the time when both the debts may be paid together.

4. On the first of January 1891, *A* finds that he owes to *B* the following debts; *viz*., £240 payable on the 3rd February, £140 on the 17th March, £250 on the 5th May, and £255 on the 18th August. If he pay off the whole debt by a single payment, when will it become due?

5. *A* owes *B* R1000 to be paid at the end of $4\frac{1}{3}$ mo.; he pays R400 after 3 mo.; when will the remaining sum be due?

6. *A* owes *B* £2000 to be paid after $5\frac{1}{2}\frac{1}{10}$ mo.: he pays however £700 at the end of 3 mo. and £800 at the end of 5 mo.; when ought the remainder to be paid?

7. On the 1st of March *A* owes *B* £640 to be paid on the 28th May: he pays £120 on the 14th of May, and £380 on the 15th of June; on what date may he pay the remainder?

8. *A* owes *B* £1500 due $8\frac{1}{3}$ mo. hence, and pays one-third of it at the end of 7 mo.; when ought the remainder to be paid?

319. **Alligation** teaches how to mix several simples of different qualities together, so that the mixture may be of some intermediate quality.

320. Alligation is of two kinds, **Medial** and **Alternate.**

Alligation *Medial* is that in which the rates and qualities of the simples being given, it is required to find the rate of the mixture. See Art. 122.

Alligation *Alternate* is the converse of Alligation Medial, and is the method of finding from the given rates or prices of the simples and the rate of the mixture, the quantity of each of the simples composing the mixture.

Ex. 1. How must a shop-keeper mix one sort of rice at *Rs*.4. 8*a*. with another at *Rs*. 4. 12*a*. a md., to make a mixture worth *Rs*. 4. 9*a*. a md.?

For every maund of the cheaper rice that the shop-keeper puts in, he gains (*Rs*. 4. 9*a*. − *Rs*. 4. 8*a*.) or 1*a*.; for every maund of the better rice that he puts in, he loses (*Rs*. 4. 12*a*. − *Rs*. 4. 9*a*.) or 3*a*. Therefore his loss on 1 md. of the second sort = his gain on 3 mds. of the first sort.

Hence for every (3 + 1) or 4 mds. of the mixture, he must put in 3 mds. of the first sort and 1 md. of the second.

Ex. 2. How should a wine merchant mix wines at 15*s*. and 20*s*. a gallon, in order to make a mixture of 30 gallons at 17*s*. a gallon?

On every gallon of wine at 15*s*. he *gains* (17*s*. − 15*s*.) or 2*s*.; and on every gallon of wine at 20*s*. he *loses* (20*s*. − 17*s*.) or 3*s*.

∴ he must put in 3 gal. of the first kind for every 2 gal. of the second. Now 3 + 2 = 5, and 30 ÷ 5 = 6; ∴ in 30 gal. of the mixture there should be 3 × 6 or 18 gal. of the first sort and 2 × 6 or 12 gal. of the second.

Ex. 3. A shopkeeper buys two kinds of sugar, one at $3a.\ 6p.$ per seer and the other at $5a.\ 3p.$ per seer; how should he mix them, so that he may gain $3p.$ on every seer by selling the mixture at $5a.$ per seer?

To sell at $5a.$ per sr. gaining thereby $3p.$ per sr., he must make a mixture costing $4a.\ 9p.$ a sr. Now $4a.\ 9p. - 3a.\ 6p. = 15p$, and $5a.\ 3p. - 4a.\ 9p. = 6p.$ Therefore he *gains* $15p$ on every seer of the first kind and *loses* $6p.$ on every seer of the second. Hence he must mix 6 sr. of the first with 15 sr. of the second, or 2 sr of the first with 5 sr. of the second.

Ex. 4 Three kinds of tea at *Re.* 1, *Re.* 1. $5a.\ 4p$, *Re.* 1. $12a$ per lb. respectively, are mixed together; if the price of the mixture is *Re.* 1. $8a.$ per lb., how many pounds of each kind have been taken?

The *gain* on the first per lb is $8a.$; the *gain* on the second per lb is $2\frac{2}{3}a.$; and the *loss* on the third per lb is $4a.$

The *gain* on 1 lb of the first = the *loss* on 2 lbs. of the third, and the *gain* on 3 lbs. of the second = the *loss* on 2 lbs of the third.

Therefore with 1 lb. of the first and 3 lbs. of the second are mixed 4 lbs. of the third; *i e.*, they have been taken in the proportion of $1 : 3 : 4$

Note. When there are more than 2 kinds, the simples may be mixed in more proportions than one Thus, in the above case, they may also be mixed in the proportion of $2 : 3 : 6$.

Ex. 5. In what proportion must wines of *Rs.* 7. $8a$, *Rs.* 10, *Rs* 13 and *Rs.* 15 a gal. respectively be mixed, so that the mixture may be sold at *Rs.* 12 a gal. with a profit of $14\frac{2}{7}$ per cent.?

Rs. $114\frac{2}{7}$: *Rs.* 12 :: *Rs.* 100 : cost price of the mixture per gallon;

$\therefore$ cost price of the mixture per gallon = *Rs* 10. $8a.$

The *gain* on the first sort per gallon = *Rs* 3; the *gain* on the second per gal. = $8a.$; the *loss* on the third per gal = *Rs* 2. $8a.$; and the *loss on* the fourth per gal = *Rs* 4. $8a.$ Thus the *gain* on three gal of the first = the *loss* on 2 gal. of the fourth, and the *gain* on 5 gal of the second = the *loss* on 1 gal. of the third.

$\therefore$ with 3 gal. of the first should be mixed 2 gal. of the fourth, 5 gal. of the second, and 1 gal. of the third; *i.e.*, the reqd proportion = $3 : 5 : 1 : 2$.

Otherwise :

The *gain* on 5 gal. of the 1st. = the *loss* on 6 gal. of the 3rd, and the *gain* on 9 gal. of the 2nd. = the *loss* on 1 gal. of the 4th.

$\therefore$ they may also be mixed in the proportion of $5 : 9 : 6 : 1$.

Examples 138.

1. A grocer mixes sugar at $4a.\ 2ps.$ per sr. with sugar at $5a.\ 2ps.$ per sr., to make a mixture worth $5a.\ 1ps.$ per sr. How must he mix them?

2. How must a grocer mix coffee at $2s.\ 3d.$ per lb. with chicory at $10d.$ per lb. so that the mixture may be worth $1s.\ 8d.$ per lb.?

3. A wine-merchant buys two sorts of wine, the first sort at $22s.$ a gal. and the other at $27s.$ a gal.; how should he mix them that a gal. of the mixture may be worth $24s.$ per gallon?

4. How should a tobacconist mix tobaccoes at 3*a.* 6*p.* and 5*a.* 4*p.* respectively per seer, to get a mixture of 66 sr. at 4*a.* 3*p.* per seer ?

5. How must a shopkeeper mix rice at R4. 12*a.* and R3. 15*a.* per md. respectively, to get a mixture of 65 mds. at R4. 3*a.* per md. ?

6. A wine-merchant buys two sorts of wine at 18*s.* and 25*s.* respectively per gallon. How must he mix them so as to make 112 gallons at 22*s.* per gallon ?

7. A *mudi* mixes ghee at 15*a.* per seer, with oil at 7*a.* 6*p.* per seer, and sells 7 mds. 2 sr. with a profit of R18. 12*a.* at 10*a.* per seer. How does he mix them ?

8. A goldsmith mixed gold at R23 per tola with silver at R1 per tola and by selling the mixed metal at R20 per tola gained a rupee on every tola sold ; how did he mix them ?

9. A goldsmith buys gold and silver at R20 and R1 per tola respectively. How must he mix them to get a mixture of 38 tolas and to gain R76 by selling them at R17 per tola ?

10. A *mudi* makes a mixture of 76 mds. with ghee and oil at R32 and R13 per maund, respectively. How must he mix them so as to gain R3 per maund, by selling the mixture at R27 per maund ?

***11.** A wine-merchant buys three sorts of wine at 15*s.*, 25*s.*, and 35*s.* a gallon respectively. If after mixing them he can sell the mixture at 30*s.* per gallon, in what proportion does he mix them ?

***12.** A grocer buys four sorts of sugar at 4*d.*, 6*d.*, 9*d.* and 11*d.* per lb. respectively, and makes a mixture worth $7\frac{1}{2}$*d.* per lb. ; how does he mix them ?

13. A wine-merchant buys some spirits at 13*s.* a gallon and some of a superior quality at 16*s.* a gallon : how should he mix them so as to gain 25 per cent by selling the mixture at 19*s.* 2*d.* per gallon ?

14. A *mudi* buys ghee at the rate of R34 per md. and oil at R10 per md. ; in what proportion must he mix them that by selling the mixture at R36 2*a.* 8*p.* per md. he may make a profit of $16\frac{2}{3}$ per cent. ?

***15.** A grocer buys 40 sr. of tea of two different kinds for R95. 10*a.* What quantity of each did he buy, the prices of the two kinds being R2. 8*a.* and R2. 4*a.* per seer respectively ?

16. A wine-merchant sells 80 gallons of a mixture of two different sorts of wine for £84, gaining thereby 20 per cent. on his outlay. The two sorts of wine respectively cost £1. 4*s.* and 16*s.* per gallon ; how was the mixture formed ?

Problems worked out.

Ex. 1. A woman buys some eggs at 3 an anna and also the same number at 5 an anna She retails them at 3*p.* each : does she gain or lose, and how much per cent. ?

Cost price of one each of the two kinds is $\frac{1}{3}a.$ and $\frac{1}{5}a.$ respectively ;
∴ average cost of an egg $= \frac{1}{2}(\frac{1}{3}+\frac{1}{5})a. = \frac{1}{2}\times\frac{8}{15} = \frac{4}{15}a.$;
and selling price " " $= \frac{1}{4}a.$

$\therefore$ loss on 1 egg, *i.e.* on $\frac{4}{15}a. = (\frac{4}{15} - \frac{1}{4})a. = \frac{1}{60}a.$

$\therefore$ $\frac{4}{15} : 100 :: \frac{1}{60}$: percentage reqd.;

$\therefore$ percentage reqd. $= \dfrac{100 \times \frac{1}{60}}{\frac{4}{15}} = \dfrac{100 \times 15}{4 \times 60} = 6\frac{1}{4}$. *Ans.*

Ex. 2. A labourer was engaged for 36 days on the condition that for every day he worked he should receive 2*s.* 6*d.* and for every day he was absent he should be fined 1*s.* 6*d.* At the end of the time, he received only £2. 18*s.*; how many days was he absent?

Had he worked all the 36 days, he would have got 2*s.* 6*d.* × 36 = £4. 10*s.*;

$\therefore$ he lost through absence, £4. 10*s.* − £2. 18*s.* = 32*s.*

But for every day of absence he *actually* loses, (2*s.* 6*d.* + 1*s.* 6*d.*) = 4*s.*;

$\therefore$ no. of days he was absent = 32 ÷ 4 = 8. *Ans.*

Ex. 3. The sum of the ages of 3 brothers is 105 years. Ten years before, their ages were in the proportion of 6 : 5 : 4; find their ages.

Ten years back, *each* brother was 10 years younger, and therefore the sum of their ages was 30 years less, *i.e.*, was (105 − 30) or 75 years. Also 6 + 5 + 4 = 15.

$\therefore$ the age of the eldest son was then $\frac{6}{15} \times 75$ yrs. = 30 yrs.; of the second son, $\frac{5}{15} \times 75$ yrs. = 25 yrs.; of the third son, $\frac{4}{15} \times 75$ yrs. = 20 yrs.

$\therefore$ the present age of the eldest son = (30 + 10) yrs = 40 yrs.;
" " " " second " = (25 + 10) " = 35 " ;
" " " " third " = (20 + 10) " = 30 " . } *Ans.*

Ex. 4. If 14 horses and 20 oxen cost *Rs.* 2800, and 21 horses and 13 oxen cost *Rs.* 3605, find the price of a horse and of an ox.

14 horses & 20 oxen cost *Rs.* 2800; $\therefore$ 42 horses & 60 oxen cost *Rs.* 8400;
21 " 13 " " " 3605; $\therefore$ 42 " 26 " " " 7210.

$\therefore$ 34 oxen cost *Rs.* (8400 − 7210) = *Rs* 1190, or 1 ox costs *Rs* 35;
and 14 horses " " (2800 − 700) = " 2100, or 1 horse " " 150. } *Ans.*

Ex. 5. If 6 men and 10 boys can do $\frac{7}{8}$ of a piece of work in 6 days, and 4 men and 13 boys can do $\frac{3}{5}$ of the same work in 4 days: in what time can each do it separately?

	6 men and	10 boys can do	$\frac{7}{8}$ of the work in	6 days;	
$\therefore$	6 " "	10 " "	$\frac{7}{48}$ " "	1 "	
$\therefore$	12 " "	20 " "	$\frac{7}{24}$ " "	1 "	
Again	4 " "	13 " "	$\frac{3}{5}$ " "	4 "	
$\therefore$	4 " "	13 " "	$\frac{3}{20}$ " "	1 "	
$\therefore$	12 " "	39 " "	$\frac{9}{20}$ " "	1 "	

$\therefore$ 19 boys can do $(\frac{9}{20} - \frac{7}{24})$ or $\frac{19}{120}$ " " 1 "

or 1 boy can do the work in $(19 \div \frac{19}{120})$ or 120 days. *Ans.*

Again 10 boys can do $\frac{1}{12}$ of the work in 1 day;

$\therefore$ 6 men can do $(\frac{7}{48} - \frac{1}{12})$ or $\frac{1}{16}$ of the work in 1 day;

$\therefore$ 1 man " the work in $(6 \div \frac{1}{16})$ or 96 days. *Ans.*

Ex. 6. Two passengers going to the same place have 8 mds. of luggage between them and are charged *Rs.* 3. 4*a.* for the same; if the whole luggage had belonged to only one of them, the charge would have been *Rs.* 3. 10*a.*; find the quantity of luggage allowed free, as well as the charge per maund.

When some luggage belongs to one man, a certain weight is allowed free; but when it belongs to two men, twice that weight is allowed free.

∴ charge on 8 mds. = *Rs.* 3. 10*a.* + charge on the free allowance,

and „ „ „ = „ 3. 4*a.* + *twice* „ „ „ „ „ ;

∴ charge on free allowance = *Rs.* 3. 10*a.* − *Rs.* 3. 4*a.* = 6*a.*

∴ *Rs.* 3. 10*a.* + 6*a.* = *Rs* 4 = proper charge on 8 mds.;

∴ proper charge per maund = *Rs.* 4 ÷ 8 = *Re.* $\frac{1}{2}$ = 8*a.* *Ans.*

Again, the charge on 1 md. = 8*a.* and that on the free allowance = 6*a.*

∴ 8*a.* : 6*a.* :: 1 md. : the free allowance;

∴ the free allowance = $\frac{6}{8}$ md. = 30 sr. *Ans.*

Ex. 7. A person starts business with a certain sum of money, and doubles his capital every year, his establishment charges which remain the same having to be met at the end of each year. At the end of 4 years he finds he has nothing left. Find the proportion of his original capital to the establishment charges.

Suppose his establishment charges to be denoted by 1.

At the end of the fourth year, his capital was 1;

∴ „ „ beginning „ „ „ „ $\frac{1}{2}$

„ „ end of the third „ „ „ $(1+\frac{1}{2})$ or $\frac{3}{2}$;

∴ „ „ beginning „ „ „ „ $\frac{3}{4}$.

At the end of the second „ „ „ $(1+\frac{3}{4})$ or $\frac{7}{4}$;

∴ „ „ beginning „ „ „ „ $\frac{7}{8}$.

At the end of the first „ „ „ $(1+\frac{7}{8})$ or $\frac{15}{8}$;

∴ „ „ beginning „ „ „ „ $\frac{15}{16}$.

∴ his original capital : the establishment charges :: 15 : 16. *Ans.*

Ex. 8. A ship is victualled for 30 days. After 20 days a storm arises and 6 men are washed overboard; two days later on, a wreck is sighted and 18 men are taken in. The men being put on five-sixth rations, the provisions just suffice for the journey: find the number of the crew.

After 20 days there are left provisions for the entire crew for 10 days.

After 2 days more „ „ „ „ 8 „

together with provisions of 6 men for 2 days, or of 12 men for 1 day.

During these 8 days, they have to provide for (18−6) or 12 additional men on $\frac{5}{6}$ rations: or for 10 men on full rations; *i.e.*, they have to provide for 80 men on full rations for one day; but they have saved provisions of 12 men for 1 day; hence they have to find 68 men's provisions for 1 day or of $8\frac{1}{2}$ men for 8 days. Now they do this by putting the crew on $\frac{5}{6}$ rations; hence $\frac{1}{6}$th of the crew's provisions for 8 days = provisions for $8\frac{1}{2}$ men for 8 days, or $\frac{1}{6}$ of the crew = $8\frac{1}{2}$ men, or the crew = 51 men.

MISCELLANEOUS EXAMPLES VI.

1. How long will a person take to walk 10 mi. if he takes 100 steps of $2\frac{3}{4}$ ft. every minute?

2. A rupee is worth 1*s.* $5\frac{1}{2}$*d.*; find the least number of rupees which will make an exact number of guineas.

3. A bill of £4. 3*s.* $1\frac{1}{2}$*d.* has to be paid by several persons in equal shares; if nine of them pay £1. 4*s.* $11\frac{1}{2}$*d*, how many were they?

4. Add the difference between ·0175 of a ton and ·32 of a cwt. to twice the difference between ·0675 of a qr. and ·195 of a lb and give the result in lbs. and the decimal of a lb.

5. A bankrupt pays 12*s.* 9*d.* in the £: if his debts are £3284 8*s.* 3*d.*, find by Practice the amount of his assets.

6. Rice is bought at R3. 6*a.* a maund, and sold at R3. 8*a.* 6*p.* a maund: how many maunds must be sold to realise a profit of R500?

7. Find the least number of months in which an exact number of half-guineas can be earned, the wages per month being £1 4*s.* 6*d.*

8. A boy after spending two-sevenths of his money, finds that three-fifths of what he has left is R2. 7*a.* 6*p.*: how much had he?

9. Express $\frac{1}{38}$ of R11. 4*a.* 6*p.*+·125 of R5. 11*a.* 6*p.* as a decimal of 14*a.*

10. A farmer pays a rent of R2. 8*a.* per bigha for 50 bighas, of R3. 4*a.* per bigha for 40 bighas, and R3. 2*a.* 8*p.* per bigha for 30 bighas: what is the average rent per bigha?

11. A man receives 5 per cent. on one-third of his capital, $4\frac{1}{2}$ per cent. on one-fifth, and 4 per cent. on the remainder; what percentage does he receive on the whole?

***12.** A servant was hired for 100 days on the condition that he should receive 8*a.* for every day he worked and forfeit 2*a.* for every day he was idle: at his departure he received R31. 4*a.* for his labour. Find the number of days he was idle.

***13.** A man embarks his whole capital in three successive ventures; in the first he clears 76 per cent., and in each of the other loses 25 per cent: shew that he has lost one per cent. on his original capital.

14. *A*, *B*, *C*, working together can do a piece of work in 12 days, which *A* could do alone in 24 days; after working together for 2 days, *A* is taken ill; how long will *B* and *C* take to finish it?

15. Find the amount of a bill for 420 yds. of linen at 6*a.* 6*p.* per yard, 330 yds. of cloth at 3*a.* 6*p.* per yard, and 260 yds. of silk at R2. 4*a.* per yard; deducting 10 per cent. for ready payment.

16. A merchant gains 25 per cent. on one-fifth, and loses 5 per cent. on the rest of his capital: what does he gain on the whole?

17. Find the cost of papering the walls of a room 10 ft. 6 in. high, 24 ft. 2 in. long, and 10 ft. broad, with paper 32 in. wide at 9*a.* per yd.

18. A mixture is made of 242 gal. of wine at 16*s.* 8*d.* per gal., 376 gal. at 26*s.* 6*d.* a gal., 96 gal. at 18*s.* 9*d.* a gal. and 186 gal. of water: at what price should the mixture be sold to get a profit of 25 per cent.?

19. Divide R7840 between 21 men, 28 women, 35 boys and 42 girls, in suchwise that a woman has one-fourth less than a man, each boy two-sevenths as much as a man and a woman together, and each girl one-fifth as much as a man, woman and a boy together.

20. The 4 P.M. mail train from Sealdah to Goalundo stops at Barrackpore 14 mi. distant, at 4-20 P.M.; the whole journey is 154 mi., and 20 per cent. of the time is lost in stoppages; when is the train due at Goalundo?

21. Simplify $\cdot 3\dot{6}4\dot{8} \times 6\cdot 29 + 2\cdot 6575\dot{6} \div \cdot 0002 - 1\cdot 2\dot{4}\dot{5}$ of $17\cdot 5$.

22. A garrison of 1200 men has just provisions enough to allow 28 oz. of bread a day to each man, for 54 days: but an engagement coming on, the garrison is strengthened by 420 men: how many ounces of bread must be given to each man, that the siege may be protracted for 30 days more?

23. Extract the square root of 5 to six decimal places: and find approximately the cost of enclosing a garden 20 bighas in area, by a fence costing 1*a*. 4*p*. per yard.

24. A reef of quartz contains ·0066 % of gold; if £8. 8*s*. worth of gold can be obtained from a ton, find the weight of a sovereign in grains.

25. Two clocks point to 6 o'clock at the same instant on the morning of New Year's day; one loses 5 second in 16 hrs. and the other gains 7 seconds in 18 hrs.: when will one be an hour before the other, and what time will each clock then [illegible]

26. A square park is bordered by a foot-path 8 yds. wide; if the park together with the path occupy 19·6 acres, find the cost of gravelling the path at 1*s*. 3*d* per square yard.

***27.** The annual consumption of tea in a country is 9,000,000 lbs. and the duty is 4*a*. 7*p*. per pound: the duty is reduced 6 per cent. and the consumption then increases 8 per cent.: how is the revenue affected?

28. Find the prime factors of 334620 and 465696: and find the smallest whole numbers which will make them perfect squares.

29. Find the cost of covering the outside of a box 2 ft. by $2\frac{2}{3}$ ft. by $4\frac{1}{3}$ ft. with cloth $\frac{3}{4}$ yd. wide 10*a* per yard.

30. In an examination in which the full marks were 800, *A* got 10 per cent. more than *B*, *B* 12 per cent. more than *C*, and *C* 25 per cent. more than *D*; *D* got 450: what percentage of the full marks did *A* get?

***31.** A shop-keeper sells rice at R3. 4*a*. a maund, and takes off 5 per cent. for cash payment: if he makes a profit of 20 per cent., what did a maund cost him?

***32.** *A* and *B* start together from Calcutta for Hughli, *A* walking $2\frac{1}{2}$ mi. an hour and *B* riding 10 mi. an hour. *B* reaches Hughli in 2 hrs. 24 min. and immediately rides back to Howrah. After how long should he again start from Howrah, that he may overtake *B* 2 mi. from Hughli?

33. Sound travels at the rate of 1125 feet a second; if a shot be fired from a ship moving at the rate of 12 miles an hour, how far will the ship have moved before the report is heard 20 miles off?

34. A map of Bengal is drawn on the scale of 10 miles to an inch. What will be the size on it of Calcutta which is 15·6 sq. mi. in area?

35. I go to the market with a certain sum of money to buy mangoes and find that if I buy at the rate of 40 a rupee I shall spend R5 too much, if at 50 a rupee, R10 too little. How many mangoes do I buy ?

36. If 20 coolies, in 3 days of 8 hrs. each, can dig a trench 40 yds. long, 7 yds. wide and 4 yds. deep, in how many days of $7\frac{1}{2}$ hrs., will 120 coolies dig a trench 150 yds. long, 8 yds. wide and 7 yds. deep ?

37. The cost of papering a room 24 ft. 9 in. long, and 20 ft. wide with paper $1\frac{1}{4}$ ft. wide at 1*a*. 6*p*. a yard, R29. 13*a*. 4*p*. : find the height.

38. *A* starts a business with R6000, and after 4 months is joined by *B* with R450. Two months later, they take in *C* with R750. At the end of the year the net profits amount to $16\frac{2}{3}$ per cent. on the whole capital invested : how should they be divided ?

39. Find by Duodecimal the area of a room 20 ft. 8′ 6″ long by 12 ft. 3′ 5″ broad, and the cost of carpeting it at 9*a*. per sq. foot.

40. The length of a room is half as much again as its width, and the height is half the length and breadth taken together : if the cost of painting the walls be R169 at 2*a*. 3*p*. a sq. ft, find its dimensions.

***41.** Three horses can in a given time plough as much as 5 oxen, but 2 horses cost as much daily as 3 oxen. If a certain field can be ploughed by 4 horses in 9 days, find the cost of ploughing it by oxen in 12 days, the daily cost of a horse being 8*a*.

42 I sold an estate at a loss of $1\frac{1}{4}\%$ thus getting R100 less than the cost price ; what should I have sold it for, to gain $12\frac{1}{2}\%$?

43. The cost of painting the faces of a cube at 1*a*. per square inch is R315. 6*a*., find its volume by Cross Multiplication.

44 A ball of iron is heated and swells by 5 per cent. of its size : what percentage of the new volume does the ball lose on cooling back to the original temperature ?

45. Two men can do as much work as 3 women, and their wages are proportional to the numbers 5 and 3. A work on which 3 men and 3 women are employed takes 16 weeks and costs R70 ; how long would it take if 4 men and 6 women were employed, and what would be the cost ?

46. By selling a piece of land for R241. 10*a*. a man loses half as much as he would have gained by selling it for R328. 3*a*. ; find its value.

***47.** *A* and *B* run round a circular course in opposite directions from the same point, *B* getting a start of 21 feet. They pass each other when *B* has run 970 feet. If the length of the course be 1951 feet, which will come first to the starting point, and how far will they be apart then ?

48. A hare sees a hound 352 yards away from her and scuds off ; 20 sec. later the hound sees her and gives chase. The hare takes in 1 min. 660 leaps of $1\frac{1}{2}$ ft. each, and the hound runs at a speed of 18 mi. an hour. When and where will the hound overtake the hare ?

49. One pendulum oscillates 7 times in 3·843 sec. and another pendulum 11 times in 8·052 sec. : if started together at 3 P.M., how often will they tick together within 9·30 P M. ?

***50.** A wine-merchant buys spirits at 7*s*. 6*d*. per gallon, and mixes so much water that by selling it at 6*s*. 9*d*. per gallon he clears 20 per cent. upon his outlay ; what proportion of water does he mix ?

51. Two cogged wheels work together, there being 40 cogs on one and 32 on the other: if the smaller wheel makes 15 revolutions per second, how often will the same cogs come in contact during 24 hours?

***52.** A man having 4 sons, left R21300 to be divided among them in the proportion of their ages at the time of his death. When he died, the eldest son was 24 years old, the second son 20, the third son 15, and the fourth 12: what difference would it have made to the sons if he had died a year later?

***53.** Two trains are running past each other in opposite directions, one at 30 and the other at 25 miles an hour. The first train contains 12 carriages and the second 15. If the carriages are each 30 ft long, the engines 36 ft. and the coupling spaces 6 ft., how long would the trains take to pass each other?

***54.** A tradesman professes to sell his goods at 20 per cent. profit, but adulterates them by mixing 20 per cent. of an inferior article which costs 20 per cent. less; how much per cent. profit does he make, and in what proportion should he mix the two kinds to gain 30 per cent.?

***55.** The income tax is reduced from $10\frac{1}{2}d.$ to $5d.$ in the £, but a man's gross receipts are at the same time reduced by 10 per cent. owing to the stoppage of a mine; find by what percentage his net income is altered.

***56.** A piece of work is done by three men in the following manner; *A* works the whole time, *B* only on the first and second days, *C* only on the third, fourth, and fifth days. The work might also have been done by *B* and *C* working together for 6 days If *B* and *C* working together for 2 days can do as much work as *A* alone can do in 3 days, find how long it would take *A*, *B* and *C* each to do the work separately.

57. How long will a column of men 3000 ft. long take to move through a defile $\frac{3}{4}$ mi long, at the rate of 75 paces of $2\frac{1}{2}$ ft. each a minute?

***58.** A horse-dealer bought a certain number of horses; he sold 20 of them at a gain of 5 per cent. and the rest at a gain of 20 per cent., thus clearing 15 per cent. on his outlay: how many horses did he buy?

***59.** A train leaves Howrah at 10 o'clock in the morning and is timed to reach Rajmehal at 3 o'clock in the evening; at some distance from Howrah it is detained by an accident for 20 min., after which the speed is increased for the remainder of the distance 5 mi. a hour, and then Rajmehal is reached at the appointed time. Where did the accident take place, Rajmehal being distant 200 miles from Howrah?

60. Two persons start at the same time from places 120 miles distant and meet after travelling 6 hours; their speeds are as 2 : 3; at what rate did each travel?

***61.** Three men *A*, *B* and *C*, divide a sum of R3000 among themselves. The share of *A* is to that of *B* as 4 to 3, and *C*'s share exceeds *A*'s by R140. What does each receive?

***62.** In 1884, the profit realized by a Railway company bore to the expenses the proportion of 13 to 15 In 1890 the gross receipts increased $3\frac{1}{3}$ per cent., while the expenses increased $1\frac{1}{3}$ per cent.: find how much per cent. the profits increased.

63. *A* and *B* join their capitals, which are in the ratio of 4 : 5; but at the end of 3 months they withdraw $\frac{2}{3}$ and $\frac{3}{4}$ of their capitals respectively: how should they divide a profit of £335 at the end of the year?

***64.** A wine-merchant bought a cask of spirits for £60, and sold 2 gal. more than three-fourths of the whole at a profit of 25 per cent. He then sold the remainder at such a price as to clear 60 per cent. on the whole. Had he sold the whole quantity at the latter price, he would have gained 175 per cent.; how many gallons does the cask contain?

65. Divide £76681 among *A, B, C* and *D* in such a manner that $\frac{3}{5}$ of *A*'s share, $\frac{5}{6}$ of *B*'s share, $\frac{7}{12}$ of *C*'s share and $\frac{11}{16}$ of *D*'s share may be all equal.

66. The length of a room is treble its breadth. The cost of flooring at 7*s*. 6*d*. per sq. yd., is £28. 2*s* 6*d*., and that of painting the four walls at $4\frac{1}{2}d$. per sq. ft., is £28. 2*s*. 6*d*. What is the height of the room?

***67.** An armourer undertakes to supply 2000 swords at 17*s*. 3*d*. each. He estimates that if 5 per cent. fail to stand the required test and are worthless, the profit will be 15 per cent. on the whole outlay. At the trial 35 per cent. prove worthless. What does he lose by the contract?

68. A grocer mixes tea at 4*s*. a lb. with tea at 3*s*. a lb., and by selling the mixture at 3*s*. 6*d*. a lb gains 5 per cent. How does he mix them?

***69.** The market price of a certain article has risen 50 per cent. during the last 20 years, but the value of money has fallen 25 per cent.; find the actual increase in the value of the article. How much should the value of money fall that its value may remain unchanged?

***70.** A watch that gains 24 sec. per hour is set to right time at 4-45 P. M. What will be the right time between 8 and 9 o'clock the next morning, when the two hands point in exactly opposite directions?

***71.** A person sold a horse so as to gain $12\frac{1}{2}$ per cent. Had he bought him for £4 more and sold him for £6 less, he would have lost $2\frac{1}{2}$ per cent. of the old cost. What did he pay for the horse?

72. How do I mix spirits worth 15*s*., 16*s* 6*d*. and 18*s*. 6*d*. a gallon, if the mixture is sold at 17*s*. 6*d*. a gallon with a profit of 5 per cent.?

***73.** Two brothers go to a fair with a certain sum of money and, after paying the entrance fee, buy with the money left 40 yards of silk; if only one brother had gone with the same money, he could have bought 42 yards. If 3 yds. of silk cost as much as 5 yds. of cambric, and 10 yds. of cambric cost as much as a yard of velvet, and $2\frac{1}{2}$ yds. of velvet can be had for R15, find the admission fee.

74. If the numbers of boys in 3 classes of a school are as 3 : 4 : 5, and the percentage of absentees 20 . 30 : 36; find the percentage of absentees in the school.

75. A man bought a certain number of mangoes at 15 for a rupee, and half as many at 20 for a rupee; he afterwards retailed them at 1*a*. 4*p*. each; what was his gain per cent.?

76. Find the cost of painting the outside of a box 6 ft. 8 in. long, 2 ft. 8 in. wide and 2 ft. 6 in. high, at $\frac{3}{4}d$. per sq. foot.

77. Two pipes are attached to a cistern which can hold 1200 gal. of water. When the first pipe is opened for 15 min. and the second for 13 min., 211 gal. of water enter into it; and when the first pipe is opened for 21 min. and the second for 30 min, 3781 gal. are poured in: when will it become full, if both the pipes are opened at once?

78. A man bought 3 boxes of oranges, each containing the same number. They cost respectively $1d.$ for 3, $1\frac{1}{2}d.$ for 4, and $2\frac{1}{2}d.$ for 6. How much per cent. would be gained by retailing the whole at $\frac{1}{2}d.$ each?

79. A man buys 5 tables and 8 chairs for R84. Next day he buys 4 tables and 12 chairs at the same rate for the same sum. Find the cost of one chair and one table.

***80**. If 12 men and 18 boys do $\frac{2}{5}$ of a piece of work in 5 days, and then 6 men and 3 boys are sent away, and if the remaining men and boys do $\frac{1}{6}$ more of the work in $2\frac{1}{2}$ days; how many men must be added, so that the remaining work may be finished in one day more?

***81**. Two passengers have together 6 mds. 30 sr. of luggage and pay luggage charges R3 8$a.$ and R2. 4$a.$ respectively. If all the luggage had belonged to one of them, the charge would have been R6. 4$a.$ Find the amount of luggage owned by each.

82. If 2 boys and 1 man do a piece of work in 4 hrs and 2 men and 1 boy can do the same in 3 hrs.; find in what time a man, a boy, and a man and a boy together, can respectively do the same.

83. The cost of carpeting a room is R36, and of papering the same room with paper at 10$p.$ per sq. foot, is R53. 2$a.$ The length of the room is 18 ft.; and if the width had been 4 ft. less, the carpet would have cost R9. less Find the height of the room.

***84**. A tea merchant had a rectangular godown for storing tea 28·8 ft. long, 21·6 ft. wide and 19·2 ft. high. What is the largest side of cubical boxes that will fill it exactly and what would be the size of each box?

***85**. A person pays an income tax of $4d.$ in the £ during the first half of the year and of $6d.$ in the £ during the second half; and finds that owing to an increase of his income his net income remains the same. If his gross income be £470, find his net income.

***86**. A Brahman worshipped at four temples one after another, offering the same whole number of flowers each time. As soon as he entered a temple, the number of flowers in his hand was trebled, but when he came away, there was none left. shew that he could not have taken less than 40 flowers to the first temple.

***87**. Ten years ago, the ages of A and B were in the ratio of 4 : 5, and in 5 years they will be in the ratio of 7 : 8 : find their ages.

88. A, B, C start from Benares for Allahabad, at 7, 8 and 8-30 in the morning, walking at the rates of $3\frac{1}{2}$, 4 and $4\frac{1}{2}$ miles an hour. When C overtakes A, he is sent back with a message to B, after delivering which he immediately comes back. Where will he again meet A?

***89**. A boat propelled by 8 oars, which take 10 strokes per minute, goes 9 miles an hour. Where and when will it overtake another boat $4\frac{1}{5}$ mi. off. which is propelled by 6 oars taking 8 strokes per minute, 6 strokes of the first crew being equivalent to 5 of the latter?

***90**. A boat gets loose at noon on Monday in the middle of high tide. It is carried up the stream for $3\frac{1}{2}$ miles, when the tide begins to ebb: and is then carried down $10\frac{1}{2}$ miles by the ebb-tide, when the tide begins again to flow. When will the boat pass a place which is 10 miles below the spot where it got adrift?

PART IV.

CHAP. XXXIV. INTEREST.

321. When a person borrows from another a certain sum of money, the money which he has to pay for the use of it is called **Interest.**

The money lent, or upon which the *interest* is reckoned, is called the **Principal.** The principal together with the interest upon it, is called the **Amount.** The interest which has to be paid for the use of a fixed sum for a certain fixed period of time is called the **Rate.** The interest on every R100 or £100 for one year is called the **rate per cent. per annum,** *per annum* meaning 'for a year.' Unless stated otherwise, by Rate per cent. is always to be understood the *rate of interest* on every R100 or £100 for **one year.**

Thus, if the debtor agrees to pay every year the sum of R5 on every R100 borrowed, the *rate of interest* is 5 *per cent. per annum* Again, if the sum of money lent be R500, and the interest upon it in a certain time is R50, R500 is the *Principal,* R50 the *Interest,* and R(500+50) or R550 the *Amount.*

322. Interest is of two kinds, **Simple** and **Compound.**

When the principal on which interest is reckoned remains the same whatever the number of years may be, it is called **Simple Interest;** but when each portion of the interest, as soon as it becomes due, is added to the principal to produce a new principal for the next period, it is called **Compound Interest**

SIMPLE INTEREST.

323. *To find the Simple Interest on a given sum of money for a given time, when the rate of interest per Re. or £ for one month is given.*

Suppose R500 is lent for 8 months at the rate of 2 pies per rupee per month. The interest on R1 for 1 mo. being 2 pies, the interest on R500 for 1 mo. $= 500 \times 2p.$; therefore the interest for 8 mo. $500 \times 8 \times 2p.$ Hence we have the following

RULE. Multiply the principal, the given time, and the rate of interest together; the product will give the required interest in the same denomination as the given rate of interest.

Ex. Find the Simple Interest on *Rs.*72 for 9 mo. at $\frac{1}{2}a.$ per *Re.* a mo.
The required interest $= 72 \times 9 \times \frac{1}{2}a. = 324a. =$ *Rs.* 20. 4*a.*

Examples 139.

Find the Simple Interest on :

1.	R	64	for	7 months	at	2*ps.*	per	rupee	per	month.
2.	„	86	„	8	„	6*p.*	„	„	„	„
3.	„	77	„	11	„	$\frac{1}{2}a.$	„	„	„	„
4.	„	183	„	15	„	3*p.*	„	„	„	„
5.	„	540	„	21	„	$\frac{1}{4}a.$	„	„	„	„
6.	„	840	„	$3\frac{1}{6}$ years		5*p.*	„	„	„	„

324. *To find the interest on a given sum of money for a given time at a given rate per cent.*

Suppose a sum of money (in R) is lent for 4 years at 5 per cent.

The interest on R100 for 1 yr. is R5 ; ∴ the interest on any sum for one yr. $= \frac{5}{100}$ of the given sum, and the interest for 4 yrs. $= 4 \times \frac{5}{100}$ of the given sum. Hence the following RULE is obtained :

RULE. Multiply the principal, the given time, and the given rate per cent. together, and divide the product by 100.

Ex. Find the Simple Interest on £250 for 3 years at 5 per cent.

$$\text{The required interest} = £\frac{250 \times 3 \times 5}{100} = £37.\ 10s.$$

325. The following will illustrate the several methods of calculating Simple Interest.

Ex. Find the interest on £720. 16*s.* 8*d.* for 8 years at $3\frac{1}{5}$ per cent., and also the Amount.

(i) The interest on £100 for 8 years $= £3\frac{1}{5} \times 8 = £\frac{128}{5}$;

∴ the interest on £1 for 8 years $= \frac{128}{5} \times \frac{1}{100} = £\frac{32}{125}$;

∴ the interest on $£720\frac{5}{6}$ for 8 years $= £\frac{32}{125} \times \frac{4325}{6} =$ £184. 10*s.* 8*d.* ;

and the amount = £720 16*s.* 8*d.* + £184. 10*s.* 8*d.* = £905. 7*s* 4*d.*

(ii) £720. 16*s* 8*d* × $3\frac{1}{5}$ (rate p.c.)

£2162. 10*s.* 0*d.*
144 3 4
£2306 13*s.* 4*d* × 8 (no. of yrs.)
£184,53. 6*s.* 8*d.* (÷ 100)
20
s. 10,66
12
d. 8,00

	£.	*s.*	*d.*
Interest	184	10	8
Principal	720	16	8
Amount	905	7	4

(iii) $8 \times 3\frac{1}{5} \div 100 = 8 \times 4 \div (5 \times 5 \times 5)$

£ 720. 16*s.* 8*d* × 8
£ 5766. 13*s.* 4*d.* × 4
£23066 13*s.* 4*d.* ÷ 5
£ 4613. 6*s.* 8*d.* ÷ 5
£ 922 13*s.* 4*d.* ÷ 5

Interest	£	184. 10*s.* 8*d.*
Principal	£	720. 16*s.* 8*d.*
Amount	£	905. 7*s.* 4*d.*

Examples 140.

1. Find the Simple Interest for one year on :

(1) R 200 at 3 per cent.
(2) „ 400 at 5 per cent.
(3) „ 356. 4*a*. at 4 per cent.
(4) „ 800 at $3\frac{1}{2}$ per cent.
(5) „ 2017. 8*a*. at 5 per cent
(6) „ 2567. 8*a*. at 5 per cent.
(7) £ 350 at 4 per cent.
(8) „ 575 at 6 per cent.
(9) „ 757. 10*s*. at 6 per cent.
(10) „ 1200 at $4\frac{1}{2}$ per cent.
(11) „ 578. 5*s*. at 5 per cent.
(12) „ 1883. 10*s*. at 5 per cent

2. Find the Simple Interest on :

(1) R 2055 for $2\frac{1}{2}$ yrs. at $5\frac{3}{8}$ p. c. (2) £ 940 for $3\frac{1}{2}$ yrs. at $3\frac{1}{2}$ p. c.
(3) „ 7520 for 5 yrs. at $4\frac{1}{4}$ p. c. (4) „ 5250 for $6\frac{1}{3}$ yrs. at $3\frac{1}{2}$ p. c.
(5) „ 327. 8*a*. for $2\frac{1}{2}$ yrs. at 4 p. c. (6) „ 333. 10*s*. for 20 yrs. at $3\frac{3}{4}$ p.c
(7) „ 744. 6*a*. for 4 yrs. at 5 p. c. (8) „ 291 2*s*. for 3 yrs. at $2\frac{1}{2}$ p.c.
(9) „ 101 4*a*. for 20 yrs. at $2\frac{1}{2}$ p.c. (10) „ 538 5*s*. for 4 yrs. at 5 p c.
(11) „ 2971 for $3\frac{1}{8}$ yrs. at 3 p. c. (12) „ 578. 15*s*. for 73 days at 5 p c
(13) „ 4691. 4*a* for 146 da. at 4 p.c. (14) „ $1266\frac{2}{3}$ for 219 days at 6 p c.

3 Find to the nearest pie or penny the Simple Interest on .

(1) R 764. 7*a* 4*p*. for 4 yrs. at 3 p.c. (2) R87910 for 219 days at $5\frac{1}{2}$ p c
(3) „ 1793. 8*a*. for 1 year 151 days at $6\frac{1}{4}$ per cent.
(4) £450 from July 17th to 14th August at 6 p. c.
(5) „ 96281. 10*s*. 11*d*. from 3rd March to 28th December at $5\frac{1}{2}$ p. c.
(6) „ 4568. 11*s*. from 24th January 1890 to 15th March 1891 at 3 p c

4. Find the amount of :

(1) R1981. 10*a*. 8*p*. for 3 yrs. at 5 p.c. (2) R4968. 12*a*. for 5 yrs at $4\frac{1}{2}$ p c.
(3) „ 3478. 5*a*. 4*p* „ 12 yrs. „ $3\frac{3}{4}$ p c (4) „ 56. 4*a*. for 2 yrs. at 4 p. c.
(5) £125. 6*s*. 3*d*. for $3\frac{1}{2}$ yrs. at $4\frac{1}{2}$ p.c. (6) £3191. 5*s*. for $9\frac{1}{3}$ yrs. at $5\frac{1}{4}$ p.c.
(7) „ 516. 13*s*. 4*d*. at $2\frac{1}{3}$ yrs. at $2\frac{3}{4}$ p.c. (8) „ 325. 10*s*. for 4 yrs. at $5\frac{1}{2}$ p c.

326. All questions on Interest involve four things, *viz.* :—the **Principal,** the **Rate of Interest,** the **Time,** and either the total **Interest** or the **Amount**; and if any three of the four be given, we can easily obtain the fourth. The RULES for finding the Interest and Amount have already been explained. The following will illustrate the cases in which the Principal, the Rate, and the time have respectively to be obtained.

Ex. 1. **What sum will produce *Rs.* 533 in 4 yrs. at $3\frac{1}{4}$ *p. c.* ?**

Rs. 100 produce in 4 years at $3\frac{1}{4}$ per cent., *Rs.* $3\frac{1}{4}\times4=$ *Rs.* 13.

Rs. 13 : *Rs.* 533 :: *Rs.* 100 : required principal ;

∴ required principal = *Rs.* 4100. *Ans.*

Ex. 2. What sum will amount to *Rs.* 9000 in 4 years at $6\frac{1}{4}$ *p.c.* ?
Rs. 100 in 4 years at $6\frac{1}{4}$ *p.c.* amount to *Rs.* $(100+4\times 6\frac{1}{4})$ or *Rs.* 125.

Rs. 125 : *Rs.* 9000 : : *Rs.* 100 : required principal ;

∴ required principal$=$*Rs.* 7200. *Ans.*

Ex. 3. In what time will *Rs.* 12750 amount to *Rs.* 15495. 8*a.* at $3\frac{4}{5}$ *p.c.* ?
The intt. on *Rs.* 12750 in the reqd. time at $3\frac{4}{5}$ *p.c.*$=$*Rs.* $(15495\frac{1}{2}-12750)$;
" " " " " in 1 year " " " $=$ " $(12750\times 3\frac{4}{5})\div 100$.

$$\text{Reqd. time}=\frac{15495\frac{1}{2}-12750}{12750\times 3\frac{4}{5}}\times 100=\frac{5491\times 100\times 5}{2\times 12750\times 19}=5\tfrac{2}{3}\text{ yrs.}\quad \textit{Ans.}$$

Ex. 4. At what rate p. c. will £250 amount to £300. 12*s.* 6*d.* in $4\frac{1}{2}$ yrs.?
The intt. on £250 in $4\frac{1}{2}$ yrs. at the reqd. rate$=$£$(300\frac{5}{8}-250)=$£$50\frac{5}{8}$:
" " " " in 1 yr. " " " $=$£$50\frac{5}{8}\div 4\frac{1}{2}=$£$11\frac{1}{4}$;

∴ £250 : £100 : : $11\frac{1}{4}$: reqd. *p.c.* ;

∴ reqd. *p.c.*$=4\frac{1}{2}$. *Ans.*

327. The following formulæ, if remembered, will be of great use.

If P, n, r, I, M, be respectively the Principal, Time, Rate, Interest and Amount, $I=\dfrac{Pnr}{100}$; $M=P+\dfrac{Pnr}{100}=P\left(1+\dfrac{nr}{100}\right)$;

$$P=\frac{100M}{100+nr}\ ;\ n=\frac{(M-P)\times 100}{Pr}\ ;\ r=\frac{(M-P)\times 100}{Pn}.$$

Examples 141.

1. What sum will produce R546 in 3 yrs. at 4 p.c. Simple Interest ?

2. On what sum will the Simple Interest be R1250 in 5 yrs. at 5 p.c.?

3. The Simple Interest on a certain sum of money for $3\frac{1}{2}$ years at 5 per cent. is R3981. 4*a.* What is the sum ?

4. What sum is that the Simple Interest on which for 6 years at 5 per cent. is £167 ?

5. What sum will produce £17. 14*s.* 6*d.* in $2\frac{1}{2}$ years at 3 p. c. ?

6. " " " amount to R7590. 3*a.* 8*p.* in $7\frac{5}{6}$ years at $3\frac{1}{4}$ p. c. ?

7. " " " " " " 1346. 10*a.* 8*p.* in 73 days at 5 p. c. ?

8. " " " " " " 1705 in $3\frac{5}{13}$ years at 4 p. c. ?

9. " " " " " £423. 2*s.* 6*d.* in 3 yrs. 8 mo. at $3\frac{1}{2}$ p.c. ?

10. What sum at $3\frac{3}{4}$ per cent. per annum, will amount to £236. 14*s.* $4\frac{1}{2}$*d.* in 7 years ?

11. What sum in 2 years 6 months at 4 per cent., will produce the same interest as R5000 will produce in 3 years at 5 per cent. ?

12. Find the sum, which will produce in $4\frac{1}{2}$ years at 6 per cent. the same income as £9720 in $7\frac{1}{2}$ years at 4 per cent.

13. In what time will R3050 amount to R3888. 12*a.* at 5 per cent. ?

14. " " " " " 4525 produce R1923. 2*a.* at $4\frac{1}{4}$ per cent. ?

15. In what time will R9690. 12*a*. yield R132. 12*a*. at 5 per cent. ?

16. " " " " 9366. 10*a*. 8*p*. amount to R11573. 11*a*. at $4\frac{5}{8}$ per cent. ?

17. In what time will £152. 12*s*. 6*d*. amount to £167. 17*s*. 9*d*. at 4 p.c.?

18. " " " " 345 amount to £438. 3*s*. at $4\frac{1}{2}$ per cent. ?

19. " " " " 452. 5fl. amount to £644. 8fl. 1c. $2\frac{1}{2}$m. at $4\frac{1}{4}$°/o?

20. At what rate per cent. will £200 amount to £204. 16*s*. in 146 days?

21. At what rate per cent. will 200 guineas amount to £259. 1*s*. 9*d*. in 5 years 6 months ?

22. In what time will R100 double itself at 5 per cent. ?

23. " " " " £400 double itself at 4 per cent. per annum ?

24. " " " " a sum double itself at $6\frac{1}{4}$ per cent. per annum ?

25. " " " " " " treble itself at 3 per cent. per annum ?

***26.** *A* borrowed of *B* at the same time R2600 at $3\frac{1}{2}$ per cent, and R3580 at 5 per cent.: and when both these sums amounted together to R6405, he cleared off his debts. How long did the loan continue ?

27. In what time will £4000 at 3 per cent. per annum produce the same income as £5000 in 4 years at 5 per cent Simple Interest ?

28. At what rate per cent. will R5026. 10*a*. 8*p*. amount to R5780. 10*a*. 8*p*. in 3 years 4 months ?

29. At what rate per cent. will R2191. 2*a*. 4*p*. amount to R2213. 0*a*. 11*p*. in 73 days ?

30. At what rate per cent. will R8127. 8*a*. amount to R13004 in 20 years ?

31. At what rate will £256. 5*s*. amount to £323. 10*s*. $3\frac{3}{4}$*d*. in 7 yrs. ?

32. At what rate per cent. will the interest of £619. 17*s*. 6*d*. amount to £185. 19*s*. 3*d*. in $7\frac{1}{2}$ years ?

33. At what rate will a sum double itself in 20 years ?

34. At what rate per cent. will a sum treble itself in 30 years ?

***35.** At what rate per cent. per annum will R6500 produce in 6 years the same interest as R4500 will produce in 10 years at $6\frac{1}{4}$ per cent. ?

***36.** Two equal sums were lent out at 5 per cent. and 4 per cent. respectively, and the joint interest amounted in 3 years to R270 ; find the sums

CHAP. XXXV. COMPOUND INTEREST.

328. When it is agreed between the lender and the borrower that the interest shall be paid in stipulated periods as soon as it becomes due, the interest is called **Compound Interest.**

In most cases, however, the borrower is either unwilling or unable to pay the interest as soon as it falls due. In such cases, the interest is considered as an additional loan on which additional interest has to be paid

For instance, suppose a man borrows R100 at compound interest, at 5 per cent. per annum. At the end of the year he is bound to pay R5 as the interest on the R100 borrowed. If, however, he does not or cannot pay this sum, then for the next or the second year, he will have to pay interest on R(100+5) or R105 at the same rate, *i.e.*, R5. 4*a*. In the third year he will have to pay interest on (R105+R5. 4*a*.) or R110. 4*a*. ; and so on.

Ex. Find the Compound interest on *Rs.* 6845 in 3 years at 4 *p.c.*

1st Principal	*Rs.* 6845	Principal *Rs.* 6845 0 0
	4	1st year's intt., 273 12 9·6
[We multiply by	273,80	
4 the rate *p.c.* and	16	
divide by 100 as	*a.* 12, 80	
in ART. 97.]	12	
	p. 9, 60	
2nd Principal	*Rs.* 7118. 12*a.* 9·6*p.*	2nd year's intt. 284 12 0·384
	4	
	Rs. 284,75 3 2·4	
	16	
	a. 12,03	
	12	
	p. ·384	
3rd Principal	*Rs.* 7403. 8*a.* 9·984*p.*	3rd year's Intt. 296 2 3·27936
	4	
	296,14 3 3·936	
	16	
	a. 2,27	Total intt. *Rs.* 854. 11*a.* 1*p* nearly.
	12	
	p. 3,27936	

Or thus :—

$4\ p.c. = \frac{4}{100}$.	*Rs.*6845·000	00	
	273 800	00	
	7118 800	00	Amount at the end of the 1st year.
	284·752	00	
	7403 552	00	" " " " 2nd year.
	296·142	08	
	*Rs.*7699·694	08	" " " " 3rd year.
Principal=	" 6845		

∴ Interest = *Rs.*854·694 = *Rs.*854. 11*a.* 1*p.*

329. When the time given involves a **fraction** of a year it is usual to calculate it for the fractional part as for a whole year at a rate equal to the same fraction of the rate.

Ex. Find the amount of £2068. 15*s.* in $2\frac{1}{2}$ years at 3 *p.c.* Compound Interest.

Half-year's interest at 3 *p. c.* per annum = a year's interest at $1\frac{1}{2}$ *p.c.* per annum.

$3\ p.\ c. = \frac{3}{100}$	£ 2068·750		
	62·062	50	
	2130·813		= Amount at the end of 1st year.
	63·924	38	
$\frac{1}{2} \times 3\ p\ c. = \frac{15}{1000}$	2194·737		= „ „ „ 2nd year.
	32·921	50	
	2227 658		= „ „ „ $2\frac{1}{2}$ years.
	= £2227. 13s. 2d.		

To obtain each year's interest we multiply by 3 and divide by 100 as in ART. 97; to obtain the interest for the last half-year we multiply by 15 and divide by 1000.

Note:—In questions on Compound Interest, it is to be understood that the interest is payable annually, unless otherwise stated.

330. When interest is due **more than once in the year** as, for instance, half-yearly, the number of years should be multiplied, and the rate divided, by 2. So when the interest is payable quarterly, they should be multiplied and divided by 4.

331. The following process may also be used, though it is not so convenient for practical calculations.

Amount of £100 at the end of one year = £104; (rate = 4 *p.c.*)

„ „ 1 „ „ „ = „ $\frac{104}{100}$;

any sum „ „ „ = „ $\frac{104}{100}$ of the sum.

∴ Amt. of any sum at the end of 1st year = of the sum;

„ „ „ 2nd „ = $\frac{4}{0} \times \frac{104}{100} = (\frac{104}{100})^2$

„ „ „ 3rd „ $= (\frac{104}{100})^2 \times \frac{104}{100} = (\frac{104}{100})^3$ „ ;

and so on:

Generally, if P denote the sum lent, n the number of years, r the rate per cent., and M the amount, then $M = \left(\frac{100+r}{100}\right)^n \times P$.

Ex. Find the Compound Interest on £1000 for $3\frac{3}{4}$ years at 4 *p.c.*

4 per cent. for $\frac{3}{4}$ year = 3 per cent. for 1 year.

Amt. at the end of 3 yrs. = £1000 × $(\frac{104}{100})^3$;

∴ „ „ „ $3\frac{3}{4}$ „ = £1000 × $(\frac{104}{100})^3 \times \frac{103}{100}$ = £1158·60992.

∴ reqd. interest = £158·60992 = £158. 12s. $2\frac{1}{4}d.$

Examples 142.

1. Find the Compound Interest, to the nearest *pie* or *penny*, on:

(1) R50000 for 3 yrs. at 4 p. c.

(2) R87500 for 3 yrs. at 4 p. c.

(3) „ 16000 for 3 yrs. at 5 p. c.

(4) „ $5333\frac{1}{3}$ for 3 yrs at 3 p. c.

(5) „ 22400 for 2 yrs. at $3\frac{3}{4}$ p. c.

(6) „ 47250 for 3 yrs. at $3\frac{1}{3}$ p. c.

(7) £7500 for 3 yrs. at $2\frac{1}{2}$ p.c. (8) £$375\frac{1}{4}$ for 2 years. at $2\frac{1}{2}$ p.c.
(9) „$5674\frac{3}{8}$ for $3\frac{1}{2}$ yrs. at 4 p.c. (10) „1485. 5s. 9d. for 3yrs. at 5p.c.
(11) „914. 12s. 6d. for 3 yrs at $3\frac{1}{2}$ p.c. (12) „ 8751. 15s. [illegible] for 3 yrs. at $3\frac{1}{3}$ p.c.

2. Find to the nearest *pie* or *penny* the Compound Interest, payable half-yearly, on :

(1) R6740 for $2\frac{1}{2}$ yrs. at $3\frac{3}{4}$ p.c. (2) £421 [illegible] $3\frac{1}{2}$ for 2 yrs at $3\frac{1}{2}$ p.c.
(3) £4500. 17s. 6d. for $1\frac{1}{2}$ yrs. at 6 p.c. (4) R45[illegible]61 for $2\frac{1}{2}$ yrs. at 2 p.c.

3 Find to the nearest *pie* or *penny* the Compound Interest, payable quarterly on :

(1) R45000 for 1 yr. at 3 p.c. (2) R[illegible]6050 for 9 mo. at 5 p.c.
(3) £4600 for 6 mo. at $4\frac{1}{2}$ p.c. (4) [illegible]26000 for 1 yr. at $2\frac{3}{4}$ p.c.

4. Find the Amount at Compound Interest of :

(1) R4624. 6a. for 2 yrs. at 6 p.c. (2) R5461. 10a. for 3 yrs. at $4\frac{1}{2}$ p.c.
(3) £5600. 10s.,, 3 yrs. „ $2\frac{3}{4}$ p.c. (4) £4620. 17s. 6d. „ $2\frac{1}{2}$ yrs. „ $3\frac{3}{4}$ p.c.

CHAP. XXXVI. PRESENT WORTH & DISCOUNT.

332. It often happens that a person is bound to pay a certain sum of money to another only *at the end of a given time.* For instance, when a paper merchant buys £100 worth of paper from the manufacturer, he may not pay the value immediately, but at some subsequent time, as at the end of 3 months. Should, however, he or somebody else in his behalf, choose to give ready money, it is evident that the sum which ought to be paid is such that *the sum together with the interest upon it for 3 months should amount to* £100. The manufacturer, in fact, by investing this sum for the interval would at the end of it get his £100, *just as it falls due.* This sum is called the **Present Worth** of the £100 ; and the difference or allowance thus obtained is called the **Discount.** Hence:

The **Present Worth** or **Present Value** of a sum of money due at the end of a given time is the sum which with its interest for the given time will amount to the given sum.

Thus, R500 at 4 per cent. would in 3 months amount to **R505** ; therefore the *Present value* of **R505** due 3 months hence, is **R500.**

Discount is the amount that is taken off from a certain sum of money, because of its being paid before it is due. It is the difference between the sum and its present value.

Thus, in the above Example, R(505 − 500) or **R5** is the *Discount* on R505 due 3 months hence.

333. It is clear from the above that the Discount is the interest on the Present Worth.

334. To find the present value and the discount of a given sum of money, we should first find what R100 would amount to in the given time at the given rate per cent. ; then

(i) Amount of £100 : given sum :: £100 : Present Worth ; and
(ii) Amount of £100 : given sum :: Interest of £100 : Discount,

or Discount = Sum − Present Worth.

Ex. Find the Present Worth and Discount of £2300 due 3¾ years hence, at 4 per cent.

£100 will at 4 p.c. amount in 3¾ yrs to £115 ;
£115 : £2300 :: £100 : *P. W.*
Present Worth = £2[illegible]

Discount = £(2300 − 2000) = £300.
Or thus :
£115 : £2300 :: £15 : Discount ;
∴ Discount = £300.

The merchant, in a case like the above, would give the manufacturer a **Bill** undertaking to pay him the value of the paper at the end of the given time. The latter, if he wants cash, has to go to a **Bill Broker** and get the Bill **discounted**; *i. e.*, take from him the Present Value of the amount given on the **face** of the Bill.

When a Bill is due 3 months hence, it is called payable at **Three months' Sight**.

Examples 143.

1. Find the Present Worth of :

(1) R810 due 2 yrs. hence at 4 p.c. (4) £430 due 2½ yrs. hence at 3 p.c.
(2) „ 331½ „ 3½ yrs. „ „ 3 „ (5) „ 733⅓ „ 16 mo. „ „ 3 „
(3) „ 9810 „ 18 mo. „ „ 6 „ (6) „ 76¼ „ 4 mo. „ „ 5 „
(7) „ 912. 8*a*. 3*p*. due 9 months hence at 4 per cent.
(8) „ 2752. 14*a*. 3*p*. due 11 months hence at 6 per cent.
(9) „ 17879. 9*a*. 4*p*. due 15 months hence at 3 per cent.
(10) £ 205. 5*s*. 6*d*. due 1 year hence at 3½ per cent.
(11) „ 224. 16*s*. 3*d*. due 7 months hence at 3¾ per cent.
(12) „ 1388. 17*s*. 6*d*. due 1 year hence at 2½ per cent.

2. Find the Discount of :

(1) R63⅓ due 4 mo. hence at 4 p.c. (4) £487[illegible] due 5 mo. hence at 3½ p.c.
(2) „ 102½ „ 6 „ „ „ 5 „ (5) „ 676[illegible] „ 6 „ „ „ 3 „
(3) „ 269 „ 3 „ „ „ 3½ „ (6) „ 550[illegible] „ 15 „ „ „ 4 „
(7) „ 126. 1*a*. 6*p*. due 3 months hence at 3½ per cent.
(8) „ 250. 4*a*. 11*p*. due 17 months hence at 5 per cent.
(9) £360. 10*s*. due 6 months hence at 6 per cent.
(10) „ 966. 13*s*. 4*d*. due 9 years hence at 5 per cent.
(11) „ 221 due 292 days hence at 2½ per cent.
(12) „ 13365. 12*s*. 6*d*. due 3½ yrs. hence at 5 per cent.

335. The time and the Rate per cent. can be found by a method similar to that explained in ART. 326 ; only the Present Worth should be taken as Principal and the Discount as Interest.

Ex. 1. If the discount on £207, due 8 months hence, be £7, what is the rate of interest?

The Present Worth of £207 = £(207 − 7) or £200. Therefore £7 is the interest on £200 for 8 mo.; ∴ the interest on £100 for 8 mo. is £$3\frac{1}{2}$;

∴ interest on £100 for 1 yr. (or rate per cent.) = £$3\frac{1}{2} \div \frac{2}{3}$ = £$5\frac{1}{4}$. *Ans.*

Ex. 2. If the Present Worth of £423. 2*s.* 6*d.* due at the end of a certain number of years at $3\frac{1}{2}$ per cent. is £375, find the time.

£$423\frac{1}{8}$ − £375 = £$48\frac{1}{8}$ is the discount on £$423\frac{1}{8}$;

∴ £$48\frac{1}{8}$ is the interest on £375 at $3\frac{1}{2}$ *p.c.* for the time required.

Also interest on £375 for 1 year at $3\frac{1}{2}$ *p c.* = £$375 \times 3\frac{1}{2} \div 100$ = £$13\frac{1}{8}$.;

∴ the required time = $(48\frac{1}{8} \div 13\frac{1}{8})$ years = $3\frac{2}{3}$ years. *Ans.*

Examples 144.

1. When is the sum due, if the

(1) Discount on R2287. 8*a.* at $4\frac{1}{2}$ p. c. be R37. 8*a.*?

(2) „ „ R7470 at $2\frac{1}{2}$ p. c. be R830?

(3) „ „ R8697. 8*a.* at $3\frac{1}{2}$ p. c. be R2572. 8*a.*?

(4) „ „ £4972. 10*s.* at 10 p. c. be £1147. 10*s*?

(5) P. Worth of „ 708. 12*s.* at $3\frac{3}{4}$ p. c. be £590. 10*s.*

(6) „ „ „ „ 1616 2*s.* 8*d.* at 4 p. c. be £1405. 6*s.* 8*d.*

2. What is the rate of interest in the following cases?

(1) When the discount on R2080 due one year hence, is R80.

(2) „ „ „ „ R1772. 12*a.* due 1 yr. hence, is R84. 6*a.* 8*p.*

(3) „ „ „ „ R2255. 8*a.* due 6 mo. hence, is R86. 12*a.*

(4) „ „ „ „ £1356. 13*s.* 4*d* due at 3 mo. is £23. 6*s.* 8*d.*

(5) When the *P. W.* of £3215. 15*s.* 8*d.* due at $4\frac{1}{2}$ yrs. is £2833. 6*s.* 8*d.*

(6) „ „ „ „ „ £4120. 8*s.* 7*d.* due 9 mo hence is £4000 8*s.* 4*d.*

336. The following solutions of Examples on Present Worth and Discount will be found useful.

Ex. 1. If the interest on £270 at $7\frac{1}{2}$ per cent. be equal to the discount on £283. 10*s.*, for the same time at the same rate of interest, when is the latter sum due?

Interest on £270 = Discount on £283. 10*s.*;

∴ £270 is the. *P. W* of £283. 10*s.*;

∴ Interest on £270 = £283. 10*s.* − £270 = £13. 10*s.* = £$\frac{27}{2}$;

∴ Interest on £100 for the reqd. time = £$\frac{100}{270} \times \frac{27}{2}$ = £5;

∴ the required time = $(5 \div 7\frac{1}{2})$yr. = 8 months. *Ans.*

Ex. 2. What is the amount on the face of a bill due $2\frac{1}{2}$ years hence, if the discount, at 4 per cent., amount to *Rs.* 350?

Amount of *Rs.* 100 for $2\frac{1}{2}$ yrs at 4 p. c = *Rs.* 110; ∴ discount = *Rs.* 10.

∴ *Rs.* 10 : *Rs.* 350 : : *Rs.* 110 : Amount of the bill;

∴ Amount of the bill = *Rs.* 3850. *Ans.*

Ex. 3. If the difference between the interest and the discount on a sum of money for 4 months at $2\frac{2}{3}$ per cent. is *Rs* 45, what is the sum ?

Interest on *Rs.*100 for 4 mo. at $2\frac{2}{3}$ p.c. = *Rs.*$\frac{8}{9}$;

∴ „ „ *Rs.*900 „ „ „ „ = *Rs.*8 ;

∴ Discount on *Rs.*908 „ „ „ „ = „ 8 ;

∴ „ „ *Rs.*900 „ „ „ „ = „ $\frac{900}{908} \times 8$.

∴ the diff. between Intt. & Disc. on *Rs.*900 = *Re.* $(8 - \frac{900}{908} \times 8)$ = *Re.*$\frac{64}{908}$.

But „ „ „ „ „ on the reqd. sum = *Rs* 45 ;

∴ the reqd. sum = *Rs.*$900 \times 45 \div \frac{64}{908}$ = *Rs.* 574593. 12*a.* *Ans.*

337. The Amount = the *P. W.* + the Discount ;

∴ Intt. on the Amount = Intt. on the *P. W.* + Intt. on the Discount.

But Intt. on the *P. W* = the Disc. on the sum ; (ART. 333.)

∴ Intt. on a sum = the Disc. on the sum + Intt. on the Disc. ;

∴ Intt. on a sum − the Disc. on the sum = Intt. on the Disc. ;

i.e., the difference between the interest and discount on a certain sum of money for a given time at a given rate per cent. is the **interest on the discount** for the same time at the given rate.

Ex. The interest on a certain sum of money is *Rs.*48, and the discount on the same sum for the same time at the same rate of interest is *Rs.* 45 : find the sum.

Rs.(48 − 45) or *Rs.* 3 is the interest on *Rs.* 45.

∴ *Rs.*3 : *Rs.*48 : : *Rs.*45 : reqd. sum ; or reqd. sum = *Rs.*720 *Ans.*

Examples 145.

1. If the interest on R1187. 8*a.* at 3 per cent. is equal to the discount on R1193 7*a.* for the same time at the same rate, when is the latter sum due ?

2. If the interest on £506. 5*s.* at 5 per cent. be equal to the discount on £514. 13*s.* 9*d.* for the same time at the same rate, when will the latter sum be due ?

3. If the interest on R5333. 5*a.* 4*p.* for 2 years is equal to the discount at the same rate on R5600 due 2 years hence, what is the rate of interest ?

4. At what rate per cent. will the interest on £1864. 13*s.* 9*d.* for 4 yrs., be equal to the discount on £2330. 17*s.* $2\frac{1}{4}$*d.* for the same time ?

5. On what sum of money, due at the end of 5 months, does the discount, at 4 per cent., amount to R37. 8*a.* ?

6. What is the amount of a bill due 9 months hence, on which the discount, at $5\frac{1}{4}$ per cent., is £188. 14*s.* $4\frac{1}{2}$*d.* ?

7. If the difference between the interest and discount on a sum of money at 4 per cent. for 3 years be R12, what is the sum ?

8. The difference between the interest and discount on a certain sum of money for 4 months at 5 per cent. is £7 ; find the sum.

9. The interest on a certain sum of money in 4 years is R800, and the discount of the same sum due 4 years hence at the same rate of interest is R666. 10*a*. 8*p*.; find the sum and the rate of interest.

10. The interest on a certain sum of money for a certain time at 6 per cent. is R1170, and the discount on the same sum for the same time at the same rate per cent. is R900; find the sum and the time.

11. If the interest on £200 for a certain time at a certain rate be £18 find the discount on the same sum at the same rate for the same time.

***12.** Five articles can be bought for a certain sum of money payable at the end of a year; and si.. articles of the same quality can be bought for the same sum with ready money; what is the rate of discount?

***13.** Four volumes of a work can be bought for a certain sum payable at the end of a year, and five volumes of the same work can be bought for the same sum in cash; what is the rate of interest?

***14.** A tradesman marks his goods with two prices, one for ready money and the other for a year's credit, allowing discount at 4 per cent.; if the credit price be R45. 8*a*., what ought to be the cash price?

15. A shawl-vendor sells a piece of shawl for R42. 8*a*. at 4 months' credit, the interest being reckoned at 6 per cent. per annum. If he be paid cash, what ought he to demand?

CHAP. XXXVII. COMMERCIAL TRANSACTIONS.

338. **Commercial Allowances.** In buying and selling goods by weight there are usually certain allowances or reductions made from the weight for the boxes, packages, &c. which contain the goods.

Tare is the weight of the boxes, &c., containing the goods; it is deducted at so much a box or cwt., &c.

The total weight of the goods is called the **gross** weight; the weight which remains after the *tare* has been deducted is called the **net** weight.

Draft is a deduction made on some goods, to allow for the loss of weight in selling by retail.

Ex. What is the net weight of 9 chests of tea, weighing 13 cwt. 0 qr. 22 lbs., tare 4 lbs. per chest and draft 18 lbs. per cwt.?

	cwt.	qr.	lbs.	
	13	0	22	gross.
4 lbs. × 9		1	8	tare.
	12	3	14	lbs.
16 lbs. = $\frac{1}{7}$ of 1 cwt.	1	3	10	
2 lbs. = $\frac{1}{8}$ of 16 lbs.			26	
	2	0	8	draft.
	10	3	6	net.

[The tare is deducted first, and the draft is calculated on the remainder by Practice.]

339. **Commercial Discount.** We have already seen how the discount on a Bill should be calculated so that neither the buyer nor the seller might be benefited to the disadvantage of the other (see ART. 332). The amount of discount obtained in this way (being the interest on the Present Worth) is called the **True Discount.** In commercial circles, however, the deduction that is *always* made is the interest on the sum due. and not this true discount, which is the interest on its present worth. Discount calculated in this way is called the **Commercial Discount.**

Thus, it was seen before that the *True Discount* on R505 due 3 months; hence at 4 per cent. interest is R5. The *Commercial Discount*, however, is the interest on R505 for 3 months at 4 per cent. and is therefore R505 $\times \frac{4}{100} \times \frac{3}{12} =$ R5 0*a*. 10*p*. nearly.

The Commercial Discount is thus always *greater* than the true Discount; and the transaction is *to the advantage of* the buyer.

In practice it is usual to allow to the buyer a further time of 3 days after a Bill has become due. These three days are called the **Three days of Grace.** Although originally a mere custom, it has now all the force of law. When months are mentioned, **Calendar Months** have to be considered

Thus, if a Bill is dated the 3rd March and is payable, at 3 months' sight, the sum of the Bill is actually payable, not on the 3rd June, but on the 6th June. The Bill is said to be nominally due on the 3rd June and legally on the 6th June. In discounting a Bill interest is of course charged for these three days also.

In calculating the number of days, it is usual to neglect the day on which the money falls due.

Ex. 1. Find the banker's discount on *Rs.* 4635 due in $2\frac{1}{2}$ yrs. at $2\frac{3}{4}$ *p.c.*

The Banker's discount is the interest on the given sum.

Interest or Banker's discount on *Rs.* 100 for $2\frac{1}{2}$ yrs. at $2\frac{3}{4}$ *p.c.* $=$ *Rs.* $\frac{55}{8}$.

$\therefore$ interest on *Rs.* 4635 $=$ *Rs.* $\frac{55 \times 4635}{8 \times 100} =$ *Rs.* 318, 10*a.* 6*p.* *Ans.*

Note. In examples like the above where no dates are mentioned, the Days of Grace need not be considered.

Ex. 2. What will a banker gain on discounting a bill of *Rs.* 12750, drawn on March 4th at 10 mo., and discounted on August 14th at 5 *p. c.*?

The Bill is nominally due on the 4th of January and therefore legally due on the 7th. Also it is discounted on the 14th August, and the number of days between 14th August and 7th January is 146

Interest on *Rs.* 100 for 146 days at 5 per cent. $=$ *Rs.* $5 \times \frac{146}{365} =$ *Rs.* 2.

Rs. 102 : *Rs.* 12750 :: *Rs.* 2 : true discount, $\therefore$ true discount $=$ *Rs.* 250;

Rs. 100 : *Rs.* 12750 : : *Rs.* 2 : banker's disc.; $\therefore$ banker's disc. $=$ *Rs.* 255.

$\therefore$ gain of the banker $=$ *Rs.* 255 $-$ *Rs.* 250 $=$ *Rs.* 5 *Ans.*

Examples 146.

[Omit fractions of a pie or penny whenever they occur.]

1. A bill of R96000, due 1 year 10 months hence at $4\frac{3}{8}$ per cent., is discounted by a banker; how much will the holder of the bill receive?

2. Find the difference between the banker's and the true discount on R1291. 12*a* 8*p*. due in 5 months, at $7\frac{1}{2}$ per cent.

3. Find the gain of a banker by discounting a bill of R7249. 6*a*. due 9 months hence, at $7\frac{1}{8}$ per cent.

4. How much less than the true present value will a banker give me for a bill of R9502. 10*a*. due 7 months hence, at $6\frac{1}{4}$ per cent.?

5. Find the difference between the true and the banker's discount, on a bill of £7310. 5*s*., due 15 months hence, at $5\frac{1}{2}$ per cent.

6. If £2700 be due 8 mo hence, what is the difference between the true and the practical discounts on it, interest being reckoned at 5 p. c.?

7. How much will a banker gain by discounting a bill of £2851. 4*s*. which has $7\frac{1}{2}$ months to run, reckoning interest at 5 per cent.?

8. What will a banker retain on discounting a bill of R51000, drawn on March 4th at 10 months' sight, on August 4th at 4 per cent.?

9. A bill of R3079. 11*a*, drawn on the 1st of April at 6 months, is discounted on the 31st of May; what does the banker gain, interest being reckoned at $6\frac{1}{4}$ per cent.?

10. What will a banker give on 1st July for a bill of R19314. 9*a*. 4*p*. drawn on the 10th May at 6 mo, interest being reckoned at $5\frac{1}{3}$ per cent.?

11. A bill of £3695. 12*s*. 6*d*. drawn on April 6th at 9 months, is discounted on 4th June; find the gain of the banker at $3\frac{1}{4}$ per cent.

12. A bill of £1088 is dated on May 23rd at 4 months, and is discounted on July 7th at $1\frac{1}{2}$ per cent.; what does the banker gain thereby?

340. **Commission and Brokerage. Commission** is a charge of so much per cent. made by an **Agent** for buying or selling goods &c., on account of another. The rate usually varies from 1 to 10 per cent.

Brokerage is a similar charge made by **Brokers** for helping others to buy or sell goods, shares, &c. The rate is usually less than 1 per cent.

341. Examples on Brokerage and Commission are worked out in the same way as Examples on Percentage.

Ex. 1. An agent buys some goods for *Rs.* 5000 for another, and receives a commission of 3 per cent.: what does he get?

The commission $= \frac{3}{100} \times$ *Rs.* 5000 = 150. *Ans.*

Ex. 2. A broker sells some shares for £5400, and is given $\frac{1}{8}$ per cent.; how much does he get?

The amount required $= \frac{1}{8} \times \frac{1}{100} \times$ £5400 = £6. 15*s*. *Ans.*

Examples 147.

1. A broker purchases R50000 worth of goods; what commission does he get at $\frac{1}{4}$ per cent?

2. An agent buys a house for R25500, and receives a commission of $1\frac{1}{2}$ per cent.; what sum does the buyer pay?

3. A broker received $\frac{1}{8}$ per cent. in buying some goods; if his brokerage amounted to £105, what was the value of the goods bought?

4. An agent sells a certain number of barrels of flour at £1. 5s. each, and gets £23 as commission at $2\frac{1}{2}$ p. c Find the number of barrels.

5. A tax-gatherer collects R57600 at $1\frac{1}{3}$ per cent.: what commission does he retain, and how much money does he pay over?

342. Invoice, Account-Sales, &c.

An **Account-Sale** is an account drawn out by a Commission Agent or broker shewing the sales he has made of goods on account of another party. It is of the following form.

ACCOUNT-SALE of 12 tierces of sugar, per *Britton* from London, on account of Charles Thomas Esq., Calcutta.

C.T.		cwt.	qr.	lbs.		cwt.	qr.	lbs.		£	s.	d.
No.	3 tierces	24	2	6	gross.	2	3	10	tare.			
	5 „	37	3	21	„	3	1	5	„			
1 to 12	4 „	31	1	14	„	2	0	19	„			
		93	3	13	„	0	3	7	draft			
	Deduct	9	0	13		9	0	13				
	Net	84	3	0	at £2. 18s. 6d. per cwt. ...					247	17	10
						cwt.	qr.	lbs				
	Overtaker	5	1	19	gross.	0	3	19	tare.			
	Deduct	1	0	5		0	0	14	draft			
	Net	4	1	14	at £2. 10s. per cwt. ...					10	18	9
									Gross proceeds ...	258	16	7

CHARGES.

	£	s.	d.	£	s.	d.
Insurance on £275, at 2 per cent. ...	5	10	0			
Policy duty	0	14	3			
Freight on 100 cwt. at $8\frac{1}{2}$*d.* per cwt. ...	3	10	10			
Primage and dock dues	8	3	9			
Other charges	0	18	7			
Insurance of fire	0	12	8			
Sale charges	1	1	4			
Commission on £258. 16*s.* 7*d.* at 2 p.c. ...	5	4	8			
„ on £275. 0 0 at $\frac{1}{2}$ p.c ...	1	7	6			
				27	3	7
Net proceeds due to Charles Thomas Esq. ...				231	3	0

London:
The 4th Feby. 1892.

ANDREW JONES.

When a merchant (James Howard) of London ships goods &c., on account of a merchant (Thomas Reid) of Marseilles, he sends with the goods a paper containing an account of the quality and quantity of the goods sent. This is called an **Invoice.** It is different in many respects from the simpler invoices described in *p.* 189, and is of the following form.

INVOICE of 30 hhds. of Tobacco shipped on board the *Adventure*. Charles Clare, master, for Marseilles on account and risk of Thomas Reid.

T.R.		£	s.	d.
	No. cwt. qr. lbs. cwt. qr. lbs.			
	1 20 3 21 gross. 1 2 10 tare.			
1 to 3	2 18 2 13 " 1 1 14 "			
	3 21 1 7 " 1 2 26 "			
	60 3 13 " 4 2 22 "			
	4 2 22			
	56 0 19 net at 8*d*. per. lb. ...	209	14	0
	CHARGES. £. s. d.			
	Bond and Custom-house entry ... 0 8 6			
	Cost of empty hogsheads 2 13 0			
	Small charges 1 3 6			
	Bills of lading 0 1 9			
	Brokerage on £210 0 0 at ½ p. c. ... 1 1 0	5	7	9
	Commission on " 215 5 9 " 2 " ... 4 6 1			
	Insurance on " 225 " 2 " ... 4 10 0			
	Commission on " 225 " ½ " ... 1 2 6			
	Policy duty 0 15 0	10	13	7
	Errors Excepted	225	15	4
	London : The 15th Nov., 1892. } JAMES HOWARD.			

343. **Insurance. Insurance** is a contract by which certain persons or insurance-offices engage to make good, to the party ensuring any loss he may sustain of ships or their cargoes at sea, or of houses or goods by fire.

The parties who take the risk are called **Insurers** or **Under-Writers,** and the party protected is called the **Insured.**

The sum paid to the Insurers is called the **Premium**; the stamped paper on which the contract is drawn up is called the **Policy of Insurance**; and the stamp-duty on the policy is called the **Policy duty.**

(1) *To find the premium on the sum insured.*

Proceed as in the following Example.

Ex. What is the cost of insuring a cargo valued at *Rs.*7547. 4*a.* 8*p.*, the premium being 3 per cent., the policy duty ¼ per cent., and the commission ½ per cent. ?

Total charge per cent. $= 3 + \frac{1}{2} + \frac{1}{4} = 3\frac{3}{4}$.

Rs. 7547 4*a*. 8*p*.	$\frac{3}{4} = \frac{1}{4}$ of 3	*Rs.* 226. 6*a*. 8·4*p* ÷ 4.
3		56 9 8·1
Rs. 226,41. 14*a*.		*Rs.* 283 0*a* 4·5*p*.
= *Rs.* 226. 6*a*. 8·4*p*.		= the expense reqd.

(ii) *To find how much must be insured, in order to cover a given sum besides paying all expenses of premium, &c.*

Proceed as in the following Example.

Ex. What sum must be insured to cover £3780 in case of loss, the premium being $4\frac{1}{2}$ *p.c.*, policy duty 5*s*. *p.c.*, and commission $\frac{3}{4}$ *p.c.*

Deduct from		£100
Premium £4.	10*s*	
Policy duty	5*s*.	
Commission	15*s*.	£ 5. 10*s*
		£94. 10*s*.

If the sum insured be £100, the sum of £5 10*s*. has to be paid for premium &c. so that only £94. 10*s*. are left. Hence £100 should be insured for every £$94\frac{1}{2}$.

£94. 10*s*. : £3780 : : £100 : reqd. sum ;

∴ reqd. sum = £4000. *Ans.*

Examples 148.

1. Find the premium on a policy for R12582. 8*a*. at $6\frac{1}{4}$ per cent.

2. A ship is worth R560000, and is insured for $\frac{3}{4}$ of its value : what does the premium amount to at 3 per cent. ?

3. How much must be paid to insure a cargo worth £7500, the premium thereupon being $1\frac{1}{4}$ per cent.

4. For how much must a person insure a cargo worth £24250 at 3 per cent., so that both the cargo and premium may be recovered ?

5. For what sum must a person insure a house worth R38200 so that in case of loss, its value and all expenses connected with its insurance may be recovered, the premium thereupon being 3 per cent., policy duty $\frac{3}{8}$ per cent. and brokerage $\frac{5}{8}$ per cent. ?

6. Cargo worth £766460 is to be insured, so that in case of loss its value, premium, and all other expenses connected with the insurance may be recovered. The premium is 3 per cent., policy-duty $1\frac{1}{2}$ per cent., and brokerage $\frac{1}{2}$ per cent. For what must the cargo be insured ?

344. Bill of Lading, Bill of Exchange, &c.

When a merchant in Calcutta ships goods to a merchant in London, he takes from the Captain of the ship an acknowledgment that such and such goods have been made over to his charge. This is called a **Bill of Lading** ; and the Captain will not deliver the goods to any one who does not present this receipt to him.

The **Shipper** or **Consignor** also draws a **Bill of Exchange** on the **Consignee** ordering him to pay to his agent or somebody else in his behalf the value of the shipment.

This Bill, together with the Bill of Lading and the policy of Insurance he may, if he chooses, forward to his agent in London for presentation to the *Drawee*; but as this will necessarily involve some delay in his getting the money, he will usually go to a bank and ask it to **discount** the bill, *i.e.*, pay over the value (less the commission, of course) on the strength of the documents. If the bank agrees, the documents will have to be **hypothecated,** *i.e*, pledged to the bank. The bank now becomes master of the goods, and realizes the money in London through its agents. When the Bill of Lading and the policy of Insurance are attached to the Bill, it is called a **Documentary Bill**; otherwise a **Clean Bill**.

When one merchant sells certain goods to another, the Bill that he draws on the other is called **Draft.** If the buyer agrees, he usually **accepts** it, *i.e.*, puts his signature just below that of the seller. The Draft now becomes a **Bill.** (See Art. 334, *p.* 280.)

345. **Exchange. Exchange** is the method by which sums in the money of one country are converted into sums of equivalent value of another.

Thus by Exchange we find the value of £3 in R, and *vice-versa.*

Par of Exchange is the sum in the currency of one country which *intrinsically* or *really* equal to a given sum in the currency of another.

Thus the quantity of gold in £1 sterling being 25·22 times the quantity of good in a franc, the *Par of Exchange* of £1 sterling is 25 22 francs.

The **Course of Exchange** at any time is the sum in the currency of one country which, at the time, is actually equal to a given sum of money in the currency of another.

Thus, owing to a variety of causes, £1 is sometimes more and sometimes less than 25·22 francs. If £1 = 25·30 francs, the *Course of Exchange* would be 25·30.

When the exchange is made by remitting bills through a third place, the rate is called the **Arbitrated Rate.**

Foreign Money (*a*) Gold Currency.

Austria	100 kreutzers	=	1 florin	= 1s. 11½d. nearly.
France	100 cents	=	1 franc	
Belgium	100 „	=	1 „	
Italy	100 centesimi	=	1 lira	= 9½d.
Spain	100 centimos	=	1 peseta	
Greece	100 lepta	=	1 drachma	
United States Canada	100 cents	=	1 dollar	= 4s. 2¾d.
Denmark Norway Sweden	100 ore	=	1 krone	= 1s. 1⅛d.
Germany	100 pfennig	=	1 mark	= 11½d.
Holland	100 cents	=	1 florin	= 1s. 8d.
Portugal	1000 reis	=	1 milrea	= 3s. 8¼d.

(b) SILVER CURRENCY.

India	192*p*. $=$ 16*a*.	$=$ 1 Rupee.	
China	100 candareens $=$ 10 mace.	$=$ 1 tael	$=$ *Rs.* 3.
Russia	100 copecks	$=$ 1 rouble	$=$ *Re* 1. 12*a*. 3*p*.
Turkey	100 paistres	$=$ 1 pound	$=$ *Rs.* 12. 12*a*.

As the value of silver for some years has been steadily declining or **depreciating** as compared with gold, the course of Exchange of the Rupee, Tael, Rouble, &c., has been continuously declining. Twelve years ago a Rupee was equal to about 2*s*.; now it is only about 1*s*. 4*d*.

Note. The application of the **Chain Rule** is the easiest method of working out Examples on Exchange. (See *p*. 142.)

Ex. 1. What is the value of £548. 18*s*. 4*d*. in Indian money, the course of exchange being 1*s*. $5\frac{1}{2}$*d*. per rupee?

Reqd. no. of *Rs.* $=$ £548. 18*s*. 4*d*. or £$\frac{6587}{12}$;

1*s*. $5\frac{1}{2}$*d*. or £$\frac{7}{96}$ $=$ *Re*. 1.

$\therefore$ no. of *Rs.* reqd. $= \frac{6587}{12} \div \frac{7}{96} = 7528$. *Ans.*

Ex. 2. A person in London owes another at Petersburg 1500 roubles, exchange at 40*d*. sterling per rouble. He remits to Paris at 24 francs for a pound sterling; thence to Lisbon at 500 reis for 3 francs; thence to Amsterdam at 20 stivers per crusado of 400 reis; and thence to Petersburg at 25 stivers per rouble; find the arbitrated rate between London and Petersburg, and the gain or loss by this circuitous mode of remittance.

Let x be the exchange value of the rouble in *d*.

reqd. no. of *d*.	$=$	1 rouble
1 rouble	$=$	25 stivers
20 stivers	$=$	400 reis
500 reis	$=$	3 francs
24 francs	$=$	240*d*.

$\therefore x = \frac{25 \times 400 \times 3 \times 240}{20 \times 500 \times 24} d. = 30d.$

$\therefore$ he gains $(40-30)$ or 10*d*. per rouble;
and his gain $= 1500 \times 10d. =$ £62. 10*s*.

Ex. 3. Convert *Rs.* 5000 into English money, when English money is at a premium of 25 per cent.; the par of exchange being 2*s*. per rupee.

English money being at 25 *p. c.* premium, is worth 25 *p. c.* more of Indian money than it would be if it were at par.

At par, 2*s*. $=$ *Re*. 1 : $\therefore$ at 25 *p. c.* premium, 2*s*. $=$ *Re*. 1 $+$ *Re*. $\frac{1}{4}$ $=$ *Rs*. $\frac{5}{4}$

$\therefore$ *Re*. 1 $= \frac{4}{5} \times 2s. = \frac{8}{5}s.$;

and *Rs.* 5000 $= \frac{8}{5} \times 5000s. =$ £400. *Ans.*

Ex. 4. The par of exchange between England and India is 2*s*. a rupee. Find the course of exchange when Indian money is at a discount of 30 per cent.; also exchange *Rs.* 25000 for English money.

Indian money being at 30 *p. c.* discount, it is worth 30 *p. c.* less in English money than it would be if were at par.

At par, *Re*. 1 $=$ 2*s*.; $\therefore$ at 30 *p. c.* discount, *Re*. 1 $= 2s. - \frac{3}{10} \times 2s. = \frac{7}{5}s.$;

$\therefore$ the course of exchange $= \frac{7}{5}s. = 1s.\ 4\frac{4}{5}d.$ }
Also *Rs.* 25000 $= \frac{7}{5} \times 25000s. =$ £1750. } *Ans.*

Examples 149.

1 If £300 be due from London to Paris when £1 is worth 25 francs what sum must be remitted when a guinea is exchanged for 27 francs?

2 The course of exchange between Calcutta and London is 1*s.* $4\frac{1}{2}$*d.* per Rupee; find the value of R12750 in English money.

3 How many pounds Flemish can I receive for £3160 sterling, the course of exchange being 35 shillings Flemish for £1 sterling.

4. An American-dollar and a rupee are respectively worth 4*s.* $2\frac{1}{2}$*d.* and 1*s.* 4*d.*: find the course of exchange, and also the value in Indian money of a bill of 2456 dollars.

5. If the exchange between Paris and London be 25 fr. for £1 and that between Amsterdam and London be 33*s.* 9*d.* for £1.; what is the arbitrated price between Paris and Amsterdam?

***6**. The rates of exchange being £1 = 25·4 francs, 3·75 francs = 105 kerutzers, 60 kerutzers = 1 florin, and the cost of travelling in Germany being $1\frac{1}{2}$ florins per German mile which is equal to $4\frac{1}{3}$ mi.: find the expense in English money, of travelling 2667 mi. in Germany

7. What is the face of a bill on Paris for which I pay R54000 in Calcutta, the course of exchange between Calcutta and London being 1*s* $4\frac{1}{3}$*d*. per R and between London and Paris 25 20 francs per £?

8. The exchange at Paris upon London at the rate of 25 francs 60 cents. for £1 sterling, and the exchange at Vienna upon Paris is at the rate of $40\frac{1}{3}$ Austrian florins for 100 francs: find how many Austrian florins should be paid at Vienna for a £150-note.

***9**. The exchange between St. Petersburg and London is 50*d.* per rouble, between St. Petersburg and Amsterdam 91*d.* Flemish per rouble, between Amsterdam and London 34*s.* 8*d.* Flemish per £ sterling. If a London merchant owes 75000 roubles to a banker at St. Petersburg, what difference will be made by the merchant being drawn upon direct of through Amsterdam?

10. The par of exchange between England and Calcutta is 2*s* per rupee. Find how much a merchant at Calcutta must pay for goods worth £2000 sterling, English money being at 25 per cent. premium.

11. The par of exchange between England and India is 2*s.* for the rupee: a merchant in Calcutta pays R100,000 for goods bought in England when the Indian money is at a discount of 25 per cent. How much should they cost in England?

***12**. A person in London owes another at St. Petersburg a debt of 230 roubles, which he has to remit through Paris. He pays the requisite sum to his broker, at a time when the exchange between London and Paris is 23 fr. for £1, and between Paris and St. Petersburg 1·2 fr. for a rouble. The remittance is delayed until the rates are 24 fr. for £1, and 3 fr. for 2 roubles. What does the broker lose or gain by the delay?

13.* A merchant in London owes 1000 pistoles to a merchant in Cadiz; find how much he gains by sending it to him through France; the exchange being £1 = 25·6 francs, 19 fr. = 1 pistole, 4 pis. = £3.

346. **Annuities.** An **Annuity** is a periodical income payable at equal intervals, as yearly, half-yearly, &c.

Annuities are called *certain* when they are to continue for a fixed number of years; they are called *perpetual* when they continue for ever; when they are to be paid only so long as one or more individuals shall live they are called *contingent* or *life* annuities.

When an annuity is payable at present, it is said to be *in possession*; but when the payment is not to commence until some time has elapsed, it is called *deferred* or *reversionary*.

An annuity is said to be worth as many **year's purchase** as there are pounds in the annuity of £1. Freehold estates are generally bought and sold at *so many years' purchase* or at so many years' rent.

Ex. 1. If a freehold estate be worth 25 years' purchase, find the rate of interest.

If the annual rent be £1, the value of the estate is £25.

∴ rate of interest $= (\frac{1}{25} \times 100)$ or 4 per cent. *Ans.*

Ex. 2. If the rate of interest be 5 per cent., how many years' purchase is an estate worth?

The interest would amount to £100 in $(100 \div 5)$ or 20 yrs.

∴ the estate is worth 20 years' purchase. *Ans.*

Ex. 3. An estate is bought at 20 years' purchase for £20000, three-quarters of the purchase money remaining on mortgage at 4 per cent. The cost of repairs averages £150 per annum. What interest does the purchaser get on his investment?

$\frac{3}{4}$ of £20000 or £15000 remains on mortgage; so he practically invests only £5000. Also the estate being bought at 20 years' purchase he gets annually $\frac{1}{20}$ of £20000 = £1000.

Now, he has to pay $\frac{4}{100} \times$ £15000 or £600 for interest on mortgage

∴ his total expenses are £600 + £150 = £750.

∴ on £5000 he gets an annual income of £1000 − £750 = £250.

5000 : 100 : : 250 : rate per cent.; ∴ rate per cent. = 5. *Ans.*

Examples 150.

1. A gentleman purchased a free-hold estate for R20000; what is the annual rent, if it is worth 16 years' purchase?

2. I paid down £1000 for a perpetual annuity; what was the yearly annuity, supposing interest to be at 5 per cent?

3. How many years' purchase must an estate be worth, that the purchaser may have $6\frac{1}{4}$ per cent. for his money?

4. The annual rent of a free-hold bought for £6265. 12*s*. 6*d*., is £250. 12*s*. 6*d*.; at how many years' purchase was it bought?

5. An estate was bought at 25 years' purchase for R30,000, two-thirds of the purchase money remaining on mortgage at 3 per cent. The cost of repairs averages R200 per annum. What interest does the purchaser secure on his investment?

CHAP. XXXVIII. STOCKS.

347. **Stock.** When money is lent to a Government, to a Railway Company, or to a Corporation, it is called **Stock.**

When, a government has some great undertaking before it such as a great war or the building of a large Railway, it generally borrows money for the purpose. If the money is borrowed from people of other nations it is called a **Foreign Loan,** or more simply, a **Loan**; but if it is borrowed from the nation itself it is usually called a **National Debt.** The way in which a government borrows money is somewhat different from that in which a private individual does so. Suppose, for example, the government of India wants to borrow 10 crores of rupees. It proceeds **to float** a loan for the purpose; *i e.*, it declares that it will borrow money up to 10 crores at 4 per cent. interest, and invites the public to make their offers for the whole or any part of this money within a certain date. Those that have money to spare come forward with their **Tenders.** It may, however, be that *at the time* money is worth more than 4 per cent. *i e*, that people may get more than 4 per cent. by investing it in other ways. In this case they will not offer so much as R100 for the 4 rupees that they would annually get, but somewhat less; on the other hand, if money is cheap, *i.e.*, if people cannot get so much as 4 per cent. by any other kind of investment, they will gladly pay more than R100 for the R4 a year. Government now accepts the offers of those that name the highest figures coming down in the list until it has got all the 10 crores it wants. Money borrowed or lent in this way is called **Stock.**

It will, therefore, be seen that the Government obtain sometimes more and sometimes less than R100 for the R4 which it undertakes to pay in the year. Suppose, in the above case, it got from *A* a tender at the rate of R102. *A* pays down in cash R102 for which he gets a **Bond** for R100 bearing interest at 4 p c. *A* will not ordinarily get back his money from Government; if he wants ready money, he must go to the market where others are prepared to buy the bond. Let us suppose that *A* offers the bond for sale 6 months after. In the meanwhile the state of the money market has very likely changed. Owing to the exigencies of trade and other causes the demand for money may have increased and merchants may now be offering a higher rate of interest, so that no one is now willing to buy the note for R102. Probably the bond is now at **99,** *i.e.*, *A* can realise only R99 by selling the bond which he had before bought for R102.

It should be carefully remembered that while the *nominal* value of the bond remains the same, *viz.*, R100, its *actual* value, as a rule, changes from day to day, according to the condition of the money market; and that, whatever the actual value of the bond may be, the holder of it will get (in two half-yearly instalments) *Rs.4 interest* in the year.

The dates of the British and the Indian Government are in the form of **Promissory Notes** . the former stock is called the **Funds** and the latter **Government Securities**. The **Consols** (abbreviated from Consolidated Annuities) are only a particular kind of the British Government stock. The **Reduced** Consols are certain consols whose rate of interest was *reduced* at some time subsequent to the date of issue.

Debentures. In many cases the whole sum required is divided into 'hundreds' or 'thousands' and instead of the people being invited to tender what amount they can, they are called upon to send in their offers for these fixed sums. For instance, in the above illustration, the 10 crores would be divided into 10 lac 'hundreds,' and 10 lac papers called **Debentures** issued, each paper stating that the India Government is *indebted* to the bearer of it to the amount of R100 and promising to pay upon it an interest of 4 p. c. per annum.

The difference between Stocks and Debentures is principally this. While stocks may be bought and sold to *any amount* and may not be bought back or **redeemed** at any time the Government merely undertaking to pay interest thereon, Debentures must be bought *entire* (no fractional part of the nominal value being ever available) and are declared to be redeemable at their nominal value at some future date.

Shares. Money wanted for private Companies is raised in a different way. Suppose certain persons to start a company for working a coal mine. These men, called **Promoters,** first settle what sum of money or **Capital** will be required for the purpose, and generally establish a **joint Stock Company.** They divide the whole amount required into a number of **Shares** and invite the public to **subscribe** to them. If it is believed that the Company will *pay i. e.*, give a fair rate of interest, the people, sooner or later, take up all the shares and become **Shareholders.** The subscribers are not, as a rule, required to pay down the whole amount at once; they have to pay in instalments, as the work progresses and **Calls** are made. Sometimes the entire capital is never required, but only a part of it, each subscriber of course paying his portion of the share. The part which is thus paid is called the **Paid-up Capital** As soon as the concern begins to yield profits, they are divided amongst the shareholders in the following manner. Suppose the subscribed capital to be 10 lacs of rupees in 10000 shares of R100 each, and the paid-up capital 6 lacs or R60 per share. If the profits at the end of a half-year be R25000, it is not usual to divide the whole of it amongst the shareholders. A certain portion of it, say R10000, would be reserved for contingencies, or carried to the **Reserve Fund** : and the remaining R15000 would be used in paying **Dividends,** *i. e.*, divided amongst the share-holders, as at so much per share or at so much per cent. In the latter case, the percentage is calculated on the paid-up capital only. Thus in the present example, the dividend would be R15000 for 10000 shares or R1. 8*a.* per share; while the percentage would be R1 8*a.* on R60 in the half-year, *i. e.*, 5 per cent. per annum.

It is clear that the market value of the shares will depend very much on the dividend which is declared; a large dividend sending the price up, and small dividend bringing it down.

Preference Shares When a company wants to increase its capital, it does not increase the number of original shares, but borrows money at a fixed rate of interest in the manner described before Such shares are called **Preference Shares,** and sometimes **debentures**: and the interest on them *must* be paid before any dividends can be declared.

Par, Premium, &c. The student must by this time have understood that the market values of Stocks, Debentures, Shares, &c, will very rarely be the same as their nominal value. When their market values are the same as their nominal values, they are said to be at **par**; when greater, at **Premium** or **above par**; when less, at **Discount** or **Below par.** Thus if R100 stock sells at R100 it is said to be *at par* if at R104, *at 4 Premium* or *above par*; and if at 97, *at 3 Discount* or *below par.*

Brokerage Stocks, Debentures, Shares, &c., are usually bought and sold through the agency of certain persons called **Stock-Brokers** or **Brokers**. They usually charge $\frac{1}{8}$ per cent. on the *nominal* value of the stock both in buying and selling. When, therefore, the market value of a hundred-rupee share is R101. 8*a*, the buyer will have to pay for it R101. 10*a* while the seller will get only R101 6*a*.

Ex. 1. What is the price of *Rs* 40000 stock in the $3\frac{1}{2}$ per cents. at 95 ?

In this case *Rs.* 100 stock costs *Rs.* 95.

∴ *Rs.* 100 stock : *Rs.* 40000 stock : : *Rs* 95 : reqd. sum of money ;

or the reqd. sum of money = *Rs.* 38000 *Ans.*

Ex. 2. What sum must be invested to purchase £2500 stock in the 3 per cents at $97\frac{1}{2}$, brokerage as usual ?

Here £100 stock costs, with brokerage, £$(97\frac{1}{2}+\frac{1}{8})$ = £$97\frac{5}{8}$.

∴ £100 stock : £2500 stock : : £$97\frac{5}{8}$: reqd. sum of money ;

or reqd sum of money = £2440. 12*s.* 6*d.* *Ans.*

Ex. 3 If I sell £2000 of 3 per cent. consols at $93\frac{7}{8}$ and have to pay $\frac{1}{8}$ *p. c* for brokerage, what sum shall I receive ?

For every £100 stock I get £$(93\frac{7}{8}-\frac{1}{8})$ or £$93\frac{3}{4}$, after paying brokerage.

∴ £100 stock : £2000 stock : : £$93\frac{3}{4}$: reqd. sum of money ;

or reqd sum of money = £1875. *Ans.*

Ex. 4 What sum must a person invest to purchase £15000 in the 4 per cent consols at $4\frac{7}{8}$ per cent. premium, the brokerage being as usual ?

The stock being at $4\frac{7}{8}$ *p. c.* premium, £100 stock costs with brokerage £$(104\frac{7}{8}+\frac{1}{8})$ or £105.

∴ £100 stock : £15000 stock : : £105 : reqd. sum of money ;

or reqd. sum of money = £15750. *Ans.*

Ex. 5. How much stock can be purchased by investing £4650 in the 3 per cents. at 93 ?

£93 money will purchase £100 stock ;

∴ 93 : 4650 : : £100 stock : reqd. amount of stock ;

or reqd. amount of stock = £5000. *Ans.*

Ex. 6. What quantity of stock can be purchased by investing £7120 in the $3\frac{1}{2}$ per cents. at $88\frac{7}{8}$ through the agency of a broker?

£$(88\frac{7}{8}+\frac{1}{8})$ or £89 will be required to purchase £100 stock.

∴ 89 : 7120 : : £100 stock : reqd. amount of stock ;

or reqd. amount of stock = £8000. *Ans.*

Ex. 7. A gentleman has £10000 stock in the 4 per cents. at $93\frac{1}{8}$; how much of it must he sell through a broker to realise £2650. 10*s.* ?

By selling every £100 stocks he receives £$(93\frac{1}{8}-\frac{1}{8})$ or £93.

∴ £93 : £$2650\frac{1}{2}$ :: £100 stock : reqd. quantity of stock ;

or reqd. quantity of stock = £2850. *Ans.*

Ex. 8. A man laid out a certain sum of money in the purchase of $4\frac{1}{2}$ per cent. Government Securities at 95, and sold them when the price rose to 97, gaining thereby *Rs.* 1000 ; what money did he invest?

By selling every *Rs.* 100 stock, *i.e. on every Rs. 95 invested,* he gained *Rs.* (97 − 95) or *Rs.* 2.

∴ *Rs.* 2 : *Rs.* 1000 :: *Rs.* 95 : sum invested :

or sum invested = *Rs.* 47500. *Ans.*

Note. In Examples like the above, *where no questions* of **Income** *comes in* the rate of interest need not be considered.

Examples 151.

1. What sum of money must be invested in the purchase of :

(1) R5000 in the 3 per cents. at 94?

(2) £4750 in the $3\frac{1}{2}$ per cents. at 96?

(3) R4760 in the $3\frac{3}{4}$ per cents. at par?

(4) £7950 in the $4\frac{1}{2}$ per cents. at $4\frac{1}{2}$ per cent. premium?

(5) R8450 in the $3\frac{1}{8}$ per cents at 8 discount?

(6) R3400 in the $3\frac{1}{4}$ per cents. at 87, brokerage as usual?

(7) £9750 in the $3\frac{3}{4}$ per cents. at $93\frac{7}{8}$, brokerage $\frac{1}{8}$ per cent.?

2. If I have to sell R7400 stock in the $3\frac{3}{4}$ per cents. at $95\frac{3}{8}$ through a broker in order to pay off a bill, find the amount of the bill.

3. How much money will a man get from the sale of R45500 Govt. Securities at 4 per cent. premium, the brokerage being $\frac{1}{8}$ per cent. ?

4. I had R15000 stock in the 4 per cents. at 97, and sold out when the price of the stock rose to par ; what did I realise, after paying brokerage at the usual rate?

5. What quantity of stock can be purchased with?

(1) R5040 in the 3 per cents. at 96.

(2) £1870 10*s.* in the $3\frac{3}{4}$ per cents. at 87.

(3) £8080 in the 4 per cents. at 1 premium.

(4) R10080 in the 4 per cents. at 4 discount.

(5) R21990 in the $3\frac{1}{2}$ per cents. at $91\frac{3}{8}$, brokerage $\frac{1}{4}$ per cent.

(6) £1425 in the $2\frac{1}{2}$ per cents. at $74\frac{7}{8}$, brokerage as usual.

6. How much of $3\frac{1}{2}$ per cent. stock at $98\frac{1}{8}$, must I sell out, so that after paying the brokerage I may obtain R4900 ?

7. A person intending to enter into a partnership with £9350, sells out 3 per cent. stock at $93\frac{3}{8}$ through a broker. Find the amount of stock sold.

8. I invest £32490 in the purchase of $3\frac{1}{2}$ per cent. stock at $89\frac{7}{8}$ and afterwards sell out at $90\frac{3}{8}$; how much do I gain, the usual brokerage being charged on each transaction ?

9. A person laid out £14076 in the 4 per cents. at $97\frac{3}{4}$; and when they had risen to 2 premium he sold out and invested the proceeds in the Turkish Bonds at 68 ; find the quantity of Bonds bought.

10. A person lays out a certain sum of money in Railway shares when they are at $5\frac{1}{2}$ per cent. discount, and sells them again when they are at $10\frac{1}{4}$ per cent. premium, gaining thereby R1666. 10*a*. 8*p*.; find the money invested.

11. How much 4 per cent. stock can be purchased by the transfer of R25000 stock from the 3 per cents at 72 to the 4 per cents. at 90 ?

12. By purchasing Railway shares at $22\frac{1}{2}$ per cent. discount, and selling out again at 9 per cent. premium, I gained R3150 ; what was the original sum invested ?

Note. A man is said to **transfer** one kind of stock to another when he sells out the first kind, and with the proceeds buys the latter kind of stock.

348. In the following Examples, the interest paid has also to be considered.

Ex. 1. What annual income will be derived by investing *Rs.* 55800 in the 3 per cents. at 93 ?

Rs. 93 will buy *Rs.* 100 stock, the interest on which is *Rs.* 3 ; therefore for every *Rs.* 93 invested, an income of *Rs.* 3 will be obtained ;

∴ *Rs.* 93. : *Rs.* 55800 :: *Rs.* 3 : annual income;

or annual income = *Rs.* 1800. *Ans.*

Ex. 2. What sum must a person invest in the $3\frac{1}{2}$ per cent. stock at 91 to have an annual income of *Rs.* 490 ?

To get *Rs.* $3\frac{1}{2}$ annually he will have to invest *Rs.* 91.

∴ $3\frac{1}{2}$: 490 :: *Rs.* 91 : reqd. investment ; or reqd. invt. = *Rs.* 12740.

Ex. 3. A person transfers *Rs.* 10000 stock from $3\frac{1}{2}$ per cents. at $97\frac{7}{8}$ to 3 per cent. railway shares at $74\frac{7}{8}$; find the alteration in his income, the usual brokerage being charged on each transaction.

Rs. 100 : *Rs.* 10000 :: *Rs.* $3\frac{1}{2}$: income from the $3\frac{1}{2}$ per cent.;

∴ income from the $3\frac{1}{2}$ per cents. = *Rs.* 350.

Brokerage being $\frac{1}{8}$ per cent., *Rs.* 100 stock yields *Rs.* $(97\frac{7}{8}-\frac{1}{8})$ or *Rs.* $97\frac{3}{4}$;

Rs. 100 : *Rs* 10000 :: *Rs.* $97\frac{3}{4}$: price of the $3\frac{1}{2}$ per cents.;

∴ money obtained from the sale = *Rs.* 9775

The price including brokerage of *Rs.* 100 stock in the 3 *p. c.* is *Rs.* 75.

∴ 75 : 9775 :: *Rs.* 3 : income from the 3 per cent.;

or income from the 3 per cents = *Rs.* 391.

∴ his income *increases* by *Rs.* (391 − 350) = *Rs.* 41. *Ans.*

Examples 152.

1. What income wiil arise from the following investment ?

(1) R51870 in the 3 p c. at 90. (2) £2500 in the 3 p c. at $78\frac{1}{8}$.

(3) „ 35119 in the $3\frac{1}{2}$ p.c. stock at $97\frac{3}{4}$; brokerage $\frac{1}{4}$ p c.

(4) £4311 8*s.* 9*d.* in the 3 p.c. at $85\frac{1}{4}$; brokerage as usual.

2. If £1138. 10*s.* be laid out in purchasing the $4\frac{1}{2}$ per cents. at 92, what annual income will be derived from this investment, after deducting an income tax of 7*d.* in the pound ?

3. If a person invest R35000 in the 3 per cent. stock at $94\frac{1}{8}$, brokerage $\frac{1}{8}$ per cent., what should be obtained from them in $4\frac{1}{2}$ years ?

4. A person invests R17100 in the three per cents. at 90, and a like sum in the $3\frac{1}{2}$ per cents. at 95 ; find his total annual income. Also find which of the investments is more advantageous, and by how much

5. Find the income which will be derived from a capital of R21000, if $\frac{3}{10}$ of it be invested in the $3\frac{1}{2}$ per cents. at par and the remainder in the 3 per cents. at 2 per cent. discount.

6. A person invests R24570 in the 3 per cents. at 90. If he invest the first year's dividend in the same stock at 91, and the second year's dividend at 96, find his income in the third year.

7. What must I invest in the $3\frac{1}{2}$ per cents. at 96, in order to obtain an income of R7112. 5*a.* 10*p.* ? How much per cents. do I get for the investment ?

8. What half-yearly dividend is derived from an investment of R31600 in the $3\frac{1}{2}$ per cents. at $98\frac{3}{4}$, after income tax is deducted at the rate of 3*p.* in the rupee ?

9. Find the alteration in income occasioned by shifting R64000 stock from the 3 per cents. at $86\frac{3}{8}$ to 4 per cent. stock at $14\frac{7}{8}$ per cent. premium, the brokerage being $\frac{1}{8}$ per cent.

10. A man lays out R21390 in the 3 per cents. at $114\frac{3}{8}$, and afterwards sells out at $135\frac{1}{8}$ and invests the proceeds in the 4 per cents. at $92\frac{7}{8}$, what is the alteration in his income brokerage being as usual ?

11. If I lay out £19400 in the 4 per cents. at 97, and after receiving the half-yearly dividend sell out when they have sunk to 95, how much do I gain or lose ?

12. A man invested R5850 in the 3 per cent. stock at 90; and having received a quarter's dividend, sold out at 93. Find how much he gained by the whole transaction and how much stock he was able to buy at par.

***13** A person invests R15000 in the purchase of 3 per cents. at 90 and $3\frac{1}{2}$ per cents. at 91. If his total annual income amount to R513, how much of each stock did he buy?

14. A 4 per cent. stock is at 95 and a 5 per cent. stock at 115 a person buys R43700 of each, another invests R43700 in each; compare their annual incomes.

15. If 3 per cent. stock be at $90\frac{3}{8}$, what sum must I invest in order to secure a yearly income of R1850 after paying an income-tax of 7*p.* in the rupee, brokerage being $\frac{1}{8}$ per cent?

349. The following solutions are instructive.

Ex. 1. A man invests his capital in the 3 per cents. at $89\frac{7}{8}$; what rate of interest does he obtain? brokerage $\frac{1}{8}$ per cent.

£$(89\frac{7}{8}+\frac{1}{8})$: £100 :: 3 : rate per cent.; ∴ rate per cent. $=3\frac{1}{3}$. *Ans.*

Ex. 2. What is the price of the $3\frac{1}{2}$ per cent. stock, when a man can buy £5000 stock by investing £4900?

£5000 : £100 :: £4900 : price of £100 stock;

∴ price of £100 stock $=$ £98. *Ans.*

Ex. 3 Find the price of 3 per cent stock, from which a man can obtain an annual income of £90 by investing £2910, brokerage as usual.

90 : 3 :: £2910 : cost of £100 stock; including brokerage;

∴ cost including brokerage $=$ £97, or price of £100 stock $=96\frac{7}{8}$. *Ans.*

Ex. 4. If the 3 per cents. pay interest at the rate of $3\frac{1}{13}$ per cent. per annum on the money invested, find the price of the stock, brokerage being paid as usual.

$3\frac{1}{13}$: 3 :: 100 : price of 3 per cent stock including brokerage;

∴ price of the 3 per cents. including brokerage $=97\frac{1}{2}$.

or price of the 3 per cents $=97\frac{1}{2}-\frac{1}{8}=97\frac{3}{8}$. *Ans.*

Ex. 5. Which is the better investment, the 3 per cents. at 20 or the $3\frac{1}{2}$ per cents. at 98?

From the 1st investment, income on £90 $=$ £3, or on £1 $=$ £$\frac{1}{30}$;

" " 2nd " " on " 98 $=$ " $3\frac{1}{2}$, or on " 1 $=$ " $\frac{1}{28}$.

Now by comparing the two fractions, we find that $\frac{1}{28}$ is greater than $\frac{1}{30}$. Therefore the $3\frac{1}{2}$ per cents. at 98 are better. *Ans.*

Ex. 6. A person invests two equal sums in the 3 per cents. at 93, and in the 4 per cents at 120 respectively, and finds that one of them yields £15 more income than the other. Find the sum laid out in each.

From the 1st he gets £$\frac{3}{93}$ or £$\frac{1}{31}$, and from the 2nd £$\frac{4}{120}$ or £$\frac{1}{30}$ per £;

∴ from the 2nd investment he gets £$(\frac{1}{30}-\frac{1}{31})$ or £$\frac{1}{930}$ more per £;

∴ he invests in each £$15\div\frac{1}{930}$ or £13950. *Ans.*

Examples 153.

1. What rate of interest may be obtained from investing in the

(1) 3 per cents. at 90 ? (2) $3\frac{1}{2}$ per cents. at 91 ?

(3) 4 per cents. at 102 ? (B. $\frac{1}{4}$) (4) 5 per cents. at $119\frac{7}{8}$? (B. $\frac{1}{8}$.)

2. A person invested £4500 in the 3 per cent. consols when they were at 90, and £5100 more in the $3\frac{1}{2}$ per cents. at 96. What rate of interest did he get on the whole sum invested ?

3. I buy R300 of 3 per cent. Government stock at 92, and R5000 of 4 per cent. Railway stock at 115. How much per cent. does my money bring ?

4. What is the price of $3\frac{3}{4}$ per cent. consols, when by investing £2775 a person can purchase £3000 stock ? Brokerage as usual.

5. If by laying out R9725 and after paying the usual brokerage I can buy R10,000 of 4 per cent. stock, what is the price of the stock ?

6. I find that I may derive an income of R787. 8*a*. annually if I invest R16380 in the $4\frac{1}{2}$ per cents. What is the price of the stock ?

7. A person invested £1037. 10*s*. in the 3 per cent. consols at 83. When the funds rose 1 per cent, he transferred his capital to the 4 per cents. ; and thereby increased his income by £6. 5*s*. Find the price of the latter stock.

8. If one's income remains untouched by shifting R32,000 stock from the 3 per cents. at $86\frac{7}{8}$ to the 4 per cents., find the price of the latter stock. Brokerage on each transaction is $\frac{1}{8}$ per cent.

9. If I lay out R19110 in the purchase of 3 per cent. stock when they are at $79\frac{1}{2}$, at what price should I sell out my stock again in order to realise on the whole a gain of R1500, after having paid $\frac{1}{8}$ per cent. commission on each transaction ?

10. A person invested £5600 in shares which are at 98 and pay $3\frac{1}{2}$ per cent. and £4800 in the 3 per cent. consols which are at 96. He sold the shares when their price had risen $15\frac{3}{4}$ per cent., and the consols when they had fallen 16 per cent. He then invested both the proceeds in the 5 per cent. Municipal Debentures and thereby increased his income by £70. Find the price of the Debentures.

11. By selling out R45000 in the 5 per cent. Govt. Stock at $112\frac{1}{2}$, and investing the proceeds in the Railway 7 per cent. stock. a person finds his income increased by R1687. 8*a*. What is the price of the latter stock ?

12. The sum of £1001 was laid out in the $3\frac{1}{2}$ per cents. at $89\frac{7}{8}$, and a whole year's dividend having been received upon it, it was sold out ; the whole increase of capital was 72 guineas : at what price was it sold out ?

13. I sold out R5000 stock when they were at $77\frac{7}{8}$, and re-investing the proceeds in the $3\frac{1}{2}$ per cent. at $87\frac{7}{8}$ found my income increased by R9 11*a*. how much dividend did the former pay, brokerage on each transaction being $\frac{1}{8}$ per cent. ?

14. The 3 per cents. are at $91\frac{1}{4}$ and the $3\frac{1}{2}$ per cents. at $96\frac{1}{4}$. A person has a sum of money which will give him £500 more of the former stock than of the latter: find the difference of the income he could obtain by investing in the two stocks. Brokerage as usual

15. A person buys 3 per cent. stock at $89\frac{5}{8}$: he receives one half year's dividend, and afterwards sells his stock at $94\frac{1}{8}$, and finds that he has gained R108 : what sum did he invest ?

16. By buying 3 per cent. consols at a certain price I find I obtain $3\frac{1}{8}$ per cent. for my money, and derive a net income therefrom, after paying an income-tax of 6*d*. in the £ of £421. 4*s*.; find the amount of stock and the price at which I bought it.

***17.** How much 3 per cent. stock at $98\frac{1}{8}$ must be sold out to purchase an estate whose rent is £172. 7*s*. 6*d*., the estate being worth 24 years' purchase and a commission of $2\frac{1}{2}$ per cent. on the purchase money being paid to the land-broker ?

18 I invest £2820 in Italian 5 per cents. at 93, sell out the stock at 95, and re-invest the proceeds in Spanish 4 per cents. at 57. If I receive a quarter's dividend on the Spanish stock, and afterwards sell out at 58, what sum shall I realize by the dividend and sale together ?

19. A man has R40,200 which he invests partly in 4 per cent. Govt. Securities at $102\frac{1}{2}$, and partly in 5 per cent. Port Trust Debentures at 103. He makes $4\frac{1}{2}$ per cent. on the whole investment. What sum does he invest in each stock ?

20. A man invests a certain sum in the ordinary stock of a railway at 90, which pays a dividend of $3\frac{1}{2}$ per cent. at the end of a year. He then sells out at 86 and invests the proceeds in a 7 per cent. stock at 98, and thus in the second year receives £647. 10*s*. more interest than in the first year. How much did he originally invest ?

21. I invest equal sums in a 3 per cent. stock and in a $4\frac{1}{2}$ per cent. stock, and get 5 per cent. for my money: the $4\frac{1}{2}$ per cents. are at 85 : what is the price of the 3 per cents. ?

22. What sum must a person invest in the $4\frac{1}{2}$ per cents at 90, in order that by selling out R5000 stock when they have risen to $93\frac{1}{3}$, and the remainder when they have fallen to $62\frac{1}{2}$, and investing the whole proceeds in the 6 per cents., at par, he may increase his annual income by R46. 4*a*.

23. A man has £3000 of a 3 per cent. stock at 125, two-fifths of which he transfers to the $4\frac{1}{2}$ per cents. at 140. What rate of interest must he get for the remainder that his income may be increased 10 p. c. ?

24. A person bought E. I. Railway shares at a premium of 15 per cent.; and after drawing the first quarter's dividend at the rate of 8 per cent. per annum, sold out when the price rose to $120\frac{3}{4}$, gaining R286. 12*a*. by the whole transaction; find the number of shares he bought, and also his original investment.

25 A person invests £7935 in Railway shares at $14\frac{3}{4}$ per cent. premium, the annual dividend on each share being £5; afterwards he sells out at $125\frac{1}{4}$ and invests in the 3 per cent. consols at $93\frac{5}{8}$ (brokerage on Railway stock at $\frac{1}{4}$ per cent. and on consols $\frac{1}{8}$) Find the change in his income.

26 A man invested the same sum in two different stocks 3 per cent. Municipal Debentures at $87\frac{1}{2}$ and $3\frac{1}{2}$ per cent. Government securities at $93\frac{1}{3}$: his income from one is R1060 more than from the other; what sum was invested in each stock ?

27. I invest R32490, half in the 3 per cents. at 90, and the other half in the 4 per cents. at 95 : when both the stocks have risen 5 per cent. I sell out and invest the proceeds of one stock in the other. The stocks falling to their original value, I again sell out Find my gain or loss.

28 A person invests £5460 in the 3 per cents at 91 : he sells out £2000 when they have risen to $93\frac{1}{2}$, and the remainder when they have fallen to 85 : he then invests the whole proceeds in the $4\frac{1}{2}$ per cents. at 102. Find the change in his income.

***29**. A person possesses R32000 stock in the 3 per cents, which he sells out at $99\frac{3}{4}$, and invests the proceeds in Railway shares at R560 a share, which shares pay 5 per cent. interest on R150, the amount paid on each share. How much is his income altered by the transaction ?

30. The difference between the incomes derived from investing a certain sum in 6 per cent. stock at 126, and the same sum in 9 per cent. stock at 210, is R2250. Find the amount.

31. A person invests R14100 in Railway shares which are at 98 and pay $3\frac{1}{2}$ per sent. and the same sum in 3 per cent. Government promissory Notes which are at 94. What difference should it have made in his annual income if he had invested the whole amount in the shares ?

***32**. Two persons invest each the same sum of money in 3 per cent. stock at 91 and $3\frac{1}{4}$ per cent. stock at $97\frac{1}{2}$, one purchasing an equal amount of each kind and the other dividing his money equally between the two stocks : which receives the larger income ? If their incomes differ by R5, how much money did they each invest ?

***33**. The annual income of a railway company is $4\frac{1}{2}$ per cent. of the total capital invested, but £100000 of this capital consisting of preference shares on which 6 per cent. is guaranteed, a dividend of only $3\frac{1}{2}$ per cent is declared : find the total amount of capital.

***34**. The annual income of a company would justify a declaration of dividend at 5 per cent. ; but as there is a certain amount of guaranteed stock, the shareholders only receive $2\frac{1}{2}$ per cent. If the preference shares are only 30 per cent. of the ordinary shares, find the guaranteed rate

***35** A man invests R168000 in $4\frac{1}{2}$ per cent. stock at $103\frac{5}{8}$, the interest on which (payable half-yearly) had been paid three months before. After 5 months he sells out, having gained $6\frac{1}{4}$ per cent. on the money invested. At what price does he sell out ? (Brokerage$=\frac{1}{4}$ per cent.)

36 A person sold 150 one-hundred-rupee shares of railway stock, which were paying 5 per cent. at 105. With the proceeds he purchased 4 per cents. at 90, and resold them at 96. He then re-invested in the railway stock which was still at 105 and paying 5 per cent. What was the change in his income ?

***37** A person has £80000 in the 4 per cents. : he sells out at $96\frac{1}{4}$ and invests the proceeds in railway shares at £56 a share which pay 5 per cent. on the sum paid up, and are at a premium of £11. How much is his income altered ?

38 A person invests £5170 2*s*. in the 4 per cents. at 97 ; and after receiving a year's dividend sells out, investing both stock and profit in the $3\frac{1}{2}$ per cents. at $66\frac{5}{8}$. What is the increase in his income ?

39. A person having invested £10500 in the $3\frac{1}{4}$ per cents. at 98 sells out at the end of 7 years He invests both stock and profits in Railway Debentures at $129\frac{3}{8}$, bearing interest at $2\frac{3}{4}$ per cent half yearly. What is the difference in his annual income ?

40. A sum is laid out in the $4\frac{1}{2}$ per cents at 96, and one half-year's dividend being received upon it, the stock is then sold at 99, and the whole increase of capital, including the half year's interest, is R3558. 5*a*. 4*p*. Find the original sum.

***41.** A man having money invested in the 4 per cent. stock from which he derives an income of R1152, sells out at 90 and invests in 100-rupee shares that pay 5 per cent. interest : if his income be now increased by R336, at what price does he buy the shares ?

42. A person sells out of the 3 per cent. consols at 99 and invests in Exchequer bills bearing interest at the rate of $1\frac{1}{2}$*d*. a day per cent., when the bills are at a premium of 7*s*. 6*d*. (on £100). What effect has this on his income ?

***43** A person wants to lay out £60600, partly in the 3 per cents. at 84 and partly in the 4 per cents at 90 ; how must he divide his money between the two stocks that he may derive the same income from both ?

***44.** If £1150 stock be sold out of the 3 per cents. at 72 and £5400 stock out of the 4 per cents. at $80\frac{1}{2}$; how should the proceeds be invested partly in the 4 per cents. at 98 and partly in the 3 per cents. at 78 that the income from the two may be as 2 : 1 ?

***45.** "Consols were first quoted at $96\frac{5}{8}$, whence they improved to $96\frac{7}{8}$." What would be the difference of stock from the investment of £10,000 at the two prices, after paying a commission of $\frac{1}{8}$ p. c. ?

***46.** A Railway stock is sold at 108, and with the proceeds 3 per cent. Govt. stock is bought at $91\frac{1}{8}$; after 6 months the Govt. stock is sold at $93\frac{3}{8}$, and the original stock repurchased at 109, leaving a profit of £109 on the transaction ; find the amount of Railway stock sold.

CHAP. XXXIX. SQUARE ROOT.

350. **Application of Geometry to Arithmetic.** By Euc. I. 47, the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the sides containing the right angle. Now, by Art. 195, the square on a

line contains a number of square units equal to the square of the number of linear units contained in the line ; hence

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2 ;$$
$$(\text{Perpendicular})^2 = (\text{Hypotenuse})^2 - (\text{Base})^2$$
$$= (\text{Hyp.} + \text{Base})\,(\text{Hyp.} - \text{Base}) ; \text{(i)}$$

Similarly, $(\text{Base})^2 = (\text{Hyp.} + \text{Perp.})\,(\text{Hyp.} - \text{Perp}) ;$ (ii)

Thus of the three sides, if any two are given, the third can be easily determined.

Ex. 1. The sides of a right-angled triangle are 4 ft. and 3 ft. : find the hypotenuse.

$$4^2 + 3^2 = 16 + 9 = 25 ; \ \sqrt{(25)} = 5.$$

$\therefore$ the required hypotenuse $= 5$ ft. *Ans.*

Ex. 2. In a certain lake the tip of a bud of lotus was seen a span above the surface of the water. Forced by the wind it gradually advanced and was submerged at a distance of 2 cubits. Compute the depth of the water.

In the right-angled triangle OCB,

$OC - OB = OA - OB = \frac{1}{2}$ cub., and $BC = 2$ cub. ;

$\therefore$ Hypotenuse $-$ Base $= \frac{1}{2}$ cubit,

and Perpendicular $= 2$ cubits.

$\therefore$ (Hyp. $+$ Base) $\times \frac{1}{2} = 2^2$, or Hyp. $+$ Base $= 8$.

But Hyp. $-$ Base $= \frac{1}{2}$.

$\therefore$ Base $= \frac{1}{2}\,(8 - \frac{1}{2}) = 3\frac{3}{4}$.

Thus the depth of the water $= 3\frac{3}{4}$ cubits. *Ans.*

351. **Application of Algebra to Arithmetic.** The following Examples will illustrate the application of Algebraical formulæ to arithmetical examples.

Ex. 1. $$\frac{1}{\sqrt{3} - \sqrt{2}} = \frac{3 - 2}{\sqrt{3} - \sqrt{2}} = \sqrt{3} + \sqrt{2} = 1\cdot732\ldots + 1\cdot414\ldots$$
$$= 3\cdot146\ldots\ldots$$

Ex. 2. $$\frac{2 + \sqrt{3}}{1 + \sqrt{3}} = \frac{\frac{1}{2}(4 + 2\sqrt{3})}{1 + \sqrt{3}} = \frac{\frac{1}{2}(\sqrt{3} + 1)^2}{\sqrt{3} + 1}$$
$$= \tfrac{1}{2}(\sqrt{3} + 1) = \tfrac{1}{2}(1\cdot732\ldots + 1) = 1\cdot366\ldots\ldots$$

Ex. 3. $$\frac{\cdot5893 \times \cdot5893 - \cdot4107 \times \cdot4107}{\cdot5893 - \cdot4107} = \frac{(\cdot5893)^2 - (\cdot4107)^2}{\cdot5893 - \cdot4107}$$
$$= \cdot5893 + \cdot4107 = 1.$$

Ex. 4. $$\frac{(\cdot05)^2 + (\cdot03)^2}{(\cdot005)^2 + (\cdot003)^2} = \frac{(50)^2 + (30)^2}{(5)^2 + (3)^2} = \frac{10^2(5^2 + 3^2)}{5^2 + 3^2} = 10^2 = 100.$$

Ex. 5. $$\frac{2\frac{1}{4} \times 2\frac{1}{4} \times 2\frac{1}{4} - 1}{2\frac{1}{4} \times 2\frac{1}{4} - 1} = \frac{(2\frac{1}{4})^3 - 1}{(2\frac{1}{4})^2 - 1} = \frac{(2\frac{1}{4} - 1)\{(2\frac{1}{4})^2 + 2\frac{1}{4} + 1\}}{(2\frac{1}{4} - 1)(2\frac{1}{4} + 1)}$$
$$= \frac{2\frac{1}{4}(2\frac{1}{4} + 1) + 1}{2\frac{1}{4} + 1} = \frac{2\frac{1}{4} \times 3\frac{1}{4}}{3\frac{1}{4}} + \frac{1}{3\frac{1}{4}}$$
$$= 2\tfrac{1}{4} + \tfrac{4}{13} = 2\tfrac{29}{52}.$$

Examples 154.

1. A rectangular field measures 225 yds. in length, and 120 yds. in breadth : what is the distance from corner to corner ?

2. There are three towns *A*, *B*, *C*, so situated that *B* lies 80 miles south of *A* and *C* 60 miles west of *A* : find the distance of *B* from *C*.

3. The Ochterloney monument is 180 ft. in height : what will be the length of a rope from the top to a peg driven into the ground, 120 ft. from the base of the monument ?

4. A ladder placed in a street 33 ft. out from the foot of the wall of a house reaches a window 44 lt. high : and its foot being retained in the same position, reaches another window 40 ft. high on the other side of the street ; what is the width of the street ?

***5.** The fly-wheel of an engine has 6 in. above a plane, while its length along the plane is 3 ft. 6 in. : find the diameter of the wheel.

6. Find the value of :

(1) $\dfrac{\cdot328\times\cdot328-\cdot172\times\cdot172}{\cdot328-\cdot172}$ (2) $\dfrac{\cdot8192\times\cdot8192-\cdot1808\times\cdot1808}{\cdot8192+\cdot1808}$

(3) $\dfrac{(\cdot011)^2-(\cdot008)^2}{(\cdot0011)^2-(\cdot0008)^2}$ (4) $\dfrac{(\cdot32)^4-(\cdot31)^4}{(\cdot0032)^2-(\cdot0031)^2}$ (5) $\dfrac{1\frac{1}{2}\times1\frac{1}{2}\times1\frac{1}{2}-1}{1\frac{1}{2}\times1\frac{1}{2}-1}$

7. Find to 3 Places of decimals the value of :

(1) $\dfrac{3+\sqrt{3}}{\sqrt{3}+1}$ (2) $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ (3) $\dfrac{2+\sqrt{(12)}-\sqrt{(27)}}{2+\sqrt{(48)}-\sqrt{(27)}}$

(4) $\dfrac{\sqrt{5}+\sqrt{7}}{\sqrt{7}-\sqrt{5}}$ (5) $\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)^{\frac{1}{2}}$ (6) $\left(\dfrac{2+\sqrt{(12)}-\sqrt{(27)}}{2+\sqrt{(48)}-\sqrt{(27)}}\right)^{\frac{1}{2}}$

CHAP. XL. CUBE ROOT.

352. The **Cube Root** of a given number is the number which multiplied twice by itself is equal to the given number.

Thus, $5\times5\times5=125$; therefore 5 is the *cube root* of 125. The Cube Root of 125 is written $\sqrt[3]{(125)}$ or $(125)^{\frac{1}{3}}$.

353. We know the cube root of 1 is 1 ; of 1,000 is 10 ; of 1,000,000 is 100 ; and so forth. Hence it follows that " the cube root of any number between 1 and 1000 must be between 1 and 10, *i.e.*, will have 1 figure in the integral part of any number between 1,000 and 1,000,000 must be between 10 and 100, *i.e.*, will have two figures in the integral part ; of any number between 1,000,000 and 1,000,000,000 must be between 100, and 1000, *i.e.*, must have 3 figures in its integral part. Therefore if a point be placed over the unit's place of the number and thence over every third figure after the unit's place the points will shew the number of figures in the integral part of the root." In extracting cube roots, we should, therefore, point the unit figures and every third figure both to the left and the right.

RULE. Divide the given number into periods of 3 figures

beginning with the unit figure, adding (in the case of decimals) cyphers, if necessary, to the right to get periods of three.

Find the greatest number whose cube is contained in the first period at the left (this period may contain 1, 2, 3, figures,) and place it on the right as the first figure of the root; then subtract its cube from the first period, and to the remainder bring down the second period.

Multiply the square of the root by 300, find how often this *trial divisor* or *partial* divisor is contained in the dividend, and place this quotient as the next figure in the required root; then multiply by it the product of the previous figure of the root by 30, and place the result before the partial divisor. Under these, place the square of this last figure and add the three together to get the *complete* divisor. Multiply this complete divisor by the last figure of the root, subtract, and bring down the next period to form the next dividend.

Square the figures of the root already found, multiply the result by 300, and find how often this partial divisor is contained in the dividend. Then multiply the figures of the root already found by 300 and the product by the last figure, and place the result below the partial divisor. Then square the figure last found and at the three together as before, the sum giving the complete divisor.

Proceed in the same way with the remaining periods, if any.

If at any stage of the operation, the dividend is found to be less than the trial divisor, annex a cypher to the root, two cyphers to the trial divisor, and bring down the next period.

Ex. Find the cube root of 843908625.

We first divide into periods of three beginning with 5.

The trial divisor 24300 goes into the dividend 114908, 4 times.

The trial divisor 2650800 goes into 133246625, 5 times

$$
\begin{array}{lr|l}
 & & 843\dot{9}08\dot{6}2\dot{5}(945 \\
 & & \underline{729} \\
9^2 \times 300 & =24300 & 114908 \\
9 \times 30 \times 4 = & 1080 & \\
4^2 = & \underline{16} & \\
 & 25396 & \underline{101584} \\
94^2 \times 300 & =2650800 & 13324625 \\
94 \times 30 \times 5 = & 14100 & \\
5^2 = & \underline{25} & \\
 & 2664925 & \underline{13324625}
\end{array}
$$

$$
\begin{array}{ll}
274 & 243 \\
\underline{8} & \underline{1096} \\
2825 & 25396 \\
 & \underline{16} \\
 & 26508 \\
 & \underline{14125} \\
 & 2664925
\end{array}
\qquad
\begin{array}{l}
843\dot{9}08\dot{6}2\dot{5}(945 \\
\underline{729} \\
114908 \\
\underline{101584} \\
13324625 \\
\underline{13324625}
\end{array}
$$

Note 1. Numbers like the above, of which the cube roots can be found *exactly*, are called *perfect cubes.*

Note 2. When $2n+2$ figures are required in the root and $n+2$ figures have been found in the above way the other n figures may be found by approximate Division. (*See* ART. 267.)

354. In extracting the cube roots of vulgar fractions, we may either convert them into corresponding decimal fractions and proceed as above ; or multiply both the numerator and denominator by a number which would make the denominator a perfect cube, and then extract the cube roots of both the numerator and the denominator.

Ex. Find the cube root of $\frac{2}{3}$.

(1) $\frac{2}{3} = \cdot 666656666\ldots$

$\therefore \sqrt[3]{(\frac{2}{3})} = \sqrt[3]{(\cdot 666656666)}$

$= \cdot 873\ldots$

(2) $\frac{2}{3} = \frac{2}{3} \times \frac{9}{9} = \frac{18}{27}$.

$\therefore \sqrt[3]{(\frac{2}{3})} = \sqrt[3]{(\frac{18}{27})} = \frac{1}{3}\sqrt[3]{(18)}$

$= \frac{1}{3} \times 2\,619\ldots = \cdot 873\ldots\ldots$

Examples 155.

1. Find the cube roots of :

(1) 1728 ; 3375 ; 373248 ; 21717639.

(2) 29791 ; 804357 ; 1822842263 ; 1334633301.

(3) 746142643 ; 526079913723 ; 219365327791.

(4) ·000002197 ; 38·958219 ; ·000781229961.

(5) 841·3 ; 2304 ; 12·74867 ; (each to 4 decimal places).

(6) $\frac{1}{8}$; $3\frac{1}{8}$; $7\frac{1}{5}$; $6\frac{1}{4}$; (each to 3 decimal places).

2. The cost of a cubic mass of metal is R8640 at R5 per cubic inch ; what are the dimensions of the mass ?

3. Find the side of a cubical vessel, which will hold as much as a cistern 4 feet long, 3 feet wide, and 2 feet deep.

4. A cubical box contains 50·653 solid feet ; find the cost of painting its sides at 2*a.* 1*p.* per square foot.

355. The **Fourth** root of a number is found by extracting the square root of its square root.

The **Sixth** root of a number is found by extracting the cube root of its square root or the square root of its cube root.

The **Ninth** root of a number is found by extracting the cube root of its cube root.

Examples 156.

1. Find the fourth roots of 65536, 1·4641, ·00002401.

2. Find the sixth roots of 15625, ·531441, 000729.

3. Extract the ninth roots of 262144, 5·159780352.

Problems worked out.

Ex. 1. Four years ago, the ages of A and B were as $2:3$; and four years hence they will be as $10:13$; find their present ages.

$3-2=1$, and $13-10=3$. Reducing the latter ratio to one in which also the difference between the terms $=1$, we get the ratio $\frac{10}{3}:\frac{13}{3}$.

In $(4+4)$ or 8 years, the ratio has changed from $2:3$ to $\frac{10}{3}:\frac{13}{3}$. Now if A's age four years before had been 2 years, it would have taken only $(\frac{10}{3}-2)$ or $\frac{4}{3}$ years to become $\frac{10}{3}$.

$\therefore$ $\frac{4}{3}:8::2:A$'s age 4 years ago;

or A's age 4 years ago $=\frac{3}{4}\times 8\times 2$ years $=12$ years.

$\therefore$ A's age $=(12+4)$ or 16 yrs., and B's age $=(\frac{3}{2}\times 12+4)$ or 22 yrs.

Ex. 2. Ram and Shyam have each a number of marbles; Ram says to Shyam, if you give me one marble, I shall have the same number of marbles as yourself; Shyam says, if you give me one, I shall have twice as many marbles as you; find how many each has got.

Since after taking one marble from Shyam, Ram has the same number as the other, Shyam has 2 marbles more than Ram.

Hence, after Ram gives one marble to Shyam, the difference becomes $2+2$ or 4. But Shyam's marbles then become *twice* as many as Ram's.

$\therefore$ they have now 4 and 8 marbles respectively.

$\therefore$ they had before $(4+1)$ or 5, and $(8-1)$ or 7 marbles, respectively.

Ex. 3. Divide *Rs.* 260 between A, B, C, D, so that *Rs.* 20 less than $\frac{3}{4}$ of A's share, *Rs.* 20 more than $\frac{2}{7}$ of B's share, *Rs.* 10 less than $\frac{5}{6}$ of C's share, and *Rs.* 10 more than $\frac{3}{5}$ of D's share may all be equal.

$\frac{3}{4}$ of A's share $-$ *Rs.*20 $=\frac{3}{4}$ (A's share $-$ *Rs.*$\frac{80}{3}$);
$\frac{2}{7}$ of B's „ $+$ *Rs.*20 $=\frac{2}{7}$ (B's „ $+$ *Rs.*70);
$\frac{5}{6}$ of C's „ $-$ *Rs.*10 $=\frac{5}{6}$ (C's „ $-$ *Rs.*12);
$\frac{3}{5}$ of D's „ $+$ *Rs.*10 $=\frac{3}{5}$ (D's „ $+$ *Rs.*$\frac{50}{3}$).

Also (A's share $-$ *Rs.* $\frac{80}{3}$) $+$ (B's share $+$ *Rs.* 70) $+$ (C's share $-$ *Rs.* 12) $+$ (D's share $+$ *Rs.* $\frac{50}{3}$) $=$ *Rs.* $(260-\frac{80}{3}+70-12+\frac{50}{3})=$ *Rs.*308.

Now if A's share $-$ *Rs.*$\frac{80}{3}=1$ B's share $+$ *Rs.*$70=\frac{21}{8}$.

C's „ $-$ *Rs.*$12=\frac{9}{10}$, and D's „ $+$ *Rs.*$\frac{50}{3}=\frac{5}{4}$;

also $1+\frac{21}{8}+\frac{9}{10}+\frac{5}{4}=\frac{231}{40}$; and *Rs.*$308\div\frac{231}{40}=\frac{160}{3}$.

$\because$ A's share $-$ *Rs.*$\frac{80}{3}=$ *Rs.*$\frac{160}{3}$, or A's share $=$ *Rs.* $(\frac{160}{3}+\frac{80}{3})=$ *Rs.*80;
B's „ $+$ „ $70=$ „ $\frac{160}{3}\times\frac{21}{8}$, or B's „ $=$ „ $(140-70)=$ „ 70;
C's „ $-$ „ $12=$ „ $\frac{160}{3}\times\frac{9}{10}$, or C's „ $=$ „ $(48+12)=$ „ 60;
D's „ $+$ „ $\frac{50}{3}=$ „ $\frac{160}{3}\times\frac{5}{4}$ or D's „ $=$ „ $(\frac{200}{3}-\frac{50}{3})=$ „ 50.

Ex. 4. Supposing that in a pasture the grass grows at a uniform rate and that 133 oxen could eat up the whole of the grass in 13 weeks, and, 112 oxen could eat up the whole grass in 16 weeks; how many oxen could eat up the whole grass on the same pasture in 14 weeks?

Original grass + 13 wks.' growth is eaten by 133 oxen in 13 wks. ;
$\therefore$ " " + 13 " " " " 1 ox " 1729 "
Again " " + 16 " " " " 112 oxen " 16 "
$\therefore$ " " + 16 " " " " 1 ox " 1792 "
$\therefore$ 3 " " " " 1 " " 63 "
$\therefore$ 1 " " " " 1 " " 21 "

$\therefore$ original grass + 13 wks. growth $= \frac{1729}{21}$ or $\frac{247}{3}$ wks.' growth.
$\therefore$ original grass + 14 wks. growth $= \frac{250}{3}$ wks.' growth.

Now 1 wk.'s growth is eaten by 1 ox in 21 wks. ;
$\therefore$ 1 " " " " $\frac{3}{2}$ oxen in 14 "
$\therefore$ $\frac{250}{3}$ " " " " 125 " in 14 "

$\therefore$ no. of oxen reqd. $= 125$. *Ans.*

Ex. 5. Suppose that in a meadow of 20 acres the grass grows at a uniform rate, and that 133 oxen could consume the whole of the grass in 13 days, while 28 oxen could eat up 5 acres of it in 16 days : how many of the oxen could eat up 4 acres of it in 14 days ?

28 oxen to 112 oxen is 5 acres to 20 acres.

$\therefore$ 112 oxen : 133 oxen :: 13 days : no. of days, $= (15\frac{7}{16}$ days).

As the grass grows on the field for $(16-3)$ or 3 days more in the second case, 112 oxen can have grass for $(16 - 15\frac{7}{16})$ or $\frac{9}{16}$ of a day more.

Therefore 3 days' growth can supply 112 oxen for $\frac{9}{16}$ day ;

$\therefore$ $\frac{9}{16}$ day : 16 days :: 3 days' growth : orig. grass + 16 days' growth ;
$\therefore$ original grass + 16 days' growth $= 85\frac{1}{3}$ days' growth ;
$\therefore$ original grass $= (85\frac{1}{3} - 16)$ or $69\frac{1}{3}$ days' growth ;
$\therefore$ original grass + 14 days' growth $= 83\frac{1}{3}$ days' growth.

Let x be the no. of oxen.

$85\frac{1}{3}$ days' growth $\downarrow$ 16 da. $\uparrow$ 20 ac. $\downarrow$ 112 ox $\uparrow$ *less* growth, *less* oxen.
$83\frac{1}{3}$ " " 14 " 4 " x " " days, *more* "

$$\therefore \quad x = \frac{16 \times 4 \times 112 \times 250 \times 3}{14 \times 256 \times 20 \times 3} = 25. \ \textit{Ans.}$$

MISCELLANEOUS EXAMPLES VII.

1. A tradesman imports silk from France which, including all expenses, costs him 7 fr. 75 cents. a metre : at how much a yard must he sell it to gain 20 per cent. by the transaction ? £1 = 25 fr. 40 cents. and a metre = 39·371 in.

2. A man buys a horse, for which he gives a bill of £100, due 4 mo. : he immediately sells the horse for £108 cash ; how much per cent. does he gain, money being at 5 per cent. per annum ?

3. Which would be more advantageous to the state, an income-tax of $2\frac{1}{2}$ per cent. or of 5*p*. in the rupee ? Calculate the difference of the amounts to be paid at the two rates on an income of R3212. 8*a*.

4. A person sold a horse at a loss of 20 per cent.; if he had received £10 more for it, he would have gained 10 per cent.: find the cost.

*__5.__ The dividend on the shares of a company is at the rate of 8 per cent. per annum: had the profits been £450 more, a dividend of $8\frac{1}{2}$ per cent. could have been declared: what is the stock?

6. The shortest distance from a house to a circular lake is exactly 4 mi.; a road proceeding in a straight line from the house past the shore of the lake touches the shore at a point 8 mi. from the house: find the area of the lake. Area of a circle $= \frac{22}{7} \times (\text{radius})^2$.

*__7.__ A sum of money is put out at Compound Interest at 5 per cent. per annum and at the end of two years £338. 10s. is added to it. At the expiration of 5 yrs. from the first investment the whole principal and interest amount to £1157. 12s. 6*d*. Find the sum originally invested.

8. *A*, *B*, *C* are travelling together. *A* pays a hotel bill amounting to 105 fr., *B* another amounting to 155 fr., while *C* spends £3. 2s. on railway tickets. In the bill that *B* pays, a sum of 20 fr. is to be divided between the accounts of *A* and *C*; but otherwise the expenses are to be shared equally. Find in English money, to what extent *A* and *C* are in *B*'s debt (£1 = 25 fr.)

9. A person sells half his stock at double the cost price and the remainder at half the cost price, and thus gains R115. 6*a*. What does he receive for the whole?

*__10.__ On a sum of money borrowed, interest is to be paid at 5 per cent. per annum. After a time £200 of the loan is paid off and the interest on the remainder is now reduced to 4 per cent: the yearly interest is thus reduced by one-third. What was the sum borrowed?

*__11.__ A merchant lost a cargo at sea which he had insured: the broker offered him a sum of money for the loss, which the merchant refused, as 10 per cent. below the estimated value of his loss. The broker then offered £375. 15s. more than he offered at first, and the sum now offered was $5\frac{1}{2}$ per cent. above the estimated value. What was the value?

*__12.__ The ages of two brothers are in the ratio 4 : 3. Ten years ago, they were as 3 : 2; find their ages.

13. Divide R420 among *A*, *B*, *C*, *D* in such a manner that *A*'s money shall be one-fourth as much again as *B*'s, one-third less than twice as much as *C*'s, and equal to *C*'s, and *D*'s together.

*__14.__ The shares in the consolidated stock of a certain railway are R500 each, and the guaranteed interest upon them 6 per cent.; what interest do they pay a person who buys in at $7\frac{1}{2}$ premium? What is the premium, when they yield a purchaser just 5 per cent.?

*__15.__ In an election, one candidate polled 40 votes more than the other: if 20 votes had been transferred from the losing to the winning side, the numbers would have been as 3 : 2: find the number of votes polled by each side.

*__16.__ Divide £113. 6s. 8*d*. among *A*, *B*, *C* so that £30 more than $\frac{2}{3}$ of *A*'s share, £24 more than $\frac{3}{4}$ of *B*'s share, and £40 more than $\frac{4}{7}$ of *C*'s share may all be equal.

17. A person by selling at 4*s.* $1\frac{1}{8}$*d.* per lb. an article which cost £21 per cwt., cleared $3\frac{1}{8}$ times more profit than if he had sold the whole for £162. How much was sold of the article ?

18. Find the difference between placing £440 out at Compound interest for $4\frac{1}{2}$ years at 5 per cent. and investing an equal amount in a stock at $46\frac{2}{3}$ premium, which yields a half-yearly dividend of $7\frac{1}{2}$ per cent.

19. How many square inches does the surface of a cube contain, the diagonal of whose face is 16 inches ?

***20.** What sum of money paid at the beginning of the year would be equivalent to a salary of R336 r. 8*a.* paid quarterly, money being worth 5 per cent. ?

21. Find to 6 places the value of $\sqrt{(325)}+\sqrt[3]{(144)}+\sqrt[4]{(576)}$.

22. Find the difference between the Simple and Compound Interest of R300 for $3\frac{1}{2}$ yrs. at $2\frac{1}{2}$ per cent. half-yearly.

23. *A* buys 4 per cent. stock at par, and *B* $4\frac{1}{8}$ per cent. stock at a premium of $2\frac{1}{8}$; which person makes the more advantageous purchase and by what rate per cent. ?

24. Two sums of money amounting to £1946. 5*s.* were invested, the smaller at 4 per cent. and the larger at $4\frac{1}{8}$ per cent. per annum. At the end of 18 months, the Simple Interest on the two sums amounted together to £125. 4*s.* $10\frac{1}{8}$*d.* What were the two sums ?

25 A person places £1000 at Compound Interest for 3 yrs. at 5 per cent. and with £1000 more purchases 3 per cent. stock at $92\frac{1}{8}$ which he holds for the same period, and then sells out at 95. What is his total increase of capital ?

26. A person has invested £1092 in the $3\frac{1}{8}$ per cents. at 78, and after receiving a year's dividend sells out at 83, and places the proceeds out at Simple Interest for $2\frac{1}{8}$ years at 4 per cent. What is the total increase of capital at the end of the time ?

27. In what proportion must a grocer mix teas at 2*s.* 6*d.*, 3*s.* and 3*s.* 6*d.* respectively, so that by selling the mixture at 3*s.* 4*d.* per lb., he may gain $14\frac{2}{7}$ per cent. ?

28. I receive $5\frac{1}{8}$ per cent. interest by investing my money in the $4\frac{1}{8}$ per cents. : how much of this stock can I purchase for R2700 ?

29. *A* and *B* are 40 and 16 years old respectively ; how many years ago was *A* 5 times as old as *B* ?

***30.** £1000 sterling is due from London to Portugal, when the exchange is $61\frac{1}{8}$*d.* per milrea : whether is it better for Portugal, to draw directly on London, or circuitously at an expense of $1\frac{1}{8}$ per cent., through Holland and France ? Exchange between Britain and Holland 11·90 florins per £ sterling, between Holland and France 10 florins for 21 francs, and between France and Portugal 480 reis for 3 francs.

***31.** In a certain meadow where the growth of grass is uniform, 20 oxen would consume the grass in $12\frac{3}{4}$ days and 21 oxen in 12 days : how many days' growth was on the field ?

***32.** If 29 oxen would eat up a field of grass in 7 weeks, and 25 oxen would eat up the same field in 9 weeks, the grass growing uniformly; in what time would 32 oxen eat it up?

***33.** If 33 oxen consume 3 acres of grass in $5\frac{1}{4}$ days and 25 oxen consume 7 acres of the same in 20 days, the growth of the grass being uniform; how many oxen would consume 8 acres in 26 days?

34. There is a bridge 60 ft. long crossing a straight canal whose sides are respectively 14 ft. and 50 ft. high. How broad is the canal?

35. Two ships sail from a certain port at the same time; one going due east at the rate of 16 mi. an hour, and the other due south at the rate of 12 mi an hour: how far are they apart at the end of 8 hours?

***36.** Divide £3010 into three sums, so that if the first be put out at Simple Interest for 3 years at 4 per cent., the second for 5 years at 3 per cent., and the third for 2 years at $2\frac{1}{2}$ per cent., the amount of the second shall be double that of the first and the amount of the third treble that of the second.

***37.** My age is 64 and my son's age 28: how long ago was my age 4 times that of my son's? And how many years hence will my age be a third of 5 times his age?

38. I gave for a certain article a bill of R730 due 1 mo., and sold it at once for R870 at 4 mo. How much per cent. did I gain Simple Interest being reckoned at $4\frac{1}{2}$ per cent.?

39. E. I. Railway shares pay an annual dividend of $3\frac{1}{2}$ per cent. A person having bought 12 shares, at such a price that they yielded $5\frac{5}{9}$ per cent. on his investment, sold out when the price rose R5 and invested the proceeds in the $3\frac{1}{2}$ per cents. at 85. Find the change in his income.

***40.** *A* owes *B* R5450, and offers to pay him immediately at a certain rate of discount, instead of at the end of two years, when the debt will be due. *B* can place out the money which he will receive at 5 per cent. interest, and will gain thereby R50. Find the rate of discount.

41. A man has R180000 in the 3 per cents.: he sells out at 98 and invests one-third of the proceeds in the $3\frac{1}{2}$ per cents. at 96. What must he lend out the rest at, so as to increase his income by R568. 2*a*.?

42. *A* and *B* having £620 and £300 respectively lose equal sums in speculation and find that *A* has just 5 times as much money as *B* has. How much does each lose?

***43.** *A* owes *B* R60000 bearing interest at 5 per cent. per annum; he pays at the end of each year for interest and in part payment of principal R10000: find the amount of his debt at the end of the fourth year.

***44.** A person offered a house at an upset price which would have yielded a profit of 24 per cent.; but being obliged to take R700 less he lost 16 per cent.; what did he receive for the house?

45. The difference between the interest of a certain sum for one year and the discount on the same sum due a year hence at 5 per cent., is £16; find the sum.

***46.** One contractor sends in a tender of £5000 for a certain work; another sends in a tender of £4850, but stipulates to be paid £500 every three months; find the difference of the tenders, supposing the work in both cases to be finished in 2 years, and money to be worth 4 per cent. Simple Interest.

47. If when 25 per cent. is lost in grinding wheat, a country has to import ten million quarters, but maintains itself on its own produce if only 5°/$_{\circ}$ be lost; find the quantity of wheat grown in the country.

48. *A* sells his goods 25 per cent. cheaper than *B* and 25 per cent. dearer than *C*: how much would a customer of *A* lose by buying R100 worth of goods from each of *B* and *C*?

49. At what rate per cent. per annum Compound Interest, will £640 amount to £937. 0*s*. 5·76*d*. in 4 years?

50. A Calcutta trader bought a gun in England, and after paying a duty of 10 per cent. and freight and insurance charges 20 per cent., sold it in Calcutta at a profit of 10 per cent. Being however unable to realise R90 he actually lost 5 per cent. upon it: find its cost in pounds. (£1 = R16.)

51. A picture-dealer bought a picture for R1200, and insured it for a certain sum of money at 5 per cent.: the picture is burnt, and the dealer finds that he loses R250 on the whole transaction; for what sum of money did he insure the picture?

***52.** Of two candidates at an election, it was calculated that *A* would be returned by a majority of 50: but as at the election *A* polled only 88 per cent. of his promises, while *B* polled 90 per cent. of his, *B* was returned by a majority of 6; how many votes had each?

***53.** *A* is timed to have run 100 yds. in $10\frac{4}{5}$ sec., *B* is timed to have run 100 yds. over another course in 11 sec.; if the distance were measured accurately within a yard either way, and the time taken correctly within $\frac{1}{5}$ of a sec. either way, shew that it is possible that *B* may beat *A* by more than 4 inches in a 100 yard-race.

54. *A* holds a certain amount of $3\frac{1}{2}$ per cent. stock at 80; *B* invests the same sum (sterling money) in the same stock, and the difference in their incomes is £12. 10*s*.: find the sum and income of each.

***55.** A license to kill game costs £3; a cartridge costs $1\frac{1}{2}$*d*.: a sportsman kills his bird once in three shots; birds are worth 4*s*. 6*d*. a brace: how many birds must be shot just to pay expenses?

***56.** A person firing at a target at a distance of 500 yards hears the shot strike after $2\frac{1}{2}$ sec.: the velocity of sound is 375 yards per sec., and the grooves of the rifle which cause the bullet to rotate make half a turn in the length of the barrel (3 ft. 3 in.); find approximately the number of turns per second the bullet rotates.

57. A boatman rowing against the tide passes a body floating with the tide, and in 9 min. afterwards is a mile distant from it: in 30 min. more he rows $2\frac{1}{4}$ mi. and then returns. At what rate per hour does he return.

***58.** A boatman rows 5 mi. with the tide in the time he would take to row 3 mi. against it; but if the hourly velocity of the current were half a mile more, he would row twice as rapidly with the tide as against it. What is his power of rowing in still water?

***59.** A man bought a house, which cost him 4 per cent. upon the purchase money to put into repair; it then stood empty for a year, during which time he was losing 5 per cent. upon his total outlay. He then sold it for £1192, by which means he gained 10 per cent. upon the original purchase money. What did he give for the house?

60. *A* contributes R40 to a fund every third day, *B* R80 every fifth day, and *C* R320 every seventh day: in how many days will their contributions amount together to R2160?

***61.** A person having to pay £10800 two years hence, invests in the 4 per cent. Transfer Loan to accumulate interest till the debt shall be paid, and also an equal sum the next year. Supposing the investment to be made when the paper is at 75 and the price to remain the same what sum must be invested on each occasion that these may be just sufficient to pay the debt at the given time?

62. A man bequeaths his property amounting to R49166 in such a way that one-third of his wife's share, two-fifths of his eldest son's, three-eighths of his younger son's and half his daughter's share are all equal. Find the share of each.

63. The 4 per cents. are at 96, the $4\frac{1}{2}$ per cents. are at 120. A person had a sum of money to invest which will give R1500 more of the former stock than of the latter. Find the difference of income he could obtain by investing in the two stocks.

64. Calculate the profit made by a bookseller, assuring that he pays 11*s.* 4*d.* for a 16-shilling book, receives 25 copies for 24, and deducts 10 per cent. for commission.

65. There are three cubical boxes; the edge of the first is 15 inches, that of the second 24 inches, and that of the third 36 inches. Find the length of the edge of a cubical box which shall contain as much as all the three.

66. A person invested equal sums of money in three per cents. at $97\frac{1}{2}$, and in the $3\frac{1}{2}$ per cents. at $102\frac{1}{2}$; his resulting income was £259. 10*s.* How much did he invest?

67. My total income from two investments which pay 3 and 4 per cents. respectively is R800; if the amounts invested are in the ratio of 4 to 5, find them.

68. The $3\frac{1}{2}$ per cents. are at 9 discount and the 4 per cents. at par: how should I divide a sum of £30800 between them, so that my income from the first may be double my income from the other?

MISCELLANEOUS EXERCISES.

1. The population of Great Britain at the last Census was 35,278,999. There were 789,001 more women than men. How many were there of each sex? Write your answer in words.

2. Each of 7 wives had 7 bags, each bag held 7 cats, each cat had 7 kittens. How many wives, bags, cats and kittens were there in all ?

3. How many times has a batsman been out, if he has made 2146 runs and his average is 58 ?

4. Divide £9652. 6*s.* by 96 (using factors) : and 1 ac. 2 ro. 8 po. 11 sq. yds. 2 sq. ft. 108 sq. in. by 367.

5. Of 3 pipes *A*, *B*, and *C*, *A* fills a cub. in. in a second, *B* a cub. ft. in a minute, *C* a cub. yd. in an hour ; if all were running together, in what time would they fill 1069 cub. in. ?

6. What was the cost of a ticket, if I had £5. 8*s.* 7*d.* left out of £16. 2*s.* 2*d.* after buying eleven ?

7. My friend who covers 2 ft. 11 in. at every stride takes 462 strides in accomplishing a certain distance. How many of my strides, each 2 ft. 6 in. long, will be required to cover the same distance ?

8. The circumference of a carriage wheel is 2 yds. 2 ft. 2 in. How often will it revolve during a journey of 27 mi. 578 yds. 2 ft. 10 in. ?

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9. A certain star is 5 995,944 times as far from the Earth as the Sun. State in *words* the number of seconds which light, travelling 186,000 miles per second, will occupy in coming from that star to the Earth, if the Earth is distant 93 million of miles from the Sun.

10. If every man lived to marry and have 8 male children, how many male great-great-grand-children could every one expect to have ?

11. In each of 25 streets, 625 letters are delivered daily. How long will it be before a million letters have been delivered ?

12. If a person purchase 12 articles, one of which costs 2*s.* ; two 4*s.* each ; two 5*s.* each ; and seven 8*s.* each ; what does he pay on an average for each article ?

13. What sum will remain when 4 bills, amounting to £5. 17*s.* 4½*d.*, £13. 4*s.* 7¾*d.*, £2. 15*s.* 1*d.*, and £10. 13*s.* 2¼*d.* respectively, have been paid out of £37 ?

14. How many seconds are there in the month of April ?

15. How much does a man save a day, whose daily wages are 3*s.* 1½*d.* and who spends £54. 15*s.* a year ?

16. *A* gives *B* 192 hats at 11*s.* 10*d.* each, and gets in return £63. 12*s.* and 750 pairs of hose ; what is the value of the hose per pair ?

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17. From the sum of six hundred millions fifty thousand and sixty and twenty-three thousand and two ; subtract the sum of fifty crores twenty-five lacs twenty thousand and nine, and two lacs and one.

18. If in Bengal there are 19173 schools and 4544001 scholars, what is the average number of scholars in each school ?

19. What weight remains after taking 15 parcels, weighing 3 qr. 13 lbs 12 oz. 9 dr. each, from a heap of 2 tons 14 lbs.?

20. A gentleman divided R40. 8*a*. among an equal number of boys and girls, giving to each girl 8*a*. and to each boy 4*a*.; how many children were there?

21. An Indian officer, whose pay was estimated in rupees, lost £41. 12*s*. 6*d*. in one year by a fall in the value of the rupee from 1*s*. $11\frac{1}{2}$*d*. to 1*s*. $10\frac{1}{4}$*d*.: what was his salary estimated in rupees?

22. Find the least number of weeks in which an exact number of half-guineas can be earned, the wages per week being 16*s*. 4*d*.?

23. If a man uses 135 words a minute, how many words will he use in a speech of 2 hrs. 45 min duration, and how many lines will the speech contain if 11 words are printed in a line.

24. On a Railway 7 first-class tickets, 19 second, and 57 third, for a certain journey cost together R240. 7*a*. 4*p*. If a first-class ticket cost 3 times, and a second-class ticket 2 times, a third-class ticket, what is the price of each ticket?

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25. Take 131 seventeen times, and add to it 117, thirty-one times repeated; add to this sum 2119, and multiply the result by 468. Express the result in the Roman Notation.

26. A man spends £84. 12*s*. 3*d*. in each of the first 5 months of the year. He does not wish to spend more than £826. 8*s*. 9*d*. in whole year; what must be his average monthly expenditure for the rest of the year?

27. Divide R2652. 13*a*. 4*p*. among 2 men, 3 women, and 4 children, so that each woman's share may be twice as much as a child's, and each man's share three times as much as a woman's.

28. A youth has served 5 yrs. 1 mo. 3 wks. 5 d., of his 7 years' apprenticeship. How much longer has he yet to serve?

29. If a grocer buys sugar at the rate of $4\frac{1}{4}$*d*. per lb., and sells it at 12*s*. per quarter, how much does he gain on each ton?

30. Resolve 999999 into its prime factors; and divide the product of 999999 and 1955 by the continued product of 37, 13, 17, 7, 23.

31. A farmer pays a rent of £2. 10*s*. per acre for 100 ac. of £3 5*s*. per acre for 80 acres, and of £4. 3*s*. 4*d*. per acre for 60 acres, what is the average rent per acre?

32. A boy swims 2 miles in $5\frac{1}{2}$ hours; if he goes 2 ft. 8 in. at each stroke, how many strokes does he take per minute?

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33. Find the sum, difference and product of six dozen dozen and two score score; and express the results in the Roman Notation.

34. Each of 156 boys uses 12 pen-nibs, and a box contains 144 nibs. How many boxes are required?

35. A gentleman left his daughter the contents of a writing table containing 13 drawers; in each drawer are 7 divisions, and in each division 3 purses, each purse containing 7 guineas, 14 crowns, 3 half-crowns and 17 pennies. Find the value of the legacy.

36. The fore and hind wheels of a carriage are 7 ft. 10 in. and 11 ft. in circumferences; how many revolutions will one make more than the other on a journey of 20 mi. 4 fur. 20 po.

37. Divide R105 between *A*, *B* and *C*, so that as often as *A* gets 4*a*. 6*p*. *B* may get 7*a*. 6*p*. and as often as *B* gets 5*a*. *C* may get 6*a*.

38. If in dividing 2*s*. among 8 boys, one gets 2*d*. more than each of the others, what does he get?

39. A mass of 18 sr. of metal is composed of 15 parts gold, 4 parts silver, and 5 parts copper; find the weight of each metal in the mass.

40. A child trots alongside of a man who takes exactly 528 steps in walking a quarter of a mile. Supposing the child to take 10 steps to every 3 of his, determine the child's stride.

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41. Divide R700 among 26 men, so as to give one of them R50 more than each of the others.

42. To every 2 sheep in a flock there are on the average 3 lambs; the owner values his flock at £871. 4*s*., putting each sheep at £3. 4*s*., and each lamb at £1. 18*s*. Find the number of lambs, and the difference in the total values of the sheep and lambs.

43. One man takes 120 steps a minute, each 2 ft. 6 in. long: another walks at 3 mi. 4 fur. an hour; if they start together, how soon will one of them be 100 yards ahead of the other?

44. A farmer has 7245 sheep and 3535 lambs. He forms them into flocks, keeping the sheep and lambs separate, and having the same number of animals in each flock. If these flocks are as large as possible, how many animals are there in each?

45. *A* and *B* have an equal share in a heap of potatoes, containing 96 maunds: *A* takes 30 maunds, and *B* the rest, paying *A* R27. 13*a*. 6*p*.: what is the cost of a maund of potatoes?

46. If 51 maunds of rice cost R160. 2*a*. 9*p*., what will 63 maunds cost when the price has risen 5*a*. 3*p*. per maund?

47. £28. 4*s*. is divided among 18 men, and a certain number of women: each man has £1. 3*s*. 6*d*., and each woman 15*s*. 8*d*. Find the number of women.

48. A field is 300 yds. long and 121 yds. broad, and is planted with potatoes in rows: the distance between every two rows is 15 inches. How many yards of potatoes are there in the field?

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49. A man has two sons Jadu and Hari: Jadu is 13 years old, and Hari is 2 years old. How old was Jadu when Hari was born? How old will Jadu be when Hari is as old as Jadu is now?

50. A tank has an outlet, which lets out 106 sr. of water in an hour, and an inlet, which pours in 202 sr. in an hour. The tank has 212 sr. of water in it at a certain time, when the inlet is set running; four hours afterwards the outlet is also set to work: find the quantity of water in the tank after 17 hrs. from the time when the outlet is set to work.

51. What is that least number which when divided by any one of the numbers between 2 and 10 both inclusive, leaves a remainder which is less than the divisor by 1?

52. Ram says to Sham and Gopal, I have R1650; Sham replies, if I had R753 more than I have, I should have as many you have; Gopal says, if I had R105 more than I have, I should have as many as both of you together. How many more rupees has Gopal than Sham?

53. The contents of a bag were divided amongst 69 boys and 37 girls; a boy had 3 times as much as a girl: and there would be just 5 pice over if the girls were given 9 pice each. How many pice did the bag contain?

54. A number increased by 15 times itself, amounts to eighty thousand six hundred and forty: find the number.

55. A grocer having 504 seers of tea, wishes to pack them in tin boxes holding 2 seers, 6 seers, and 10 seers; there are to be the same number of boxes of each kind. How many boxes can he fill up?

56. A farmer gave R58800 for a certain number of horses, and sold some of them for R35100 at R650 each, but by so doing lost R50 a head: for how much a head must he sell the remainder so that he may gain R6600 on his outlay?

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57. What is the least debt in dollars worth 4*s*. 2*d*. each, which can be paid in moidores?

58. 10 sheep or 15 lambs can eat 40 bushels of turnips in 7 days; how long will it take 6 sheep and 18 lambs to eat 36 bushels?

59. If it take $1\frac{1}{4}$ oz. of shot for the charge of a gun, how many shots can a boy have for 1 shilling's worth of shot at $2\frac{1}{4}d$. a lb.? How much shot will be left in his pouch after the last shot?

60. A cistern is $\frac{5}{9}$ full of water and after 50 sr. of water are drawn off, it is found to be $\frac{5}{11}$ full; what quantity of water can the cistern hold?

61. A person going to a grocer's shop with a certain sum of money and buying 20 lbs. of tea at 4*s*. 6*d*. per lb., finds only $\frac{5}{8}$ of his money left. How much had he when he entered the shop?

62. *A*, by working $\frac{1}{6}$ as fast again as *B*, is able to earn 10*d*. more per day; how much does each earn per day?

63. A man, a woman, and a boy do a piece of work, which the man can do alone in $\frac{1}{2}$ an hour, the woman in $\frac{3}{4}$ of an hour and the boy in an hour. If the man begins $\frac{1}{8}$ of an hour before the other two and afterwards all three work together, find in what time the work will be done.

64. Shew by taking an example that the product of any four consecutive numbers together with 1 is a perfect square.

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65. Find the difference between the greatest and least of the numbers that can be formed by combining the figures 2, 5, 8, 3, 1.

66. What least number added to 734641 will make it divisible by 344?

67. Find the longest chain that will exactly measure 3 mi. 4 po. and 1 mi. 3 fur. 6 po.

68. The distance between the two extreme telegraph-posts is 4 miles; find the number of posts, their centres being 44 yds. apart.

69. A boy receiving 2*s.* 6*d.* a week, has 6*d.* stopped every third week: if there are 38 wks. in the school year, how much should he get in 3 yrs.?

70. A gentleman having 46285 mangoes kept 277 for himself and distributed the rest among the boys of a school, giving 324 to each. Find the number of boys in the school.

71. A man buys 10 horses at R400 each and 4 others at R600 each. He keeps them for 6 months, during which each costs him R15 per month At what average price per head must he sell, so as to make a profit of R3568?

72. Two friends on their way home met some beggars, among whom they equally divided R48 and R64 respectively, giving to each entire rupees. Find the largest number of beggars possible.

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73. *A*, *B* and *C* went on a tour, each with R400 in his pocket, and agreed to divide their expenses equally. When they returned, *A* had R70. 10*a.* 6*p.*, *B* R40. 12*a.*, and *C* R10 12*a.* 9*p.* How did they square their accounts?

74 £4850 is to be raised ratably from three towns, whose inhabitants number 2300, 3200, 4200 respectively: how much is each town to pay?

75. What factor is wanting in 6336 to make it a perfect square?

76. *A* is 70 years old and *B* 42. Ten years ago their ages were four times that of *C*. How old is *C* now?

77. If when labour is 9*p.* an hour, the cost of building a house is R3960, what would be the cost if labour were 8*p.* an hour? Also find how long a gang of 88 men working 10 hrs. a day would take in building it.

78. Shew by taking an example that the product of any four consecutive odd numbers increased by 16, is a perfect square.

79. The telegraph posts on a railway are 77 yds. apart: find the least number of entire miles corresponding to an exact number of posts.

80. How long would it take to count a crore of rupees at the rate of 100 a minute?

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81. *A* and *B* together started business with R576895; if *A*'s capital were R15731 more than *B*'s, find their respective capitals.

82. What number multiplied by 94, will give the same product as 216 multiplied by 470?

83. *A* was born on the 15th of April 1853; how old was he on the 19th September, 1874?

84. A man bought 15 yds. of silk at R2 4*a.* per yard, 20 yds. of flannel at R1. 6*a.* per yard, 25 yds. of damask at R2. 5*a.* a yard, and 12 yds. of velvet at R3. 3*a.* per yard. At what average price per yard must he sell them that he may gain R18. 3*a.* on his outlay?

85. A man sells goods worth R54. 7*a.* 6*p.* on the first day, R55. 7*a.* 6*p.* on the second R56. 7*a.* 6*p.* on the third and so on, increasing by R1 every day; find his total takings in 40 days.

86. How many times will a foot-ball turn round, when kicked along a space of 80 yds., the ball being 18 in. round?

87. A gentleman divided two equal sums of money among 207 men and a certain number of boys, each man received R5. 7*a.*; and each boy R1. 2*a.* less; how many boys were there?

88. Find the number which being divided by 44, the quotient increased by 92, the sum diminished by the sum of 10 and 30, the remainder multiplied by 4, and the product divided by 24, leaves a quotient 10.

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89. A boy asked his father for 63 marbles, and was told that there were not enough by five and a score; how many were there?

90. The population of a town in the beginning of the year 1880 was 5 lacs. If births be one in 25, and deaths one in 50 annually; find the population of the town at the end of the year 1881.

91. A train 88 yds. long passes a signal post every 4 seconds; how many miles an hour is it running?

92. A man has three plots of ground containing 260, 325, and 403 bighas respectively. He intends to let them in pieces of equal size, each tenant getting as much as possible. Find the number of tenants.

93. A man spends in 3 months as much as he earns in 2. If he saves £257. 14*s.* 8*d.* at the end of a year, find his monthly earnings.

94. How many times did a clock, which chimes quarters, strike and chime in the year 1896? How many times in 1900?

95. In a rectangular field 220 yds. long and 140 yds. broad, cabbages are planted a foot apart in rows, the distance between the rows being 18 in. Find the number of cabbages in the field.

96. A point will contain 9000 barley corns, and three of these placed one after another, would reach an inch: how far would they all reach?

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97. In 5 whole oranges, how many sevenths of an orange are there?

98. Simplify $\dfrac{7(1\frac{1}{6} \text{ of } \frac{3}{14})}{\frac{1}{6}\left\{\dfrac{3}{3\frac{1}{2}} \text{ of } 7\right\}} \div \dfrac{9}{14} \times \dfrac{\frac{1}{4}+\frac{1}{3}+\frac{1}{4}}{\dfrac{1}{2\frac{1}{2}}+\dfrac{2}{3\frac{1}{3}}+\dfrac{3}{3\frac{3}{4}}}$

99. The price of 8 yds. of cloth is as much above R2, as that of $3\frac{3}{4}$ yds. is below R2; find the cost per yard.

100. How many pears at 4*d.* a dozen, may be exchanged for 150 apples at 8*d.* a score?

101. A cistern has two pipes attached to it; one can fill it in 20 min. and the other in 15 min. If the latter be opened 6 min. after the former, in what time from the opening of the first will the cistern be filled?

102. A man starts from *A* to walk to *B* at the rate of five miles an hour; after he has been out for half an hour, a coach starts the same place and overtakes him after $22\frac{1}{2}$ min. At a distance of 25 miles from *A*, he meets the coach returning from *B*, where it had stayed for three quarters of an hour. Find the distance between *A* and *B*.

103. A clock gains $4\frac{1}{2}$ min. a day. At 4 P. M. on Monday, it is 25 min. too slow; when will it indicate true time?

104. A man whose weekly earnings are 17*s.* 6*d.* saves $\frac{1}{5}$ of the sum of money every fortnight. In what time will he have saved 50 guineas?

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105. Five bells tolling at intervals of $2\frac{2}{3}$, $3\frac{1}{3}$, $3\frac{1}{9}$, $4\frac{1}{6}$, and $4\frac{3}{8}$ sec. respectively begin tolling together: how soon will they again be tolling together, and how often will it so happen in 14 hours?

106. What least number of rupees worth 1*s.* $4\frac{1}{2}$*d.* each, may be paid off by an exact number of guineas?

107. An ignorant fellow said that he would spend half his income, save a third, and devote a fourth to his business. His income was R7800 a year: point out his blunder, and divide his income in the above proportion.

108. *A* and *B* have R180 and R120 respectively; and if *A* gives *B* $\dfrac{14\frac{1}{4}}{28\frac{1}{2}}$ of the difference of $\dfrac{6\frac{9}{13}}{40\frac{2}{13}}$ of their respective sums, and $\dfrac{5\frac{3}{4}}{80\frac{1}{2}}$ of *A*'s present money be added to $\frac{1}{24}$ of $2\frac{2}{5}$ of *B*'s, *C*'s money will be two-thirds of this sum; what sum of money has *C*?

109. $\frac{5}{8}$ of a ship worth R49,105 is possessed in equal portions by seven persons; what is the value of each man's share?

110. The distance between *A* and *B* is 18 miles, and the road goes over a hill whose summit is 5 mi. from *A*. Two men start from *A* and *B* at the same time. The first man walks 3 mi. an hour uphill and $4\frac{1}{2}$ down; the other walks $2\frac{1}{4}$ mi. an hour uphill and $3\frac{1}{3}$ down. Where will they meet?

111. A man leaves £16409 to be divided among his four sons in the proportion of $\frac{1}{3}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{5}{12}$. Find the share of each.

112. A man after paying 7*d.* in the £ for income-tax and 10*d.* in the £ on the remainder for rates, has £17863. 6*s.* 8*d.* left. Find his gross income.

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113. What fraction must $\dfrac{5\frac{5}{8}}{11\frac{1}{3}}$ be divided by, to make the quotient 5 ?

114. *A* has an income of $\frac{7}{10}$ of $\left(\dfrac{8\frac{1}{8}-3\frac{7}{8}}{3\frac{3}{4}+7\frac{1}{2}} \div \dfrac{2\frac{1}{2}}{7\frac{1}{2}}\right)$ of *B*'s income. If *B* after spending R3640 a year saves $\frac{109}{200}$ of his income, find *A*'s income.

115. The population of a town increases by one-tenth every year. It was 394944 at the beginning of 1892 ; what was it at the commencement of the year 1890 ?

116. Sound travels at the rate of 1142 feet per second ; how far off is a thunder-cloud, if the sound follows the flash after $8\frac{1}{2}$ sec. ?

117. At the mint 3000 coins can be struck off at one press in an hour ; supposing four of these are at work, coining rupees, half-rupees, quarter-rupees, and two-anna pieces, respectively ; what would be the whole sum coined in a week, the days being $7\frac{1}{2}$ hours long ?

118. *A* and *B* go to bed at the same time daily ; *A* rises at a quarter past 6 and *B* at 8, in the morning ; how much of working life will *A* have more than *B* in 30 years, paying attention to the leap-years ?

119. If I cut half a piece of cloth into 6 equal parts and the remainder into 9 equal parts and then cut each of the 6 equal parts into 5 equal parts, and each of the 9 equal parts into 6 equal parts, and then give to 15 boys one of each of these small parts ; what fraction of the whole will still be left.

120. The expenses of a family amount to R80 when rice sells at 12 seers for the R and to R77 when rice may be had at 15 seers a R. What will the expenses amount to, when rice is 18 seers per R ?

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121. A number diminished by $\frac{3}{4}$ of itself, when divided by 517, gives a quotient 713 and a remainder 324. Find the number.

122. Divide £1332 among *A*, *B* and *C*, so that 6 times *A*'s share, 7 times *B*'s share, and 11 times *C*'s share may be all equal.

123. The sum of two numbers is 420, and $\frac{3}{8}$ of the larger number is half of the smaller ; what are the numbers ?

124. The distance between two stations *A* and *B* is 88 mi. A train starts from *A* for *B* at the rate of 12 mi. an hour ; another train starts from *A* an hour and a half after the former, at the rate of 16 mi. an hour ; find the interval between their times of arrival at *B*.

125. If 5 oz. of silk can be spun into a thread $2\frac{1}{2}$ furlongs long, what weight of silk will supply a thread of 20 miles ?

126. The population of a town consisting of Christians and Mahomedans is 520000, and the number of Christians exceeds that of the Mahomedans by three-fifths of the number of Mahomedans ; find the number of Christians and of Mahomedans.

127. How many dollars each worth $41\frac{3}{8}d.$, are equal to £74. 9*s*. 6*d*. ?

128. A man must get $\frac{1}{3}$ of the total marks to pass an examination; he answers half the questions, but on $\frac{5}{6}$ of his answers gets on an average only $\frac{2}{5}$ of the marks, and thus gets 25 marks too few to pass. How much does he get altogether?

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129 Find the value of

$$\frac{6s.\ 8d}{£5.\ 12s.}+\frac{4\text{ mi.}}{4\text{ mi. }4\text{f. }32\text{p.}}\times\frac{10\text{ cwt. }3\text{ qr.}}{2\text{ tons }13\text{ cwt. }3\text{ qr.}}+\frac{8\text{ hrs. }12'-3\text{ hrs. }15'}{24'+45'}.$$

130. Divide R871 among A, B, C, and D, so that A's share $=2\frac{2}{5}$ of B's, C's $=1\frac{1}{7}$ of B's and D's $=B$'s $+\frac{1}{2}$ of C's.

131 A person sets out to walk 80 miles; for one-fourth of the distance he goes at the rate of 4 miles an hour, for half the remaining distance at $3\frac{3}{4}$ miles an hour, and $3\frac{1}{3}$ miles an hour for the remaining distance. Find the time taken to perform the journey.

132. A flock of sheep was put into 3 fields: in the first field there were 24 less than one-third of the whole number; in the second half as many again as in the first; and in the third there were 128. How many sheep were there in the flock?

133. I buy a set of watches at R50 each, and sell them at a profit of $\frac{1}{10}$th of the cost. but in consequence of ready payment I throw off $\frac{1}{20}$th of the purchase money. What do I gain on the cost of each?

134. In a theatre the number of seats in the stalls is $\frac{3}{5}$ of the number of seats in the boxes, and they are together twice as numerous as the seats in the pit. The number of seats in the pit is 4 times the number in the gallery; together they afford as many seats as the boxes. The number in the gallery is 200; find how many persons the house will accommodate and the number of seats in each portion.

135. If 8 men or 12 women can do a piece of work in 30 days, how long will it take 2 men and 5 women to do the same piece of work?

136. After paying away $\frac{1}{5}$ of my money to one person and $\frac{2}{5}$ of the remainder to another, I have R400. 12*a*. 8*p*. left. How much had I at first?

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137. Shew that the sum of the squares of six thousand and twenty-one and eight thousand and twenty-eight is equal to the square of ten thousand and thirty-five.

138. If I buy 25 eggs at 5 for 2*d*. and break 5, at what price must I sell the remainder to gain 6*d*. on the whole transaction?

139 If the shadow of a man, whose height is 5 ft. 10 in., be 9 ft. 2 in. long, what will be the length of the shadow of a steeple 84 ft. high?

140. A Rupee is worth 2*s*. 0½*d*. and a dollar 4*s*. 4½*d*.; find the least number of rupees which makes an exact number of dollars?

141. The polar diameter of the Earth is 41,707,796 ft.; reduce this to miles, furlongs, &c.

142 Multiply 3 mi. 5 fur. 17 po. 5 yds. 1 ft. 10 in by 7; and 1574 hhd. 2 gal 3 qt. 0 pt. 2 gills by 27. Reduce 23 ac. 1 ro. 27 sq po. 14 sq. yds. 6 sq ft. 103 sq. in. to square inches.

143. A, B, C play cricket. A's runs are to B's and B's to C's as 3 : 2. They get altogether 342 runs. How many did each get?

144. Simplify $\frac{(\frac{1}{2}+\frac{1}{5})(\frac{1}{2}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{3})(\frac{1}{2}-\frac{1}{4})}-\frac{(\frac{1}{3}+\frac{1}{5})(\frac{1}{3}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{3})(\frac{1}{3}-\frac{1}{4})}+\frac{(\frac{1}{4}+\frac{1}{5})(\frac{1}{4}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{4})(\frac{1}{3}-\frac{1}{4})}$.

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145. A man has an annual income of £974. 1s. $10\frac{1}{2}d$. How much can he spend a day in an ordinary year? How much in a leap year?

146. Simplify (a) $\frac{8\frac{3}{5}-7\frac{3}{4}+5\frac{2}{3}-4\frac{1}{2}}{13-11\frac{9}{10}+10\frac{7}{8}-9\frac{17}{20}}\times\frac{12}{121}\div\frac{36}{385}$;

(b) $\frac{(1\frac{1}{2}+\frac{3}{5}+2\frac{1}{4})\times(2\frac{1}{7}+\frac{5}{14}+3\frac{1}{2})}{8+\frac{1}{7}\times 1\frac{2}{5}\times 3\frac{1}{7}+\frac{1}{14}}$.

147. A workman was absent from his work on 36 days in 1883 besides Sundays, and his wages amounted to £65. 15s. 9d. How much would he have had, if he had missed only the Sundays?

148. How many pages of a magazine will a manuscript occupy, which has 120 pages, each of 28 lines with 7 words per line on an average, while a page of the magazine has 42 lines, with 10 words per line on the average?

149. The sum of £11. 7s. 6d. is made up of a certain number of sovereigns, twice as many half-crowns, five times as many shillings, and ten times as many three-penny pieces. Find the number of each coin.

150. A person on being asked what time it was, answered that the time past noon was $\frac{3}{5}$ths of the time till midnight. What was the time?

151. On a stream the station B is intermediate to and equidistant from A and C; a boat can go from A to B and back in 5 hrs. 15 min.; from A to C in 7 hrs. How long would it take to go from C to A?

152. A person has a certain distance to go on a river which flows at 1 mi. per hour, and finds that he takes 3 hrs. more to go than to come back. If the speed of his boat be 3 mi. an hour, find the distance.

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153. How many dollars, worth R1. 8a. each, are there in 3000 guineas? (A guinea = R12. 8a.)

154. R1000 has to be divided among 27 persons. find the share of each to the nearest pie.

155. After giving away $\frac{1}{8}$ of his income in charity and paying $\frac{1}{12}$ in taxes, a person has R570 left. What is his gross income?

156. A servant is hired at £51. 2s. per annum. Beginning work on 13th Oct. 1873, he leaves on 1st May 1874. What should he get?

157. A train is moving 27 mi. an hour. There are telegraph posts on the line 60 yds. apart. How many will the train pass in 10 minutes?

158. If the price of the 4lb.-loaf is $5\frac{1}{2}d.$ when wheat is $5s.$ per bushel and $6\frac{5}{8}d.$ when wheat is $6s.\ 3d.$ per bushel, find the cost of the workmanship per loaf.

159. Reduce to the simplest form :

$$(a)\ \frac{1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}}{1+\frac{1}{2}-\frac{1}{3}-\frac{1}{4}}; \qquad (b)\ \frac{\frac{3}{4}\text{ of }3\frac{1}{5}+\frac{5}{6}}{\frac{9}{10}-\frac{11}{18}\text{ of }1\frac{1}{3}}\div\frac{(\frac{1}{2}-\frac{1}{3})(\frac{1}{3}+\frac{1}{4})}{(\frac{1}{2}+\frac{1}{3})(\frac{1}{3}-\frac{1}{4})}.$$

160. A ship's crew consists of a captain, 3 officers, and 36 men ; the captain gets $4\frac{1}{2}$ times as much prize-money as each officer, and the officers together 12 times as much as each man. What do they get respectively out of a prize of the value of £1055. 9s ?

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161. How many seconds were there in February 1800 ?

162. Reduce the following to their simplest forms :

$$\frac{\frac{1}{3}+\frac{1}{5}+\frac{1}{7}}{\frac{1}{2}+\frac{1}{4}+\frac{1}{6}}, \quad\text{and}\quad \frac{528\text{ tons }13\text{ cwt. }3\text{ qr }27\text{ lbs. }1\text{ oz.}}{674\text{ tons }6\text{ cwt. }1\text{ qr. }2\text{ lbs. }1\text{ oz.}};$$

also find the value of $\left(\dfrac{2\frac{1}{4}-\frac{2}{3}\text{ of }\frac{11}{6}}{\frac{1}{5}\times\frac{10}{3}+\frac{13}{36}}-\dfrac{1}{2\frac{1}{2}}\right)\times\dfrac{1}{1\frac{4}{5}}$ of £1.

163. How many sheep must a person buy at £7 each, so that after paying one shilling a score for folding them for a night, he may gain £79. 16s. by selling them at £8 each ?

164. A tank has 3 pipes A, B and C. A and B can fill it in 8 hrs. and 4 hrs. respectively, C can empty it in 2 hrs. If A, B and C are opened at 1, 3, and 5 o'clock respectively, find when the tank will be full and when empty again.

165. If a gallon contains $277\frac{1}{4}$ cub. in., and a gallon of water weighs 10 lbs. find approximately the weight of a cub ft. of water in ounces.

166. A person buys a piece of land at R30 a bigha, and by selling it in allotments finds the value increased threefold so that he clears R150, after reserving 25 bighas for himself ; how many bighas were there ?

167. A clock which was 1 min. 24 sec. too fast at 4-15 P.M. on March 12th, is 6 min. too slow at 9 A. M. on March 15th. When was it right ?

168. I agree to pay a man £5. 13s. 9d for working at a certain rate during a certain time He is absent $\frac{1}{7}$ of the time and, when present, works at only half the rate agreed upon. What should I pay him ?

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169. Find the cost of a ton of sugar at $\frac{3}{4}d.$ per lb., of an ounce tea at $5s.\ 8d.$ per lb., and of a peck of flour at 60s. per quarter.

170. A farmer has 1134 sheep and 630 lambs. He forms them into flocks, keeping sheep and lambs separate, and having the same number of animals in each flock. If these flocks are as large as possible, how many are there in each ?

171. Simplify $\frac{91}{122}\times(4\frac{1}{8}\text{ of }6\frac{3}{7}+\frac{3}{14})\div\{4\frac{1}{8}\text{ of }(6\frac{3}{7}+\frac{1}{14})\}$.

172. If 3 ducks be worth 4 chickens and 2 geese be worth 7 ducks, find the value of a goose when a pair of chickens can be had for $3s.\ 9d.$

173. A sum of R290 is to be divided among 100 workmen, so that some may receive R2. 12*a*. and others R3. 4*a*. each. How many will receive the larger sum ?

174. *A* and *B* walk over the same course from opposite sides. *A* walks at the rate of $3\frac{3}{4}$ miles an hour, and *B* 4 miles an hour. They meet $1\frac{1}{4}$ min. before *A* reaches half-way. Find the length of the course.

175. Of the 840 candidates at an examination, $\frac{3}{4}$ were boys and $\frac{1}{3}$ of the candidates failed to pass. How many boys must have passed and what is the greatest number of boys who can have passed ?

176. A man starts on a journey, walking 4 mi. an hour, but afterwards gets a lift in a carriage going 10 miles an hour. He is out 6 hrs., and accomplishes 38 miles. How many of these did he walk ?

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177. A shopkeeper's drawings for the first 5 days of the week are £5. 7*s*. 5*d*., £4. 18*s*. 7*d*., £6 11*s*. $9\frac{1}{2}$*d*., £6 16*s*. 5*d*., and £6. 14*s*. 2*d*. How much must he draw on Saturday that his average daily drawings may be £7 ?

178. The year 1879 began on a Wednesday ; on what day of the week will the nineteenth century end, and how many times, in the course of the twentieth, will New Year's Day fall on a Saturday ?

179. A woman bought eggs at 2 a penny, as many more at 3 a penny and again as many more at 4 a penny. She sold them all at 5 for 2*d*. and gained 7*d*. How many did she buy each time ?

180. If 20 marks are worth £13. 6*s*. 4*d*. and an English sovereign is worth 18 Danish crowns ; find the value of 1000 marks in Danish crowns.

181. Simplify $\dfrac{2\frac{1}{3}+5\frac{1}{2}}{2\frac{1}{5}-1\frac{1}{6}}+\dfrac{1\frac{7}{9}+3\frac{1}{7}\text{ of }1\frac{3}{11}}{4\frac{1}{5}\text{ of }3\frac{4}{7}-1\frac{2}{9}}+\dfrac{2\frac{1}{8}\text{ of }\frac{3}{17}\text{ of }2\frac{2}{3}}{\frac{1}{2}+\frac{2}{10}\text{ of }28\frac{1}{2}}-\dfrac{2\frac{1}{7}\text{ of }\frac{3}{11}\text{ of }1\frac{2}{5}}{2\frac{3}{8}+\frac{3}{4}+\frac{1}{2}\text{ of }\frac{3}{4}}$.

182. In a cricket match the highest score was $\frac{1}{6}$ of the whole, the next four scores amounted together to $\frac{3}{7}$, the other six to $\frac{12}{35}$, and there were 5 extras. What was the whole score ?

183. A man wished to give 15*s*. each to a number of poor people, but found he had 10*s*. too little ; he gave them 14*s*. 6*d*. each, and had 2*s*. over. Find the number of poor people, and the sum of money he had.

184. Distribute R1000 among 4 persons *A*, *B*, *C*, *D* in such a manner that *B* may receive half as much again as *A*, *C* as much as *A* and *B* together, and *D* as much as *A*, *B*, *C* together.

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185. In Paris a sovereign is worth 25 fr. 10 cents. and in London a 20-franc piece is worth 15*s*. 3*d*. What is gained or lost by changing 100 sovereigns in Paris and changing their French equivalent back into English money in London ?

186. What is the difference between $\dfrac{3\frac{3}{8}+3\frac{1}{4}}{7\frac{1}{8}-1\frac{5}{8}}$ and $\dfrac{3\frac{2}{7}+4\frac{1}{5}\text{ of }\frac{10}{9}}{6\frac{4}{5}-1\frac{5}{6}\text{ of }\frac{40}{33}}$?

187. A merchant bought for R61 four boxes of oranges each containing the same number. They cost respectively 1*a*. for 3, $1\frac{1}{3}$*a*. for 4, 2*a*. for 5 and $2\frac{1}{2}$*a*. for 6. How much per rupee would he gain by retailing the whole at $\frac{1}{2}$*a*. each ?

188. A man has R3000 in hand, having lost a quarter of his property in speculation, and purchased a share in a mine with three-quarters of the remainder. What was he worth at first ?

189. A person travels 400 miles, the first half by railway, and the remainder by coach. On arrival he finds that his average speed has been 16 miles an hour. The average speed of the train was 40 miles an hour. At what rate did the coach travel ?

190. A man dies leaving R13800 between 3 sons and 3 daughters; each son is to have one-third more than the eldest daughter, and the other daughters each R100 less. Find what each receives.

191. *A* has twice as much money as *B*. They play together for a certain stake, and at the end of the first game *B* wins from *A* one-third of *A*'s money. What fraction of the sum that *B* now has, must *A* win back in the second game that they may have exactly equal sums ?

192. The pipe which supplies water to a cistern is $\frac{1}{4}$ of the size of the stop-cock for letting it off If, when the cistern was full, the pipe and the stop-cock were both opened, and the supply were cut off at the end of 3 hrs. the cistern would then be emptied in another hour. Find in what time the full cistern would be emptied when there is no supply.

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193. Determine the values of 1636 of 16*s*. 8*d*. ; $\cdot\dot{2}\dot{7}$ of R1. 6*a*. ; and multiply $\cdot$155873 by 0021.

194 Find the value of 11 chests of tea, each containing 39 lbs., at 5*s*. $7\frac{1}{4}$*d*. a pound.

195. *A* can do a piece of work in $10\frac{2}{3}$ days, *B* in $9\frac{3}{5}$ days, and *C* in $5\frac{9}{11}$ days ; in how many days can *A*, *B*, *C*, working together, complete it ?

196. If 5 men or 7 women can do a piece of work in 37 days, in what time will 7 men and 5 women can do a piece of work twice as great ?

197. Simplify $(4\frac{5}{7}\times 5\frac{1}{3}-1\frac{3}{14})+\{4\frac{5}{7} \text{ of } (5\frac{1}{2}-1\frac{3}{14})\}\times\frac{407}{420}$.

198. A gentleman divided £4. 18*s*. among 150 school children, giving the girls a shilling each, and the boys six-pence : how many boys and girls were there ?

199. The average age of the 25 boys in the fifth form of a school is 15 6 yrs., and the average age of the 35 boys in the sixth form is 16 8 yrs. what is the average age of the sixth and the fifth forms together ?

200. *A*, *B* and *C* can do $\cdot$8 of a piece of work together in 24 days. Had either *A* or *B* been absent, the two others would have done $\frac{5}{8}$ of the work in 28 days. In what time can they do it working singly ?

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201. If a sovereign weighs 5 dwt. 8 grs., what is the value of 106 lbs. 9 oz. 16 dwt. of the same metal ?

202 Find the value of $3\frac{4}{5}+9\frac{5}{8}+7\frac{3}{4}+4\frac{11}{125}+1\frac{2}{3}$. both by Vulgar Fractions and by Decimals : shew that the results coincide.

203. Two cog-wheels, one with 15 teeth and the other with 28 teeth, work together. If the former turns round 16 times in $7\frac{1}{2}$ seconds, how many times will the latter turn round in 21 seconds ?

204. Three men working all day can plough a field in 20 days : but one of them having different engagements can only work half time : how long will it take them to complete the work ?

205. Of a regiment of soldiers $\frac{1}{5}$th are killed or disabled in the first battle, $\frac{7}{24}$ths of the remainder in the second, $\frac{5}{13}$ths of the remainder in the third, and there are 340 left. How many were there at first ?

206. One pendulum oscillates six times in 3·2 sec, another pendulum nine times in 4·05 sec. ; if started simultaneously, how often will they tick together in an hour ?

207. In a book on Arithmetic an example was printed thus :

"Add together $\frac{1}{14\frac{2}{7}}$, $\frac{1}{19\frac{1}{4}}$, $\frac{1}{\quad}$, $\frac{1}{13\frac{3}{4}}$." The answer given at the end of the book was $\frac{11}{28}$: required the missing denominator.

208 A's rate of working is to B's as 4 : 3, and B's to C's as 2 : 1. How long will it take C to do what A would in 6 days ?

209. (i) Multiply and (ii) divide, 57·8214 by ·00045. Find what decimal 3 yds. 2 ft. 2 in. is of $2\frac{1}{2}$ rods

210. Multiply the difference between 12 cwt. 3 qr. 17 lbs. 15 oz. and 5 cwt. 2 qr. 23 lbs. 13 oz by 528. Divide the sum of the same two quantities by 324.

211. ·3 of a certain sum of money is spent in buying lands, ·45 of the remainder in buildings, ·09 of what then remains in furniture, and the rest which is R35035 in buying Railway shares. Find the sum.

212. A mixture is made of 297 gal. of wine at 17*s*. 9*d*. per gallon, 132 gal. at 22*s*. 6*d*. per gallon, and 121 gal. at 19*s*. 4*d*. per gallon ; find the value of a gallon of the mixture.

213. A man rode a bicycle from A to B, a distance of 108 mi. at an average rate of 8 mi. an hour ; another man started from A on horseback an hour later and arrived at B 30 min. earlier. Compare their speeds.

214. Brussels carpet is 2 ft. wide, costs 7*s* 6*d*. a yard, and will last 5 years ; Kidderminister carpet is $2\frac{1}{2}$ ft wide, costs 5*s*. a yard, and will last 3 years : find the ratio of their costs.

215. Find to 5 places of decimals the value of

$$\frac{1}{9}+\frac{1}{3\cdot9^3}+\frac{1}{5\cdot9^5}+\frac{1}{7\cdot9^7}+\ldots\ldots$$

216. What sum of money can be divided between A and B in such a manner that as often as A gets 15*s*. B shall get 9*s*., and A's share is 50 guineas more than B's ? Find also their respective shares.

217. In London a pound-note (6 in. × 4 in.) is said to cover ground equal to its own value. What is the value of an acre?

218. Find the number of yards in :

$\cdot 675$ mi. $+ 1\cdot 045$ fur. $- 2\cdot 75$ of 5 yds. $+ \cdot\dot{1}4285\dot{7}$ of 14 ft. $- 29\cdot 45$ of 22 in.

219. A wine merchant buys spirits at 6*s.* 3*d.* a gallon, and after adding water, retails it at 6*s.* 8*d.* per gallon, thereby gaining 1*s.* on every 5*s.* of his outlay; what proportion of water does he add to the spirits?

220. What is the least weight which can be expressed either by a whole number of Troy penny-weights or by a whole number of Avoirdupois ounces? Give your answer in grains.

221. Find to 7 places of decimals the value of

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 3^2}+\frac{1}{5\cdot 3^3}+\frac{1}{7\cdot 3^4}+\ldots\ldots$$

222. A man travels 60 miles in 3 hours, partly by rail and partly by coach. If he had gone all the way by rail he would have arrived at his destination an hour earlier, and would have saved two-fifths of the time he was on coach. How far did he travel by coach?

223. Coals rise from 15*s.* 9*d.* to 18*s.* 6*d.* per ton What is the extra expense to a firm buying in 24 tons, an amount which is $\frac{1}{6}$th less than their usual supply?

224 A legacy of R30000 remained unclaimed for 3 yrs. Find what it will amount to at 4 p. c., at (1) Simple Interest, (2) Compound Interest.

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225. Find by Practice the cost of 3 tons 11 cwt. 3 qr. $10\frac{1}{2}$ lbs. of iron wire at £8 6*s.* 8*d.* per ton.

226. If the cost of provisioning a gun-boat carrying 84 men be £598 10*s.* when the ship is at sea for 95 days what will it cost to provision for 33 days a ship carrying a crew of 110 men?

227. *C* does half as much in a day as *A* and *B* can do together, and *B* does half as much again as *A*. If all three working together can mow 20 acres of barley in 16 days, how long would each, working by himself, take to mow 5 acres?

228. What percentage of the contents of a cask of spirits must be replaced by water so that a merchant may gain 25 per cent by retailing the mixture at the same price per gal. as he paid for the undiluted spirits?

229. *A*, *B* and *C* start in a business together: *A* puts in R5000, *B* R6000. and *C* R9000. At the end of 3 mo. *C* leaves, and at the end of 7 mo. more *B* leaves, both taking out their capitals with them; but the profits are not divided till the end of the year, when *C* receives R90. How much will *A* and *B* receive?

230. An article of commerce passes successively through the hands of three dealers, each of whom in selling adds as his profit 10 per cent. of the price at which he bought it What did the first dealer pay for goods which the third dealer sells for R110. 14*a.* 8*p.*?

231. The area of each of the 64 squares of chess-board is 4·2025 sq. inches, and the outer rim of the board is ·3 in. wide; find the length of a side of the board.

232 A person invests £1362. 10*s*. 3 per cent. in Consols, which he sells when they have risen $\frac{5}{8}$ per cent, thereby gaining 15 guineas. At what price did he buy them?

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233. Find by Practice the cost of a fence 3 fur. 11 po. $3\frac{1}{4}$ yds. long at £183 6*s* 8*d*., per mile.

234 Simplify $\frac{1684}{1408}$ of {$\frac{7}{10}$ of £$3\frac{3}{14}$ + $6\frac{2}{3}$ of £3. 0*s*. 9*d*. − $4\frac{13}{31}$ of £3. 2*s*.}; multiply the result by ·004 and divide the product by 6·4; and express as a Vulgar Fraction the product of $\dot{2}\dot{7}$ and $\cdot 91\dot{6}$.

235 Divide ₨523. 12*a* among three persons *A*, *B* and *C*, so that *A* may get one-third of what *B* receives and *C* may receive half as much again as *A* and *B* together.

236. A square field contains 22 ac. 2 ro.; how long will it take a man to run round the boundary, at the rate of 12 mi. an hour? If the field be increased by 9 ac so as to form a rectangle whose shorter side is the former side of the square, at what rate does a man run who runs round it in 1 min. 15 sec. longer than was taken in running round the square field?

237. Find the least number of things that can be bought at £5·3625 each, for an exact number of sovereigns.

238. A grocer buys 15 lbs. of tea. he sells 8 lbs. at 2*s*. $7\frac{1}{2}d$ per lb., and the rest at 2*s*. 9*d*. per lb.; and finds that he has made 15 per cent. profit. What rate per cent. profit would he have made, if he had sold it all at 2*s*. $9\frac{1}{4}d$. per lb.?

239. A journey of 560 miles was made by rail, steamer and coach. The distance by coach was one fourth, and the distance by sea three-fourths of that by rail. The fare per mile by coach was double, and by sea four-fifths of that by rail. What was the expense of the whole journey, railway fare being $1\cdot\dot{5}7142\dot{8}$*d*. per mile?

240. What will £3255 4*s*. 2*d*. amount to in a year and a half, if put out at Compound Interest at 8 per cent. per annum, the interest being added at the end of every half-year?

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241. Find by Practice the cost of warming a building for 11 days 17 hrs. 28 min., if the cost is ₨4. 8*a*. per day.

242. Find the value of ·8125 of 2 tons 4 cwt.; and the difference between $3\cdot\dot{0}69\dot{3}$ of 8*s*. 5*d*. and £·6426.

243. A rectangular cistern $10\frac{1}{4}$ ft. long $6\frac{1}{4}$ ft. wide and $3\frac{1}{2}$ ft. deep, contains $140\frac{5}{8}$ cub. ft. of water: what is the least number of bricks, each 9 in. long, by $4\frac{1}{2}$ in. wide, by 3 in. thick, that must be thrown into the cistern to make the water rise to the top, a brick being found to absorb water to the extent of one-fifth of its volume.

244. In a hundred yards' race A can beat B by 4 yds., in a quarter of a mile race C can beat A by 11 yds.; by how much can C beat B in a mile race, supposing that the average speed of each man when running a hundred yards, a quarter of a mile, and a mile, are as 9 : 8 : 7 ?

245. If a sovereign be worth 62·5 Dutch guilders, 202 thalers worth 350 guilders and 10 thalers worth 7 35 francs: how many francs should be received for 320 guineas ?

246. A can do a piece of work in 6 days, B in 8 days, and C in 12 days. B and C work together for 2 days, and then C is replaced by A. Find when the work will be finished.

247. The debts of a bankrupt amount to £1067. 5*s*. 3*d*. and his assets consist of property worth £458. 7*s*. 8*d*. and an undiscounted bill of £256. 10*s*. due 4 months hence, Simple Interest being reckoned at 4 per cent. How much in the pound will he pay ?

248. By paying an income tax of 9*d*. in the pound, a man's income is reduced to £1212. 15*s*. Find the sum of money which must be invested in $4\frac{1}{2}$ per cent stock at $110\frac{3}{8}$ to produce that income, brokerage of $\frac{1}{8}$ per cent being charged on the purchase of the stock.

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249. How many rings each weighing 4 dwt. 18 grs. can a gold-smith make from a mixture of 1 lb. 10 oz. 1 dwt. 18 grs. of pure gold with 7 oz 7 dwt. 6 grs. of alloy ?

250 A wheel 17 155 ft in circumference makes 5461·75 revolutions in passing from one end of a street to the other. How long is the street ?

251. If 6 compositors in 16 days of $10\frac{1}{2}$ hrs each can set in type 720 pages, each of 60 lines with 40 letters in a line; in how many days of 7 hrs. each will 9 compositors set 960 pages, each of 45 lines with 50 letters in a line ?

252. A rectangular field which is twice as long as it is wide costs $1·01\dot{3}8$*s*. per square yard to turf. If the whole cost be £191. 17*s*. $0\frac{3}{4}$*d*., find the lengths of the sides of the field.

253 Find the whole annual cost of the house of which the rent is £840: the poor-rate being 3*d*. in the pound, the gas-rate $\frac{7}{9}$ of the poor-rate, and the paving rate $\frac{5}{7}$ of the gas-rate.

254 A man sells a horse for R12 more than he gave for it, and realizes a profit equal to $\frac{2}{7}$ths of its cost price. What was its cost price ?

255 When 52 lbs. of coffee are worth as much as 12 lbs of tea, 22lbs. of tea cost as much as 5 72 lbs. of sugar, a cask of sugar costs 2 guineas, and 1 cwt. of coffee costs £8. 8*s*, what is the weight of a cask of sugar ?

256. A man has R2000 of a 3 per cent. stock, which he sells out at 90 and then invests R1000 of the proceeds in 5 per cedts. at 125; what rate of interest must he get for the remainder in order that his income may be unaltered.

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257. How many times can a vessel which can hold 5·0125 sr. of water be filled from a water tub containing 71·67875 mds. of water ?

258 At a bankrupt's valuation his bullocks were put at R50 a head. They were sold, one-half at R44. 8*a.* a head and the other half at R90 a head; and thus R138 were realized more than was expected. How many bullocks had he ?

259. If 11 per cent. be lost by selling 230 yds. of cloth for R852. 14*a*. 8*p*., at what price must it be sold per yd. to gain 17 per cent. ?

260. If R416. 15*a*. 4*p*. be equally divided among 63 persons, what decimal of the whole will remain ?

261. How many pipes each 2 ft. in length will be required for draining a square field containing 11 ac. 4 po. with parallel drains; $5\frac{1}{2}$ yds. being the distance between each two drains and also the distance between the extreme drains and the fences ?

262. A bill of R9990 is due in such a time that R800 would in the same time amount to R832 8*a*. What discount should be allowed for ready payment ?

263. What price ought to be marked on the cover of a book, so that after allowing a discount of 35 per cent. it may be sold for 6*a*. 6*p*. ?

264. A man invests a certain sum in $4\frac{1}{2}$ per cent. Consols at 104. The price falling to 101, he sells out and loses £1500 by the transaction. Find the sum invested

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265. A boat is rowed down a river at the rate of a quarter of a mile in 1 min. 30 sec., and up the river at the rate of ·5 of a mile in 5 min., the crew working equally hard: find the velocity of the current.

266 The length of a room is 16 ft.: the cost of papering the walls at 8*a*. a sq yard is R40; and that of carpeting the room at R2. 4*a*. a sq. yard is R56 Find the breadth and height of the room.

267. The rates of the express and mail trains on a railway are 30 and 21 miles an hour respectively. What time may a passenger save by taking the former for a journey of 287 miles ?

268 A person buys tea at R3 a lb. and also some at R2 a lb. In what proportion must he mix them so that by selling his tea at R2. 10*a*. a lb., he may gain 20 per cent on each lb. sold ?

269. In a distribution of alms each child had 15*a* 6*p*., each woman twice as much, and each man thrice as much. The whole sum distributed was R13950, and men, women and children were taken in equal numbers. How many were there in all ?

270. Twelve years ago, *A* had £1300, and *B* had £910. *A* has been more fortunate than *B*, and gained 3 per cent. where *B* gained only 1 per cent. *B* has now doubled his capital: how much has *A* got ?

271. A sum of money put out at Compound Interest amounts in 1 year to £1660, and in 3 years to £1786. 16*s*. $8\frac{1}{4}d$.: find the rate of interest and the sum.

272. *A* invests R10200 in the 3 p. c. at $89\frac{1}{4}$, and *B* the same sum in the $3\frac{1}{2}$ per cents. at $95\frac{5}{8}$, who has the greater income, and by how much ?

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273. In the centre of a room 21 ft square, there is a square Turkey carpet : the rest of the floor is covered with oil-cloth The carpet and oil-cloth cost respectively 16*s*. 6*d* and 8*s*. 6*d*. per sq. yard : and the whole cost of carpet and oil-cloth is £35. 4*s*. 6*d*. Find the width of the oil-cloth border.

274. In the Centigrade thermometer the freezing point is zero, and the boiling point is 100° : in Fahrenheit's the freezing point is 32°, and the boiling point is 212° ; what degree C corresponds to 68° F ?

275. The cost price of a book is R1. 14*a*. ; if the expense of sale be 5 per cent. upon this, and the profit 25, what would be the retail price ?

276 A man who has sold tea at R2 8*a* per seer, making a profit of 25 per cent., lowers his profit so as to gain only 2*a*. per seer. In what ratio must his monthly sale increase that he may make twice as much as before ?

277. *A* and *B* buy mangoes at 10 for 12*a*. : *A* retails them at 9 for 12*a*., and *B* at 17*a*. for a dozen. Compare their gains on selling the same number of mangoes.

278. A boat whose speed was $9\frac{3}{4}$ miles an hour sailed from *A* to *B* a distance of 65 miles ; and a second boat, which left *A* $2\frac{1}{4}$ hrs after the first, arrived at *B* 5 minutes before. Compare their rates of sailing.

279. The discount on a certain sum due in 3 years is R300, and the interest on the same is R336. Find the sum and rate per cent.

280. The receipts of a Railway Company whose capital is R400000 are R50000 in a certain year. If it has borrowed R100000 at 4 per cent. per annum and the working expenses be 40 per cent of the receipts, and if R6000 be retained as a reserve fund, find the dividend paid.

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281. *A*'s rate of working is to *B*'s as 5 to 3, and *B*'s to *C*'s as 3 to 2. How long will it take *C* to do what *A* would do in 6 days ?

282. *A* can do a piece of work in 25 days, *B* in 20 days and *C* in 34. They work together for 2 days, and then *A* and *B* leave ; but *C* continues, and after $8\frac{3}{4}$ days is rejoined by *A*, who brings *D* along with him, and the three finish the remainder of the work in 3 days more. In what time would *D* alone have done the whole work ?

283. A bag contains 160 coins consisting of half-crowns, shillings, six-pences, and four-pences, and the values of the sums of money represented by each kind of coin are the same : how many of each are there ?

284 A merchant mixes 54 gallons of wine at R24 per gallon, with 36 gallons at R14 per gallon. How much water must be added in order that by selling at R20 per gallon he may gain 10 per cent. ?

285 If a person whose income is R3650 a year spend R88. 2*a*. a week for the first 20 weeks, to what must he limit his daily expenditure for the rest of the year so as not to be in debt ?

286 A rectangular field is half as long again as it is wide : the cost of levelling it at 6*a* a sq. yd. is R1764 ; find the cost of enclosing it with a fence at 5*a*. per yard.

287. Find the true discount on a bill for R16222. 8*a.* drawn on February 4th, at 10 mo. and discounted July 14th at $7\frac{1}{2}$ p.c. per annum.

288. The 3 per cents. are at $85\frac{3}{8}$: what price should the $3\frac{1}{2}$ per cents. bear, that an investment may be made with equal advantage in either stock ? And what income would be derived by so investing R7350 ?

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289. If the hands of a clock coincide every $65\frac{1}{3}$ min., how much does the clock gain or lose in a day ?

290. The external length, breadth and height of a wooden box are 18, 10, and 6 inches respectively, and the thickness of the wood is half an inch : when the box is empty, it weighs 15 lbs, and when filled with sand 100 lbs. : compare the weights of equal bulks of sand and wood.

291. Find the distance in miles of a thunder cloud when the thunder is heard 16 sec. after the flash is seen : supposing that sound travels 1140 ft. in a second and that the flash is seen instantaneously.

292. A shed rests upon two walls (each 20 ft. long) built 9 ft. apart, and 10 ft and 11 65 ft high respectively. Find the area of the shed, and also the cost of constructing it at R5 per sq. foot.

293. If a publisher, in selling a book for cash, rates it at 25 per cent. below the publishing price and then charges for 13 copies as for 12, what credit could he allow, so that on the principle of true discount at 4 per cent. per annum, the sum to be received for a book might be just 29 per cent. below the publishing price ?

294. *A* pays R137. 8*a.* more rates than *B*, though their incomes are equal : living in different places, they are rated at $1\frac{3}{8}a.$ and $1\frac{1}{16}a.$ in the rupee respectively : find their income.

295. The interest on a given sum of money for one year is £5. 8*s.* 4*d.*: the Compound interest for two years is £11. 1*s.* Find the rate per cent.

296. Find the alteration in income occasioned by shifting R34000 Government securities from the 4 per cents. at $98\frac{3}{8}$ to the $4\frac{1}{2}$ per cents. at $101\frac{7}{8}$; the brokerage being $\frac{1}{8}$ per cent. in each case. What will be the amount of the Government securities in the latter case ?

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297 If gold is worth £4. 3*s.* 4*d.* per ounce, what is the value of a bar weighing 4lbs. 5 oz. 5 dwt. 21 grs. ?

298. A field of 10 acres is twice as long as it is broad : find the length and breadth in yards to three decimal places.

299. An article is first sold at a profit of 10 per cent. ; the purchaser then sells it for 4*s.* 7*d.* and his gain is 15 per cent. of the selling price. How much did the article originally cost ?

300. If 12 men dig 12 roods of ground in 12 days working 12 hrs. a day, how many roods could 8 men dig in 8 days working 8 hrs. a day ?

301. How much will it cost to paper a room 18 ft. 6 in. broad by 20 ft. long and 11 ft. high, with paper costing 9*d.* per sq. yd ? The room has a door 4 ft. broad by 7 ft. high, and a window 5 ft. broad by 8 ft. high.

302. A debtor owing £11500 paid a composition of 6*s.* 3*d.* in the pound. How much would a creditor lose to whom $\frac{2}{3}$rds. of the whole was owing?

303. A legacy remained unclaimed for 2 years, at the end of which it amounted to R4134. 10*a.* at 5 per cent. per annum; what was the amount of the legacy?

304. I invest equal sums in a 4 per cent. stock and in a 3 per cent. stock, and get 5 per cent. for my money: the 4 per cents. are at 90; what is the price of the 3 per cents?

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305. Find the cost of painting the four sides and the bottom of a tank $2\frac{1}{3}$ yds long, 4 ft. wide and $4\frac{1}{2}$ ft. deep, at 4*d.* per sq. foot.

306. A reduction of 40 per cent. in the price of half-crown tea enables me to buy for my money 6 lbs. more than I could buy for a sovereign: what amount have I?

307. A sum of money is to be divided amongst 11 men and 18 boys: and 5 men are to receive as much as 9 boys. When 3 men and 3 boys have received their shares, what fraction of the whole sum will remain?

308. Find the square root of 549081 and of $5\frac{4}{9}$.

309. A man after a tour in Switzerland found that he had spent every day half as many shillings as the total number of days he had been from home. His tour cost £57. 12*s.*; how many days did it occupy?

310. A bankrupt's liabilities are £6235. 10*s.*, and he pays his creditors 5*s.* 6*d.* in the pound. Find by Practice the amount of his assets.

311. If the true discount on a bill of £14270 be £4270 at $5\frac{1}{13}$ per cent per annum, Simple Interest; how many years has the bill to run?

312. Shew that at 5 per cent. the interest on £650 for three months is equal to the true discount on £495. 12*s.* 6*d.* due in 4 months' time.

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313. A farmer bought 4 horses and 7 cows for R2380, the prices of a horse and of a cow being as 5 : 2; how much did he pay for each?

314. Find by Practice the value of 1297 trucks of coal at £4. 17*s.* $11\frac{1}{2}$*d.* per truck.

315. On heating a piece of metal its volume is increased 2.4 per cent. by what percentage of the new volume does the metal decrease on cooling again to its original temperature?

316. A rectangular courtyard is 100 feet long and 60 feet broad. Two paths cross it at right angles, one from end to end and the other from side to side. Each of these is 5 feet wide. Find the cost of laying down the remaining area with turf at 6*d.* per square yard.

317. Two rooms contain equal quantities of air. The area of the floor of one of them is 340 sq. ft and the height 12 ft. Find the area of the floor of the second, whose height is 17 ft.

318. If the manufacturer makes a profit of 20 per cent., the wholesale dealer a profit of 25 per cent., and the shopkeeper a profit of 50 per cent.; what was the cost of the manufacture of an article bought at a shop for R9. 12*a*.

319. At what rate of interest will a sum of money treble itself at Simple Interest in 25 years?

320. A man has £7000 which he invests partly in a 3 per cent. stock at 96 and partly in a 6 per cent. stock at 108, and makes $3\frac{1}{2}$ per cent. on the whole investment. What sums did he invest in each stock?

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321. The average temperature for Monday, Tuesday, and Wednesday was 56°; that for Tuesday, Wednesday and Thursday was 58°. The temperature of Monday is 52°. What was it on Thursday?

322. A 7-feet strip along two adjacent sides of a rectangular garden forms a border for flowers, if the dimensions of the garden be 42 ft. × 34 ft. what is the exact proportion of the strip to the whole area?

323 With a sugar worth 5*a* a seer a dealer mixes an inferior quality worth 3*a*. a seer. In what proportion must he mix them so that by selling the mixture at the higher price, he may get 16 per cent.?

324. *A* walks to a place at $4\frac{1}{2}$ mi. an hour. At 10 mi. from his destination he meets *B* and turns back with him (walking at *B*'s rate) for a mile: if *A* is half an hour late at his destination, what is *B*'s rate; and at what rate should *A* have walked after parting with *B* so as to have arrived at the proper time?

325. A watch, set on Friday at 9 P.M., gains 45 seconds in 12 hours. What time does it show on Monday next at 3 P.M.?

326. A man sells a horse at a loss for 40 guineas; had he sold it for 50 guineas, his gain would have been $\frac{3}{4}$ of his former loss; find the cost price.

327. What sum at Compound Interest will amount to R1200 at the end of the first year and to R1250 at the end of the second?

328. A person invested equal sums of money in the 3 per cents. at $97\frac{1}{2}$ and in the $3\frac{1}{2}$ per cents. at $102\frac{1}{2}$: his resulting income was £259. 10*s*. How much did he invest?

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329. *A* works for 6 days at the rate of 8 hrs. per day; *B* works for 5 hrs on the first day, and on each of the subsequent days 1 hr. longer than on the preceding: *A* does as much in 4 hrs. as *B* does in 5 hrs. If the total sum paid to *A* and *B* be £2. 2*s*., how much should each receive?

330. If 10 sheeps or 15 lambs can eat 40 baskets of turnips in 7 days, how long will it take 6 sheep and 18 lambs to eat 36 baskets?

331. If it cost £49. 14*s*. 6*d*. to decorate a wall-space measuring 69 ft. 4 in. by 6 ft. 9 in., what will it cost for one measuring $22\frac{1}{8}$ yds. by $3\frac{1}{6}$ yds., the style of decoration used in the second case being half as expensive again as in the first case?

332. By selling a horse for R850 a man lost $6\frac{1}{2}$ per cent.; find what would have been his gain or loss per cent., if it had been sold for R965.

333 If 3 kilometres are as much under 2 miles as 5 kilometres are over 3 miles, what is the length of a kilometre?

334. If 7 men and 2 women earn £51 in 8 wks., and 4 men and 2 women earn £46. 10*s*. in 12 wks., what are the weekly wages of a man and a woman?

335. A sum of £1640 is borrowed, to be paid back in two years by two equal annual payments, allowing 5 per cent. Compound Interest. Find the annual payment.

336. A person has a certain amount of 4 per cent. stock. He sells it at $117\frac{1}{4}$, invests half the proceeds in 2 per cent. stock as 96, and the rest in 3 per cent. stock at 99. He then finds that his annual income is reduced by £20; what amount of original stock had he?

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337. A person receives as a legacy $\frac{3}{11}$ of an estate of which he transfers $\frac{1}{3}$ to his son, who pays £5 10*s*. for income-tax at 6*d*. in the £; find the value of the estate at 20 year's purchase.

338. Of two watches one loses 5 sec. and the other gains 4 sec. in 12 hours; they are both right at noon on a certain day; when will one be 5 min. behind the other, and what time will each then shew?

339. If 5 men or 7 women can do a piece of work in 37 days, in what time will 7 men and 5 women do a piece of work twice as great?

340. A plot of land is valued at £1936 per acre: what is it worth in francs per square metre? (£1 = $25\frac{1}{7}$ fr., a metre = $39\frac{3}{8}$ in.)

341. A dealer buys a machine for £525 cash, and sells it after 9 mo. for £423. 10*s*. How much does he lose, considering money worth 6 per cent. per annum?

342. If 9 oz. of gold (10 carats fine) and 5 oz. of 11 carats fine be mixed with 6 oz. of unknown fineness, and the fineness of the resulting mixture be 12 carats, what was the unknown fineness?

343. A house depreciates in value each year at the rate of 10 per cent of the value at the beginning of the year, and its value at the end of 3 years is R109350: find its original value.

344. Investing a certain sum of money in a Railway paying £6 per £100 share, at a time when each share costs £125, I find my income £10 14*s*. 6*d*. better than when I invested it in 3 per cent. Consols at 95. Determine this sum.

345. By selling 15 articles at 6*s*. 8*d*. each, a shopkeeper loses $\frac{3}{7}$ of the cost price; to what must he raise the price per article that by selling 12 more he may just recover the loss?

346. The cost of painting the four walls of a room 32 ft. long and 18 ft. broad is R150: had the height been less by 3 ft., the cost of painting would have been R121. 14*a*. Find the height of the room.

347. In a bag there are 320 coins, guineas and half-guineas. If they amount to £254. 2s. find the number of each

348. Eleven boys fire 12 shots each at a target and score 350 : 22 bull's-eyes are made and 10 misses ; how many centres and outers are made, supposing a bull's-eye scores 4, a centre 3, and an outer 2 ?

349. In 1890 the sum of the ages of *A*, *B* and *C*, was 120 years : in 1878 their ages were as the numbers, 6, 7, 8. Find their respective ages.

350. A man has a certain number of mangoes to sell · to the first customer he sells one-third the number and 3 more, to the second one-third the remainder and 3 more, to the third one-third the remainder and 5 more, to the fourth one-third the remainder and 2 more, and to the fifth one-third the remainder and 2 more, by which time he has only 6 left. How many has he ?

351. A shopkeeper mixes rice at R4. 4*a*. per md. with rice at R3. 12*a*. per md. ; if by selling the mixed rice at R4 3*a*. per md. he gains 1*a*. per each maund sold, how did he mix them ?

352. A person has R10000 in the $3\frac{1}{2}$ per cents. ; how much must he have also in the 3 per cents that his whole income may be R2000, and what sum would he realize by selling out at $83\frac{3}{4}$ and $77\frac{1}{8}$ respectively ?

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353 A farmer sells to one person 10 horses and 11 cows for £465, and to another 7 horses and 17 cows for the same sum. Find the price of each.

354 If it takes 5904 tiles, each measuring 6 in. by 10 in to cover the floor of a room 120 ft long · what is the width of the room and what will be the cost of laying down the tiles at 2*a*. 6*p*. the sq. yard ?

355 If 120 men, half the number working 3 hrs. and the remaining halt 5 hrs. a day can in 3 days raise an embankment 30 yds. long 2 ft. broad, and 4 ft. high ; how many men will be required for an embankment 50 yds. long, 6 ft. high ; and $1\frac{1}{2}$ yds. broad, in 9 days of 5 hrs. each ?

356. Two cogged wheels, of which one has 16 cogs and the other 27 cogs, work into each other ; if the latter turn 80 times in three quarters of a minute, how often does the other turn in 8 seconds ?

357. A labourer was engaged for 120 days, on the condition that for every day he worked he should receive 7*a*. 6*p*., and for every day he played he should forfeit 3*a*. 6*p* ; at the end of the time he received R21. 14*a*. ; how many days did he play ?

358. The price of rice being raised 50 per cent., by how much per cent must a house-holder reduce his consumption of that article so as not to increase his expenditure ?

359. The discount and interest on a certain sum for the same time are £22 and £24, respectively ; find the sum.

360. A share-holder in a commercial company receives one year a dividend of 5 per cent. on his shares. The next year he receives a dividend of $7\frac{1}{2}$ per cent., and finds that he is R412. 8*a*. richer. Find the amount of his shares.

361. There are three pendulums. The first makes 35 beats in 36 sec., the second 36 in 37 sec, and the third 37 in 38 sec.: supposing they commence together, how many times will they beat together in 24 hrs.?

362. If 4 men and 3 boys do a piece of work in 75 days, how many men assisted by an equal number of boys can do the same in 9 days? 2 men can do as much work as 3 boys.

363. A train, having to perform a journey of 250 mi., is obliged after 103 mi. to reduce its speed by a fifth. The result is that the train reaches its destination 70 min. behind time. What is the ordinary rate?

364 A person increased his capital annually by one-third and at the end of 4 years, the year's interest at $4\frac{1}{2}$ per cent. amounted to R4050. What capital did he start with?

365. A *mudi* buys 230 maunds of ghee, and sells 207 mds. for the same amount that he gave for the whole. The rest he sells at a profit of 12 per cent. If the cost of each maund is R30, find the gain per cent.

366. I buy two horses for R900, and sell one so as to lose 4 per cent. and the other so as to gain 5 per cent.; on the whole I neither gain nor lose: what did each horse cost?

367. *A* sells *B* a house; *B* gives *A* a bill at 3 months for £5050. At the expiration of one month *A* gets the bill discounted at 6 per cent.; what ready money does he receive?

368. The sum of £13200 is invested in the 3 per cent. consols when the price of stock is 88 per cent.; what will be the income produced, and how will the income be affected if the price of stock rise to 92 and the stock be then sold out and the money put out to interest at 4 per cent.?

369. Water flowing at the rate of 6 ft. per second through a pipe, the area of whose section is 1 sq. foot, will fill a certain tank in 2 hrs. The tank is empty at the beginning of the monsoon, and receives all the rain that falls on an area of an acre. Find the depth of the rain (in inches) which must fall in order to fill the tank.

370. A boat propelled by 8 oars which take 30 strokes per minute, travels at the rate of $9\frac{1}{7}$ miles per hour; find the rate of a boat propelled by 6 oars which take 28 strokes per min., the work done by each oar during one stroke in the latter case being a quarter as much again as in the former case.

371. A man buys 111 gallons of spirit for R1080; he loses 3 gallons owing to leakage and has to sell 18 gallons at R2 under the selling price per gallon; at how much per gallon must he sell the remainder, in order that he may gain R45 by the bargain?

372. *A* is twice and *B* is just one and a half times as good a workman as *C*. The three work together for two days, and then *A* works alone for half a day, and *B* for one day. How long would it have taken *A* and *C* together to complete the work which the three will have thus performed?

373. A wine merchant buys 552 gallons of wine for R1669. 8*a*.; what quantity of water should he mix with it, if he wishes to sell it at R2. 10*a*. a gallon at a profit of R36. 12*a*.?

374. Three persons *A*, *B*, *C* traded together. Their capitals were in the proportions of 3, 4, 5. Out of the profits *A* spent R113. 12*a*., *B* R180. 13*a*. 4*p*. and *C* R106. 3*a*. How should they square their accounts, the whole profits having been spent?

375. A man invests £9256. 10*s*. partly in 5 per cent. stock at 150, and partly in $2\frac{3}{4}$ per cent. stock at 99. What sum must he invest in each stock in order that his income from each may be the same?

376. By selling out 4 per cent. stock at 96 and investing the proceeds in a railway stock which pays dividends of 7 per cent. per annum, a man finds that he can increase his income by one-half. What is the price of the railway stock?

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377. A certain number of articles was bought for £25 7*s*.; the total number of articles was the same as the number of farthings each cost. How many articles were bought and what was the price of each?

378 Divide R5435 among *A*, *B* and *C* so that *A* may receive R100 more than a third of what *B* receives, and *C* may have what *A* and *B* jointly have.

379. In a mile race *A* wins, *B* being 11 yds. behind *A* and *C* 53 yds. behind *B*; how much start should *B* allow *C* in a race of a mile and a half that they might run a dead heat?

380. A number of rupees is divided amongst four men. *A* gets $\frac{2}{3}$rds of the whole, *B* $\frac{3}{4}$ths of the remainder, *C* $\frac{5}{8}$ths of what then remains, and the number of rupees given to *D* is the square root of the whole number to be distributed. What sum does each receive?

381. A down train usually travels at the rate of 30 mi. an hour and meets an up train 50 mi. from the terminus. On one occasion, owing to an accident, it went at the rate of only 20 mi. an hour and met the up train $41\frac{2}{3}$ mi. from the terminus. Find the speed of the up train.

382. If the duty on a certain commodity were reduced by 20 per cent. of the present amount; by how much per cent. must the consumption be increased that the same revenue may be derived from it?

383. A tradesman marks his goods with two prices; the one for ready money, the other for 6 months' credit, the rate of interest being 5 per cent. per annum. If the credit price be R10. 4*a*., what is the cash price?

384. The 4 per cents. are at $98\frac{1}{3}$, and the $4\frac{1}{2}$ per cents. at $103\frac{1}{2}$. A person has a sum of money to invest which will give him one thousand rupees more of the former stock than of the latter. Find the difference of income he could obtain by investing in the two stocks.

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385. Divide R377. 8*a*. among *A*, *B* and *C*, giving *B* half as much again as *A*, less R5, and *C* as much as *A* and *B* together.

386. A gentleman paid R69 for 3 articles consisting of a chair, a table and a table-cloth; the table cost 3 times as much as the chair and 5 times as much as the table-cloth. How much did he pay for each?

387. The coxwain of an eight-oared boat weighs 9 stone. The average weight of the crew exclusive of the coxwain is greater than their average weight inclusive of him by $\frac{1}{4}$ of a stone. what is their whole weight?

388. A pudding consists of 2 parts of flour, 3 parts of raisins, and 4 parts of suet; 8 lbs. of flour is of the same value as 3 lbs. of suet or 4 lbs. of raisins, and costs 2*s.*; find the cost of the several ingredients of the pudding when the total cost is 2*s.* 7*d.*

389. Ash saplings after 5 years' growth are worth 1*s.* 6*d.*, and increase in value 1*s.* 6*d.* each year afterwards. Each is allowed 40 sq. yards of ground; if they are cut after 20 years' growth, what will then be the value of an acre?

390. A trader asked a price for a watch which was 40 per cent. above cost, and gave the purchaser 10 per cent. discount on the price asked, gaining thereby R5. 6*a.* 8*p.*; find its cost price.

391. The difference between the Simple and Compound Interest of a certain sum for 2 yrs. at 4 per cent. per annum is £3. 4*s.*; find the sum.

392. A man's income from the 5 per cents. after paying income-tax at 6*d.* in the £ is £585; he sells out at 115, and invests in the 4 per cents. at par. Find the change in his net income (allowing for income-tax).

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393. The wages of *A* and *B* for 45 days amount to the same sum as the wages of *A* alone for $77\frac{1}{7}$ days. For how many days will this sum pay the wages of *B* alone?

394. Two boats row a race over a straight course 1 mi 995 yds. long, their rates of speed being 12 mi. and $11\frac{3}{20}$ mi. an hour respectively. Assuming that sound travels at the rate of 1140 ft. in a second, find how much the faster boat will be ahead of the other when the sound of the gun fired at starting is heard at the winning post.

395. A farmer buys a flock of sheep at the rate of £7 for every 5 sheep; he afterwards loses 9, and sells the remainder at the rate of £16 for every 11, thus getting £24 more than the cost: how many sheep were there?

396. The expense of carpeting a room 18 ft. long was £7. 4*s.*; but if the breadth of the room had been 4 ft. less than it was, the expense would have been £5. 8*s.*; what was the breadth of the room?

397. A merchant has teas worth 4*s.* 6*d.* and 3*s.* 6*d.* per lb respectively, which he mixes in the proportion of 2 lbs. of the former to 1 lb. of the latter and sells the mixture at 4*s.* 4*d.* per lb.; what does he gain or lose per cent.?

398. A wine which costs £100 the cask is mixed with a wine which costs £70 the cask. In what proportion must they be mixed so that if the mixture be sold at £92 the cask, a profit may be made upon each cask sold of 15 per cent.?

399 One Company guarantees to pay 5 per cent. on shares of £100 each; another guarantees at the rate of $4\frac{5}{8}$ per cent. on shares of £7. 10s. each; the price of the former is £$124\frac{1}{2}$ and of the latter £8. 10s.; compare the rates of interest which they return to the purchaser.

400. What sum must a person invest in the 3 per cents. at 90 in order that by selling out £1000 stock, when they have risen to $93\frac{1}{2}$, and the reamainder when they have fallen to $84\frac{1}{4}$, he may gain £6. 5s by the transaction? If he invests the produce in the 4 per cents. at par, what will be the difference in his income?

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401. If a gallon contain 277·274 cub. in., and a cub. ft. of water weighs 1000 oz.; what quantity and weight of water will fill a rectangular cistern 5 ft. long, $3\frac{1}{2}$ ft. wide and $2\frac{3}{4}$ ft. deep?

402. Two boats start to row in a race at 3 o' clock. The winning boat comes in at $6\frac{3}{4}$ min. past 3, 40 yds ahead of the other. At 4 min. past 3, the losing boat was 1140 yds. from the winning-post Find the length of the course, and the speed of the winning boat in miles per hour.

403. An article when sold at a profit of 5 per cent. yields 2s. 1d. more than when sold at loss of 5 per cent.; find the prime cost.

404. The price of gold is £3 17s. 10d. per oz.; the value of a mixture of silver and gold weighing 216 oz. is £177. 12s.; but if the weights of the silver and gold were interchanged, the value would be £708; find the weight of the silver in the mixture and the price per ounce.

405. The gross receipts of a railway company in a certain year are apportioned as follows.—43 per cent. to pay the working expenses, 52 per cent. to give the shareholders a dividend at the rate of $3\frac{1}{4}$ per cent. on their shares, and the remainder £25000 is reserved find the paid up capital of the company.

406. *A* and *B* at the opposite extremities of the diameter of a circular area 135 miles in circumference, start to go round it at the same time in the same direction, *A* at the rate of 11 miles in 2 hours and *B* at the rate of 17 miles in 3 hours. How many rounds will each take before the one will overtake the other, and how long will the chase continue?

407. A person has stock in the 3 per cents. which produces him an income of R2400 a year. He sells out one-fourth of the stock at $87\frac{1}{4}$, and invests the proceeds in railway stock, when a 1000-rupee share is worth R1745. What dividend per cent. per annum ought the railway to pay on its stock, so that he may increase his income R400 a year by the transaction?

408. *M* and *N* set out from the same place in the same direction. *M* travels uniformly 18 miles per day, and after 9 days turns and goes back as far as *N* has travelled during those 9 days: he then turns again, and pursuing his journey overtakes *N* at the end of $22\frac{1}{2}$ days after the time they first set out. Find the rate of *N* in miles per hour.

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409. A man, walking a distance of 32 mi., finds at the end of 3 hr. 30 min. that the distance which he has walked is seven-ninths of the remaining distance. Find the rate of walking per hour.

410. Ten years ago, A had £1300 and B £910. A has been more fortunate than B, and made £3 where B gained only £1. B has now doubled his capital; how much has A got?

411. A man having a certain sum of money in his purse, gave R5 more than one-sixth of it for a piece of shawl, one-third of the remainder less R2, for half a dozen pictures, and R12 more than one-fourth of what then remained to his wife for family expense; finally he had R27 left. What sum had he in his purse?

412. A viaduct consists of 3 series of arches built upon each other, the breadth of the arches in each being respectively 8 yds. 2 ft., 6 yds. and 5 yds., whenever the piers in all the series are vertically above each other: there occurs a mass of masonry 4 yds. wide; of such there are 3; find the length of the viaduct.

413 The interval between the firing of two guns, at a railway station, was 6 min: and a passenger in a train approaching the station at a uniform rate, heard the second report 5 min. 51 sec. after hearing the first. Supposing the sound of the train's approach to have become audible at the station when the train was 2 mi. off, how soon after that did the train pass the station, sound travelling 1125 ft. per second?

414. There is a leak in the bottom of a cistern. When the cistern is in thorough repair, it would be filled in $2\frac{1}{2}$ hours. It now takes 30 min. longer. If the cistern is full, how long would it be in leaking itself empty?

415 A man invests in the 3 per cent. railway stock from which he derives an income of R8640, sells out at 90, and invests in shares that pay 5 per cent interest; if his income be now increased by R3360, at what price does he buy the shares?

416. The tip of a reed was 8 inches above the surface of a lake; but, forced by the wind, it gradually advanced and was submerged at a distance of 28 in. Find the depth of the water.

417. Divide 99 into four parts, so that the first shall contain 3 for every 4 in the third and every 5 in the fourth, and $\frac{1}{3}$ of the second may be $\frac{1}{4}$ of the sum of all the rest.

418. Six coins of equal weight, made of gold and silver mixed, were melted together and recast. In one the gold and silver were in the ratio of 2 : 1, in two others, of 3 : 5; and in the rest of 7 : 5. In what ratio will the gold and silver be mixed in the new coins?

419 A cistern has two supplying pipes A and B, and a tap C. When the cistern is empty, A and B are turned on, and it is filled in 4 hours; then B is shut and C is turned on, and the cistern is quite emptied in 40 hours; when, lastly, A is shut and B turned on, and in 60 hours afterwards the cistern is again filled. In what time could the cistern be filled by each of the pipes A and B singly?

420. A person rows a distance of $1\frac{1}{2}$ miles down a stream in 20 minutes; but without the aid of the steam, it would have taken him half an hour; what is the rate of the steam per hour? And how long would it take him to return against it?

421. What is the cost of paper for the walls of a room 30 ft. long, 15 ft. broad, and 15 ft. high, the paper being $1\frac{1}{2}$ yds. wide, and its price $4\frac{1}{2}d.$ per yd.? What would be the cost for a room twice as long, twice as broad, and twice as high, the paper being twice as wide and costing twice as much per yard as before?

422. Incomes below £150 a year being subject to $5d.$ in the pound income-tax, and income above £150 to $7d.$ in the pound; find what income above £150 a man must have that he may be just $7\frac{3}{4}d.$ a year poorer than a man who has £149. 10*s.* a year?

423. *A* and *B* engage in trade, their capitals being as 7 : 11; at the end of 3 mo. *A* withdraws one-third of his capital, and 1 mo afterwards *B* adds twice as much as *A* has withdrawn. How should a profit of R3373. 12*a.* be divided at the end of the year?

424 A man wants to invest R11505, partly in the 4 per cents. at 106 and partly in the $3\frac{1}{2}$ per cents. at 99: how should he divide his money between the two stocks so that he may derive the same income from both?

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425. *A* has £90,000 stock in the 3 per cent. South Sea Annuities and is offered by Government the choice of being paid off at par at the end of the year, or of receiving £110 of a new $2\frac{1}{2}$ per cent. stock for each £100. He chooses the former alternative; and on being paid off, is able to invest his money in the 3 per cent. consols at 92. Find the amount of his stock in consols, and the excess of his income above what it would have been if he had agreed to the proposed conversion.

426. A certain sum of money is laid out in the $3\frac{1}{2}$ per cents. at 96, and $\frac{5}{8}$ths of the same sum is invested in the 3 per cents. at $93\frac{1}{8}$; the interest received at the end of the half year after paying income-tax at the rate of $4d.$ in the £ amounts to £304 16*s.* 8*d.* Find the sum. (Brokerage $= \frac{1}{8}$ p. c.)

427. A *naib* is engaged to collect the rents of a Taluk on the condition that he should receive 6*p.* in the rupee on the net annual receipts. The rates amount to 3*a.* in the rupee on the total annual collections, and an income-tax is paid at the rate of 5*p.* in the rupee on the remainder. The zamindar receives R1177. 8*a.* 3*p.* Find the total annual cost of the Taluk.

428. A pound Troy of standard silver (37 parts of pure silver and 3 parts of copper) is coined into 66 shillings. A rupee weighs 180 grains, and is $\frac{11}{12}$ fine. Find the relation between the shilling and the rupee as determined by the intrinsic value of the two coins

429. When I left home after 10 o'clock P.M. in a hurry I could not see the exact time. On my return between 6 and 7 o'clock P.M. I saw the hands of the clock changed places. What was the time of my departure?

430. Two trains were running on parallel lines in the same direction. One was 240 yds. long and running at a speed of 45 miles per hour. It overtakes the second train 200 yds. long and leaves it in a minute. Find the speed of the second train.

CHAP. XLI. THE METRIC SYSTEM.

356. At the end of the eighteenth century, soon after the French revolution, the **Metric System** of weights and measure was first introduced in France. Now it is in use in almost all the countries in Europe.

357. The **Metric System** is a decimal system, so that each sub-unit is 10 times the next small sub-unit. The Tables are formed by taking as *Auxiliary* units, 10 times, 100 times and 1000 times the *Principal* unit for multiples and tenths, hundreds and thousands of this unit for sub-multiples.

358. The multiples are all denoted *in order* by the Greek prefixes deka, hecto, kilo, myria, and the sub-multiples by the Latin prefixes deci, centi, milli.

GREEK PREFIXES.			LATIN PREFIXES.		
Deka	means	10 times	**Deci**	means	10th part of
Hecto	,,	100 ,,	**Centi**	,,	100th ,, ,,
Kilo	,,	1000 ,,	**Milli**	,,	1000th ,, ,,
Myria	,,	10000 ,,			

359. The unit of **Length** is the **Metre.** It is defined to be the length of the straight line joining two marks made on a certain platinum bar preserved at Paris. All other units of this system depend upon the **Metre.** It is called Metric System from the name Metre (Greek *metron*, measure).

1 Metre = 39·37079...inches = 39⅜ in. approximately.

Note. The **Metre** was originally taken to be the *ten-millionth* part of the distance between a pole and the equator, measured on the surface of the ocean and along a circle of longitude. An error has been found in the measurement of the distance between pole and equator, but the length of the metre has not been altered. Therefore the **Metre** is not a natural unit as was supposed.

10 Millimetre (m.m.)	= 1 Centimetre.	10 Dekametres	= 1 Hecto-metre.
10 Centimetres (c.m.)	= 1 Decimetre.	10 Hectometre	= 1 Kilometre.
10 Decimetre (d.m.)	= 1 Metre.	10 Kilometre	= 1 Myriametre.
10 **Metres**	= 1 Dekametre.	1 Kilometre	= ⅝ mile (nearly).

360. The unit of **Surface** is the **Are.** It is equal to a square, each of whose sides is 10 metres.

Therefore the Are = 100 square metres.
= 1 square dekametre.
= 119·6046 sq. yds. nearly.

10 Centiares	= 1 Deciare.	10 **Ares**	= 1 Dekare.
10 Deciares	= 1 **Are.**	10 Dekares	= 1 Hectare.

The area of farms, etc., is in France usually expressed in hectares. One hectare = 2·47.. acres.

361. The unit of **volume** is the **cubic metre**. In measuring wood it is called a stere.

1 cubic metre = 1·308...cub. yds.

362. The unit of **Capacity** is the **Litre.** The **litre** is equal in volume to a cube, each of whose sides is equal to 10 centimetres.

Then 1 **litre** = 1000 cub. cms. = 1·76...Pints.

10 Centilitres = 1 Decilitre.		10 Dekalitres = 1 Hecto-litre.	
10 Decilitres = 1 **Litre**.		10 Hectolitres = 1 Kilolitre.	
10 **Litres** = 1 Dekalitre.			

363. The unit of **Weight** is the **Grame.** A **gramme** is the weight *in vacuo* of a cubic centimetre of pure water at its greatest density *i. e.* when the temperature is 4°C.

A gramme = 15·4323487...grains.

10 Milligrams = 1 Centigram	10 Dekagrams = 1 Hectogram
10 Centigrams = 1 Decigram	10 Hetograms = 1 Kilogram
10 Decigrams = 1 **Gram**	10 Kilograms = 1 Myriagram.
10 **Grams** = 1 Dekagram	

The kilogramme (1000 grammes) is defined, in practice, as the weight of a standard platinum cylinder, which is kept in the Archives at Paris. 1 litre of pure water at 4°C weighs 1 kilogramme.

1 kilogramme = 2·20462 lbs. Avoir.

364. The unit of **money** is the **Franc.** It is about $9\frac{1}{2}d$.

25 **Francs** = £1. Its submultiples are the *decime* (never used) and the centime (cent.); so that

10 Centime (*c*) = 1 Decime, 10 Decimes = 100 Centimes = 1 Franc (*fr.*)

Note. The **Franc** is a Silver Coin composed of 9 parts of silver and 1 part copper and weighs 5 grams. The Napoleon is a gold coin = 20 francs. A sou (a copper coin) = 5 Centimes.

Franc is so called from the device *Francorum rex* king of the Franks or French, on the coin as first struck by king John in 1360.

Dollar is the unit of money in the United States and Canada.

1 Dollar = 100 cents.
= 4*s*. $1\frac{1}{4}d$. English.

Ex. 1. Express ·0324 kilograms in grams.

·0324 kilograms = ·324 hectograms
= 3·24 dekagrams
= 32·4 grams.

Ex. 2. Express 4565 millimetres in metres.

4565 millimetres = 456·5 centimetres
= 45·65 decimetres
= 4·565 metres.

Ex. 3. Express 233 yds. 2 ft. $9\frac{1}{2}$ in. in the Metric System of measurement, [1 metre=39·371 inches].

233 yds. 2 ft. $9\frac{1}{2}$ in. $=8421\cdot5$ inches $=\frac{8421\cdot5}{39\cdot371}$ metres $=213\cdot901$ metres

$=2$ hectometres 1 dekametre 3 metres 9 decimetre 1 millimetre.

Ex. 4. Express 3 cwt. 2 qrs 12 lbs. in kilograms, grammes &c.—

[1 kilogramme=2·2046...lbs. Avoir].

3 cwt. 2 qrs. 12 lbs. $=404$ lbs.

$=\frac{404}{2\cdot2046}$ kilograms $=183\cdot2804$ kilograms $=183$ kilograms 280 grams 4 decigrams.

Ex. 5. Express 3821 kilograms 120 grams in tons, cwt. &c.—

3821 kilograms 120 grams $=3821\cdot120$ kilograms

$=3821\cdot120\times2\cdot2046$ lbs.

$=8424\cdot04$ lbs. $=3$ tons 15 cwt. 24 lbs.

Examples 157.

1. Express in terms of *Metres* :—

(i) 42 kilometres, 7 hectometres and 9 dekametres.

(ii) 14 Decimetres 3 centimetres and 4 millimetres.

2. Express in terms of a kilogram :—

(i) 428 grams, 72 centigrams.

(ii) 3 dekagrams 7 grams, 4 decigrams and 5 centigrams.

3. Express in terms of *Are* :—

(i) 45 sq. decimetre 25 sq. metre and 525 sq. millimetres.

(ii) The area of a square each of whose sides is 2 dekametre 7 metre and 42 centimetres.

4. The sides of a cube are each equal to 17 metres, 45 centimetres, find its volume in litres &c.

5. Each stick is 13·497 decimetres in length ; how many such sticks will be required to cover a distance of 44·99 dekametres.

6. A train is moving at the rate of 72·0918 kilometres an hour. How long it would take to travel 8010200 metres ?

7. A man bought 250 metres of wire at 25 centimes per dekametre and 82256 metres at 2 franc per kilometre. Find his total cost.

8. Find the cost of paving a road 82 kilometres long at 2 fr. 45 centimes per dekametre.

9. Find the cost of papering the walls of a room 42 metres long 28 metres broad and 10 metres high at 4 fr. per centiare.

10. The circumference of a wheel is 3 hectometre 2 dekametres and 5 metres. How many revolutions it makes in passing through a distance of 30 kilometres ?

11. The radius of the wheel of a carriage is 5 decimetres. If the carriage is moving at the rate of 5 kilometres per hour. How many revolutions the wheels make per minute ?

12. If 20 men earn 117 francs working $3\frac{1}{2}$ hours per day, what is the earning of 62 men working 5 hours daily ?

13. A merchant bought 15 kilograms of sugar at 375 francs. What would be the selling price of 1 gramme of sugar so that he might gain 10 per cent on his outlay ?

14. In what time will 5250 francs 50 centimes amount to 5773 francs 55 centimes at 4 per cent per annum ?

15. If in $2\frac{1}{2}$ years 525 francs 63 centimes amount to 700 francs, what is the rate of interest ?

16. A person invests 3490 francs in the 5 per cents. at 87·25. How much stock does he hold and what is his annual income ?

17. Find the number of *metres* in a *mile* approximately.

18. Express in English system of measurements :—

(i) 5 kilometres 3 hectometres 2 dekametres 4 metres.

(ii) 12 kilograms 324 grammes.

(iii) 525 francs 62·5 centimes.

(iv) 29 dollars 52 cents.

(v) 156 dekalitres 9 litres 42 centilitres.

19. Express an acre in square metres.

20. Express 1 ton in kilograms, grams, &c.

21. Find the number of cubic inches in 50 litres.

22. Find the value of 1 million francs in English money.

23. Atmospheric pressure is $14\frac{3}{4}$ lbs. per sq. inch. Express this in grams per square centimetre.

24. A cubic foot of water at 4°c weighs 62·5 lbs. Express the weight of 10 litres of such water in tons.

25. The second's pendulum is 3·2616 ft. long. Find its length in metres.

26. In France price of tea is 18 francs per kilogram. Find to the nearest penny the price of the tea per lb in English money.

27. A barometer indicates 745 m.m. What is the reading of a barometer in the same place at the same time which is graduated in inches ?

28. A liquid is worth 30 francs per litre. What is the price of it in English money per pint. A litre being equal to ·52 gallon.

29. In England the third class railway fare is a penny per mile. What charge in centimes per kilometre is equivalent to this ?

30. Express 1 lb in grams and 1 gram as the decimal of a lb.

31. Express 1 ft. in centimetres.

32. Acceleration due to gravity is 32·2 ft. per sec. units. Express this in centimetre per sec. units.

33. Reduce 15 gal. 3 qts. 1 pt. to litres. 1 gal. = 277·27 cu. inches and 1 metre = 39·37 inches.

34. Express a rupee in terms of Francs, Centimes. [1 shilling = 12 as.]

35. Express a dollar in terms of rupees annas and pies.

36. An Indian merchant bought 1800 clocks from France at 22 francs 50 centimes per piece and 1000 clocks from America at 5 dollars per piece. How many rupees did he pay for the clocks?

37. A cubical vessel contains 1728 kilograms of water. What is the length of an inner edge?

38. A bag contained equal number of Napoleons, Francs and Centimes. The total amount of money in the bag was 1053 Napoleons and 10 francs. Find the number of each coin in the bag.

39. 850 Napoleons are to be divided amongst 2400 boys and girls. Each boy is to receive 10 francs 33 centimes and each girl 5 francs 46 centimes. Find the number of boys.

40. Each edge of a cubical box is 2 ft. 3 in. Find its volume in litres. How much of pure water at 4°C does this box contain?

41. Find the cost of fencing a square field of 40 acres at 2 francs 50 centimes per metre.

42. Average daily expense of an Englishman is 4*s*. 5*d*., of a Frenchman 3 francs 25 centimes, of an Indian 4 as. If the incomes be proportional to the average daily expenses, compare the income of an Englishman and a Frenchman with that of an Indian.

43. A litre of water weighs a kilogram, a litre of another liquid weighs 1·340 kilograms. A mixture of the two weighs 1·270 kilograms per litre. Determine the volume of each in a litre of the mixture. (c. s.)

44. A mixture is made of two liquids, one of which weighs ·68 gram per cubic centimetre and the other 1·04 gram per cubic centimetre, and the mixture weighs ·82 gram per cubic centimetre. Find the percentage (by volume) of each liquid in the mixture. (c. s.)

45. At the point where a stream passes a bridge it is 18 metres wide and has a uniform depth of 3·6 metres. If the stream flows uniformly at the rate of 1·75 kilometres per hours, how many litres of water pass through the bridge in one minute? (c. s.

46. A wine-merchant buys in Bordeaux a cask of claret, and the freight to London is £1. It contains 208 litres of wine at 75 centimes the litre, and for the cask itself he has to pay 12 francs. He has to pay duty 1*s*. 3*d*. per gallon, and bottles the wine at a cost of 2*s*. a dozen bottle. If he sells it at 18*s*. a dozen, what percentage of this price is profit?

(£1 = 25 francs ; 6 bottles = 1 gallon) (c. s.).

47. Cycling from A to B I ride at 15·75 kilometres an hour except up one hill ·75 kilometre long, when I dismount and push the machine at 4·5 kilometres an hour. On returning I ride all the way at 15 kilometres an hour and take the same time over the journey. What is the distance from A to B? (c. s.)

48. One cubic centimetre of concentrated sulphuric acid weighs 1·842 grams. If 70 parts by volume of this acid are mixed with 30 parts of water, it is found that one cubic centimetre of the mixture weighs 1·615 grams. Find how much per cent. the mixture has shrunk. (c. s.)

49. A hollow ball, of external diameter 6 centimetres and of thickness half a centimetre, weighs 162 grams when it is filled with liquid. Find to the nearest hundredth of a gram the weight of a cubic centimetre of the liquid, a cubic centimetre of the substance of the ball weighing 2 grams. [If a sphere is inscribed in a cube, then volume of sphere : volume of cube :: 11 : 21]. (c. s.)

50. The monthly expense of a French family is 87 Napoleons 10 francs when wine is 2 francs a bottle. The monthly expense of the same family is 90 Napoleons when wine is 2 francs 10 centimes per bottle. What would be the monthly expense of the family when wine is 2 francs 25 centimes per bottle. (The other expenses remaining constant).

365. **Proposed Decimal Coinage.** It has been proposed to introduce into England a decimal coinage, in which the unit of money should be £1, with sub-multiples *florins*, *cents*, *and mils*.

10 mils (m) = 1 cent (c.)
10 cents = 1 florin (fl.)
10 florins = 1 pound (£1.)

In this system, in a decimal of £1. the first figure in the decimal place denotes florins, the second cents., and the third mils.

Ex. 1. Reduce £25. 8 florins 6 cents. 5 mils to mils; and 15287 mils to £. fl. &c —

£25. 8 florins 6 cents. 5 mils.
= £25·865 = 25865 mils.

15287 mils = £15·287
= £15. 2fl. 8c. 7m.

Ex. 2. Reduce £3. 6*s.* 10*d.* to £, fl., &c., and £3. 8fl. 5c. 6m. to £. *s.* *d.*

£3. 6*s.* 10*d.*
$= £\left(3+\frac{6}{20}+\frac{10}{240}\right)$
$= £(3+{\cdot}3+{\cdot}04166\ldots)$
$= £3{\cdot}34166\ldots$
= £3. 3fl. 4c. 2m. nearly.

£3. 6fl. 5c. 6m.
= £3·856
20
s. 17·12
12
d. 1·44
∴ the sum = £3. 17*s.* 1½*d.* nearly.

Ex. 3. Find the value of £5. 3 fl. 4c. 2m. − £1. 4 fl. 8c. 7m. + £12. 8 fl. 1c 3m. − £2. 6 fl. 5c. 1m. − £3. 2 fl. 8c 8m. − £4. 8 fl. 3c. 3m.

£5. 3 fl. 4c. 2m. = £5·342
£12. 8fl. 1c. 3m. = £12·813
£18·155
£12·259
£ 5·896

£1. 4fl. 8c 7m. = £1·487
£2. 6fl. 5c. 1m. = £2·651
£3. 2fl. 8c. 8m. = £3·288
£4. 8fl. 3c. 3m. = £4·833
£12·259
= £5. 8fl. 9c. 6m.

Ex. 4. Divide £15. 8fl. 7c. 5m. × 12 by 20.

£15. 8fl. 7c. 5m. = £15·875
12
£190·500

20)190·500
9·525 = £9. 5fl. 2c. 5m.

Ex. 5. Express £5. 6fl. 2c. 5m. as the decimal of £12. 8fl.

£5. 6fl. 2c. 5m. = £5·625
£12. 8fl = £12·8

∴ $£5·625 \div £12·8 = \dfrac{5·625}{12·8} = ·439453125$

∴ The required decimal = ·439453125 *Ans.*

Examples 158.

1. Reduce to *mils* :—

(i) £1. 2fl. 3c. 4m. (ii) £20. 8fl. 7c. 8m.

(iii) £8. 7fl. 2c. 5m. (iv) £10. 1c. 8m.

2. Reduce to £. fl. c. m. .—

(i) 5284 mils. (ii) 525c. (iii) 18·25fl. (iv) 345·5c.

(v) £11·685.

3. Add together £11. 3fl. 2c. 3m.; £9. 7fl. 3c. 4m.; £7. 8fl. 4c. 2m.; £15. 8fl. 6c. 5m. and £17. 7fl. 3c. 4m.

4. Subtract £25. 7fl 8c. 5m. from £68. 3fl. 2c. 4m.

5. Multiply £32. 5fl. 8c. 2¼m. by 120; and £72. 3fl. 6c. 5m. by 1250.

6. Divide £30763. 7fl. 1c. 1½m. by (1) 99; (2) by £1. 5fl. 8c. 4m.

7. Reduce to *decimal money* :—

13*s.* 4*d.*; 9*s.* 3¾*d.*; £11. 9*s.* 1½*d.*

8. Express in £. *s.* *d.*; £1·2302; £10·6523; £12. 8fl. 9c 2½m.; £120. 5fl. 7c. 0¼m.

9. How often is £17. 5fl. 3c. 2m. contained in £438. 3fl.?

10. Reduce £1. 7fl. 3c. 5m. to the decimal of £216. 8fl. 7c. 5m.

11. Find the value of £9. 3fl. 7c. 4m. + £12. 4fl. 6c. 8m. − £11. 2c. 3m. − £11. 2fl. 7m. − £5. 7fl. 6c. 8m. + £4. 6fl. 2m. + £5. 9fl. 8c. 3m.

12. Find the value of $\frac{3}{10}$ of £7. 2fl. 2c. 5m. + ·325 of £185. 5fl. 6c. 5m. − $\frac{1}{16}$ of £10. − $\frac{3}{25}$ of £12. 5fl. 5c. + $\frac{1}{8}$ of £2. 3fl.

13. Find the interest on £425. 6fl. 2c. 5m. for $2\frac{3}{4}$ year at 5 p. c. per annum.

14 Discount on £16. 2fl. 5c. is 6fl. 2c. 5m. for $1\frac{1}{4}$ yrs. Find the rate of interest.

15. A person invested £4250. 7fl. 5c. in 3 per cents. stock at 85·015. Find his annual income and amount of stock he bought.

CALCUTTA ENTRANCE PAPERS.

SELECTED QUESTIONS 1858-90.

1. Multiply R18957. 13*a*. by R568. $11\frac{3}{4}$*a*.; and divide the same sum by the same sum. Shew that one of these operations is absurd and impossible, and perform the other.

2 A plate of metal is beaten to the thickness of $\frac{1}{8}$ of an inch, and the weight of a circular medal cut from it, whose diameter is $1\frac{1}{6}$ inches, is $1\frac{1}{4}$ oz. Troy. If the same plate be beaten to the thickness of $\frac{1}{9}$ of an inch, what will be the weight of a medal cut out of it of the diameter of $1\frac{1}{7}$ inches, (the areas of circles being proportional to the squares of their diameters)?

3. What do you mean by a *prime number*, a *factor*, a *ratio*. Resolve 30 and 132 into their prime factors, and find their ratio in its simplest terms.

4. If a person get a bequest of $\frac{5}{7}$ of an estate of 2000 acres and sell $\frac{2}{5}$ of his share, how many acres does he retain?

5. How much must be paid for £1250 stock when it sells at 108 per cent?

6. A shopkeeper purchased $250\frac{3}{4}$ yds. of cloth for R900 and paid expenses amounting to R103: what must he charge per yard in order to make a profit of 50 per cent.?

7. State the rules for pointing in the multiplication and division of decimals; and multiply ·256 by ·0025, and divide ·0036 by ·4 and 4 by ·00001.

8. If the price of bricks depends upon their magnitude, and if 100 bricks, of which the length, breadth and thickness are 16, 8 and 10 inches respectively, cost R2. 9*a*, what will be the price of 921,600 bricks, which are one-fourth less in every dimension?

9. Explain the method of pointing in extracting the square roots of whole-number and decimals. Find the square root of 57,214,096 and also the square root of ·5 to four places of decimals.

10. A tea-dealer buys a chest of tea containing 2 mds. and 16 srs. at R4. 2*a*. per seer, and two chests more each containing 3 mds. and 24 srs. at R4. 10*a*. per seer: at what rate per seer must he sell the whole in order to gain R576.

11. If the wages of 18 coolies for a month amount to R85 when rice is 24 seers per rupee,—what ought the daily pay of a coolie to be in proportion, when the price of rice is R2. 10*a*. 8*p*. per maund?

12.* *A* and *B* run a race. *A* has a start of 40 yards, and sets off 5 minutes before *B*, at the rate of 10 miles an hour. How soon will *B* overtake him if his rate of running is 12 miles per hour?

13. If a man can perform a journey of 170 miles in $4\frac{1}{4}$ days of 11 hrs. each, in how many days of $8\frac{3}{4}$ hrs will he perform a journey of 470 miles?

14. What sum of money will produce £43 interest in $3\frac{1}{4}$ years, at $2\frac{1}{2}$ per cent. Simple Interest?

15 How many paving stones, each measuring 14 in. by 12 in., are required to pave a verandah 70 ft. long and 9 ft. broad?

16 A company guarantees to pay 5 per cent. on shares of R1000 each, another guarantees to pay $4\frac{3}{8}$ per cent. on shares of R75 each: the price of the former is R1245, and of the latter R85. Compare the rates of interest which the shares return to purchasers.

17. Find the value of $11\frac{3}{5}+14\frac{5}{8}+21\frac{7}{40}+32\frac{3}{625}$, both by Vulgar Fractions and by Decimals, showing that the two results coincide; and reduce $25^{\circ}\ 36'\ 45''$ to the decimal of 75°.

18. How many yards of matting 3·5 ft. wide will cover the floor of a room 85·3 ft. long and 40·5 ft. broad; and how much will it cost, at R2. 10*a*. 8*p*. per *square* yard?

19. If the wages of 25 men amount to R766. 10*a*. 8*p*. in 16 days, how many men must work 24 days to receive R1335, the daily wages of the latter being one-half those of the former?

20. What principal in 3 years 73 days will amount to R100. 15*a*., at $6\frac{1}{4}$ per cent. Simple Interest? A bill for R5035. 4*a*. drawn on September 12th at 5 months, was discounted on January 16th at 4 per cent.; what was the discount charged?

21. Three gardeners working all day can plant a field in 10 days; but one of them having other employment can only work half time. How long will it take them to complete the work?

22. Find the Compound Interest of £55 for one year, payable quarterly at 5 per cent. per annum.

23. Reduce $3^{\circ}\ 45'\ 36''\cdot25$ to the decimal of 36°.

24. What is the expense of paving a rectangular verandah whose length is 42 ft. and breadth 15 ft. with Burdwan paving stones, 18 inches square and which cost R15 per score?

25. The 3 per cents. are at $85\frac{1}{8}$; what price should the $3\frac{1}{2}$ per cents. bear, that an investment may be made with equal advantage in either stock? And what interest should be derived by so investing £5,000?

26. The driving wheel of a locomotive is 226 inches in circumference and makes 91 revolutions per minute: at what rate per hour is the engine travelling?

27. If £450 amount to £523. 10*s*. in 1 year 8 months, calculate the rate per cent.

28. If one man walks 165 miles in 6 days, how far will another man walk in 15 days, if the first man walks $3\frac{3}{4}$ miles in the same time that the other man walks 4 miles?

29. Three equal glasses are filled with a mixture of spirits and water; the proportion of spirits to water in each glass is as follows :—in the first glass as 2 : 3, in the second as 3 : 4, and in the third as 4 : 5. The contents of the three glasses are poured into a single vessel : what is the proportion of spirits to water in it.

30. A room is 37 ft. 2 in. long by 25 ft. 8 in. broad and 22 ft. 6 in. high; find the cost of covering its four walls with paper $1\frac{1}{4}$ yds. wide, at 1*s.* $1\frac{3}{4}$*d.* a yard.

31. In what time will £563. 13*s.* $4\frac{1}{2}$*d.*, amount to £901. 17*s.* $5\frac{4}{5}$*d.*, at $3\frac{3}{4}$ per cent. ?

32. Find the cost of matting a room whose floor is 8 yds. long by $7\frac{1}{4}$ yds. wide, with mats 2 ft. wide and $9\frac{1}{2}$ ft. long, at the rate of 9*a.* 2*p.* per mat. If the same room be $15\frac{1}{2}$ ft. high, find how many cubic feet it will contain

33. Distinguish between a Vulgar Fraction and a Decimal Fraction. Multiply $999\frac{4843}{4946}$ by 999.

34. Two gangs of six men and nine men are set to reap two fields of 35 and 45 acres respectively. The first gang complete their work in 12 days; in how many days will the second gang complete theirs ?

35. Find which is the better investment, $3\frac{1}{2}$ per cent. stock at $98\frac{1}{4}$ or $3\frac{3}{4}$ per cents. at 105.

36. What sum of money will amount to R3,761. 14*a.* in $3\frac{1}{3}$ years at $4\frac{1}{2}$ per cent. per annum. Simple Interest ?

37. A merchant bought goods which cost him R9,810. In the first day he sold to the amount of R992. 8*a.* 6*p.*, in the second to that of R1,992. 8*a.* 3*p.*, and in the next three days to an amount equal to twice the two former. Finding that he had one-fourth of the goods left, he calculated his profits in the five days. How much were they ?

38. If the interest of R1,000 in five years be R250, what will be the interest of R3,500 for 1 year and 6 months ?

39. What is the expense of matting a room 31 ft. 5 in. long by 20 ft. 4 in. wide, the mat costing 14*a.* per 12 sq. hâth (linear hâth = 18 inches) ?

40. In what time will R8,500 amount to R15,767. 8*a.* at $4\frac{1}{2}$ per cent. per annum ?

41. A person owes the sums of R31,500 and R8,500, and his property only amounts to R14,125. How much is he able to pay in the rupee; and what is the loss upon the second debt ?

42. When rice is 10 seers the rupee, nine persons can be fed for 30 days at a certain cost. For how many days can six persons be fed at the same cost, when rice is 14 seers the rupee ?

43. A wooden box 3 ft. 8 in. long, 2 ft. 3 in. high, and 2 ft. 4 in. wide is made of board one inch thick. Find the quantity of wood used; and the cubical contents of the box.

44. What sum will amount to a *lakh* of rupees in ten years at 5 per cent. Simple Interest ?

Find the discount on R1,308 due two years hence, at $4\frac{1}{2}$ P. C. per annum.

45. A person received on the death of his aunt $\frac{1}{10}$ of her property and spent $\dot{5}\dot{4}$ of it in paying off his debts; what fraction of his aunt's property did he then possess?

46. A room is 30 ft. long, 22 ft. wide, $18\frac{1}{2}$ ft. high, and has 5 doors and 3 windows; find the expense of colouring the walls at *3a.* per sq. yd. deducting 30 sq. ft. for each door and window.

47. Find the *Present Worth* of R19,021 due 4 years hence at $3\frac{3}{4}$ P. C.

48 If R16,430 be invested in the Government $4\frac{1}{2}$ per cent. loan at 106, what is the monthly income derived? Supposing that the loan is paid off at par in 10 years, what would be the rate of Simple Interest (per cent. per annum) on the sum invested?

49. An equal number of men, women, and boys earned R39. *6a.* in seven days. Each boy received *2a.* a day, each woman *3a. 6p.* and each man *4a. 6p.* How many were there of each?

50. How many yards of matting 2 ft. 4 in. wide will be required for a square room, whose side is 9 ft. 4 in.? And what will be the price of it at *2a. 3p.* per yard?

51. If 4,000 men have provisions for 190 days, and if after 30 days 800 men go away, find how long the remaining provisions will serve the number left.

52. At what rate per cent., Simple Interest, will R1,462. *8a.* amount to R1,725. *12a.* in 4 years?

53. Find the discount on £453. 15*s.* due 6 years hence at $3\frac{1}{2}$ per cent.

54. A man sells 3 per cent. stock at 75, and invests the proceeds in 5 per cents.: at what rate must he buy them in order that his income may be the same as before?

55. If 7 men and 5 boys can reap 168 acres in 18 days, how many days will 15 men, and 5 boys take to reap 700 acres, one man being able to do three times as much work as a boy?

56. In a rectangular area, 100 yds. long and 50 yds. broad, there are two paths crossing one another each parallel to one side of the rectangle and each 4 yds. broad. Find the cost of paving the area with stone at *12a.* per sq. yd. and of covering the paths with gravel at *6a.* per sq. yd.

57. Calculate to 5 places of decimals the square root of $1+(\cdot 067)^3$.

58. R49 was divided amongst 150 children; each girl had *8a.* and each boy *4a.*; how many boys were there?

59. A tank 75 yds. long, 50 yds. broad, and 11 ft. deep is full of water: how many times can each of 16 water-carts, length 5 ft breadth 5 ft. and depth 27 in., be filled from the tank before the water in it falls 6 in.?

60. If 17 men can build a wall 100 yds. long, 12 ft. high and $2\frac{1}{2}$ ft. thick in 25 days, how many will build a wall twice the size in half the time?

61. Find the change of income when a person transfers £2,616. 5*s.* from the 5 per cents. at $95\frac{1}{8}$ to the 4 per cents. at 83, brokerage as usual.

62. In a game of skill A can give B, and B can give C, 10 points out of a game of 50; how many should A give C?

63. Express each of the figures composing the number 123·456 as a multiple or sub-multiple of 10.

64. Divide £127. 8*s* among 2 men, 3 women and 7 boys, giving each of the boys one-third of what a woman receives, and each of the men twice as much as a woman.

65. A leaky cistern is filled in 5 hrs. with 30 pails of 3 gallons each but in 3 hrs. with 20 pails of 4 gallons each, the pails being poured in at intervals. Find how much the cistern holds, and in what time the water would waste away.

66 A race-course is half a mile long; A and B run a race and A wins by 10 yds.; C and D run over the same course and C wins by 30 yds.; B and D run over it and B wins by 20 yds.; if A and C run over it, which should win, and by how much?

67 A tradesman puts two prices on his goods; one for ready money, the other for 6 months' credit, interest being calculated at $12\frac{1}{2}$ per cent. per annum. If the credit price of an article be R26. 9*a*., what is its cash price?

68. A man who has a certain capital calculates that if he invest it in $3\frac{1}{4}$ per cent. stock at 91, his income will be £25 more than if he invest it in 3 per cent. stock at 88. What is his capital?

69. A besieged garrison consists of 300 men, 120 women, and 40 children, and has provisions enough for 200 men for 30 days. If a woman eats $\frac{2}{3}$ as much as a man, and a child $\frac{1}{2}$ as much, and if after 6 days 100 men with all the women and children escape, for how long will the remaining provisions last the garrison?

70. A person begins to speculate with a certain sum of money; in his first transaction he loses $\frac{1}{7}$th of this sum; in his second he gains 10 per cent on his investment; in his third he loses $\frac{6}{11}$ths of the sum invested; in his fourth he gains $66\frac{2}{3}$ per cent. If he then has R10,000, with what sum did he start?

71. If it costs R200 to build a wall 6 ft. high by 1 ft. 3 in. broad by 166 ft. 8 in. long, what will be the cost of building a wall $3\frac{1}{2}$ ft. by $1\frac{1}{2}$ ft. by 115 ft.?

72. When will the interest amount to the principal at $3\frac{1}{2}$ per cent. per annum? What will the interest on R150 at 1*a*. per rupee per month amount to in 5 years, and how much is that rate per cent. per annum?

73. If 27 men can perform a piece of work in 15 days, how many men must be added to the number that the work may be finished in three-fifths of the time?

I buy a horse for £40 and sell it for £45 at a credit of 8 months. What do I gain per cent. reckoning money worth 6 per cent. per annum?

74. Which is the better investment, bank stock paying 10 per cent. at 319 or 3 per cent. consols at 96?

What will be the cost of £1,500 3 per cent. consols at $89\frac{3}{8}$, brokerage being $\frac{1}{8}$ per cent.? What rate of interest will such investment obtain?

75. What sum of money at 4 per cent. Simple Interest will secure the same income as R25,475 at $4\frac{1}{2}$ per cent.?

76. If a rupee is equivalent to 1s. $6\frac{3}{4}d.$, what is the price of a sovereign in rupees? If, after buying 250 sovereigns at this price, I sell them, again when the rupee is equivalent to 1s. 6d., how much shall I gain or lose by the transaction.

77. If 50 men can do a piece of work in 12 days, working 8 hours a day, how many hours a day would 60 men have to work in order to do another piece of work twice as great in 16 days?

78. If R450 amount to R540 in 4 years at Simple Interest, what sum will amount to R637. 8a. in 5 years at the same rate?

79. A man can walk 600 miles in 35 days, resting 9 hours each day, how long will he take to walk 375 miles if he rests 10 hours each day, and walks $1\frac{1}{2}$ times as fast as before?

80 A man holds $15\frac{1}{4}$ shares of a bank, and receives £19. 1s. 3d., per quarter. If the interest he receives be 5 per cent. per annum, find the value of a share.

81. A piece of work can be done in 72 days by 17 men working together. If after 9 days of work, these are joined by 4 others, in how many days will the work be finished?

82. Find the price of $4\frac{1}{2}$ per cent. Govt. Promissory Notes when an investment of R59,422. 8a. produces a monthly income of R213. 12a.

1891.

1. Simplify the following expressions:—

(a) $\dfrac{\frac{3}{5}-\frac{2}{7}}{\frac{5}{[illegible]}-\frac{3}{[illegible]}}+\dfrac{\frac{5}{7}-\frac{1}{[illegible]}}{\frac{1}{[illegible]}-\frac{1}{11}}$.   (b) $\dfrac{1}{4-\dfrac{1}{2-\dfrac{1}{1-\frac{5}{13}}}}$.

2. Find the value of $2{\cdot}4607\times{\cdot}06-3{\cdot}75\times{\cdot}012+2{\cdot}163\div1{\cdot}03$.

3. Find the value of 15 cwt. 3 qr. 9 lbs. at R25. 12a 7p. per cwt.

4. If a man walking at the rate of $3\frac{1}{2}$ miles an hour, walks to a place in 4 hours 30 minutes, how long will it take a man, walking at the rate of $3\frac{1}{4}$ miles an hour, to walk there and back?

5. A man invests a certain sum in $4\frac{1}{2}$ per cent. Government Paper at 104 The price falling to 101, he sells out and loses R600 by the transaction, exclusive of brokerage. Find the sum invested.

6. *A* gives *B* 10 yards' start and *C* 15 yards' start in a race of 100 yards; how much should *B* give *C* in 150 yards?

1892.

1. Simplify $\dfrac{3\frac{5}{6}-1\frac{7}{8}\text{ of }\frac{2}{3}\frac{4}{5}}{11\frac{2}{3}\text{ of }\frac{9}{14}\text{ of }\frac{3}{5}}-\dfrac{4\frac{1}{3}-7\frac{5}{9}+3\frac{8}{9}}{\frac{5}{9}\text{ of }12}$.

2. Find, to the nearest integer, the value of $\dfrac{39\cdot37 \times 760 \times 13\cdot596}{1\cdot293 \times 12}$.

3. Find the square roots of $\cdot097344$ of $\cdot009604$, and of $\cdot996004$.

4. Find the interest on 10 lakhs of rupees for 10 days at $4\frac{1}{2}$ per cent.

5. £3,000, which I held in the Four per cents, was sold for me when they were at $82\frac{3}{4}$ by a broker whose commission is $\frac{1}{4}$ per cent.; and the proceeds were re-invested by him in the Four and a half per cents. at $98\frac{3}{4}$. What amount of the latter stock did he purchase?

1893.

1. Simplify: (1) $1+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$;

(2) $\dfrac{8\frac{1}{7}-1\frac{1}{6}}{\frac{4}{5}+1\frac{1}{2}}-\dfrac{1}{5\frac{3}{4}-1\frac{1}{8}}$.

2. Divide $1\cdot84626$ by $23\cdot4$. Express $\cdot4\dot{5}\dot{6}$ and $\cdot6\dot{5}\dot{4}$ as Vulgar Fractions reduced to their lowest terms, and their sum as a Circulating Decimal.

3. Find the cost of 73 cwt. 3 qrs. 14 lbs. at 4*l.* 13*s.* 6*d.* per cwt.

4. Distinguish between true discount and bankers' discount.

Find the former in the case of a bill for R3486. 6*a.* 8*p.* due 16 months hence, the rate of interest being $5\frac{1}{2}$ per cent. per annum.

5 A man invests R163000, part in Govt. 4 per cent. stock at 108 and the remainder in Municipal 5 per cent. debenture stock at $109\frac{1}{2}$. Find how much he must invest in each that he may have an equal income from the two sources.

1894.

1. In a compound metal containing tin and copper only, the proportion of tin to copper is 7·75 to 92·25. Find to the nearest penny the value of 8 cwt. 3 qrs. of it. Tin costs 140*l.*; copper 80*l.* per ton.

2. A rectangular court is 50 yards long and 30 yards broad. It has paths joining the middle points of the opposite sides of 6 feet in breadth and also path of the same breadth running all round it. The remainder is covered with grass. If the cost of the pavement be 1*s.* 8*d.* per sq. ft. and the turf. 3*s.* per sq yd., find the cost of laying out the court.

3. Find the value of $\cdot267\dot{1}87\dot{5}$ of 3*l.* in *s. d.*, and decimal of a penny.

4. Find the square root of $1-(\cdot0678)^2$ to four places of decimals.

5. At a cricket match a contractor provided luncheon for 24 and fixed the price to gain $12\frac{1}{2}$ per cent. on his outlay. Three persons were absent. The remaining 21 paid the fixed price and the contractor lost 2 rupees. What was the charge?

6. Find the simple interest on R12345 for 134 days at $2\frac{3}{4}$ per cent.

1895.

1. Find the square root of $1+\cdot\frac{1}{3}(\cdot0345)^2$ correctly to 4 places.

2. Find the sum of money which put out at simple interest at $2\frac{3}{4}$ P. C. will in 134 days exactly produce R124-10-$1\frac{1347}{1825}$. (1 yr. = 365 da.)

3. If one pound sterling be worth twenty-five francs and sixty centimes: and also worth six thalers and twenty silver groschen: how many francs and centimes is one thaler worth? (N. B. One thaler = 30 silver groschen. One franc = 100 centimes).

4. Simplify $\dfrac{1\frac{1}{4}-\frac{5}{12}}{1\frac{1}{4}+\frac{5}{12}}+\dfrac{7}{9}$ of $\dfrac{9\times5}{14\times3}-\dfrac{11\frac{1}{4}}{15}$.

5. I invest R12805 in the 4 per cents. at $98\frac{1}{4}$, and when they have risen to $102\frac{5}{9}$, I sell out and invest in the $4\frac{1}{2}$ per cents. at $105\frac{3}{8}$: what is the change in my income? (Brokerage $\frac{1}{4}$ per cent. on all transactions.)

Or convert $\frac{1247}{1925}$ into a decimal fraction, pointing out accurately the recurring portion (if any).

1896.

1. What greatest number and what least number can be subtracted from 23759143 that the remainders may be divisible by 24, 35, 91, 130, and 150?

2. (1) Simplify

$$\frac{5\frac{5}{8}}{6\frac{1}{7}} \text{ of } \frac{6\frac{7}{11}}{9\frac{1}{8}} \div \frac{5}{9}\left(2\tfrac{7}{11}+\tfrac{13}{22}\right) \text{ of } \frac{7s.\ 6d.}{12s.\ 6d.}$$

(2) Divide ·0023465 by ·03125.

3 Extract the square root of $5\frac{1}{7}$ correct to 4 places of decimals.

4 Find the simple interest on R4235. 12*a*. $9\frac{3}{5}$ pies for 5 years and 7 months at $3\frac{1}{2}$ per cent. per annum.

5. If by selling a horse for R11000, I lose 18 per cent.: how much per cent. should I have gained or lost, had it been sold for R1420?

6 A man invested the same sum in two different stocks, $3\frac{1}{2}$ per cent. Government Securities at $103\frac{1}{2}$ and 4 per cent. Municipal Debentures at 105: his income from one is R93 more than from the other; what sum was invested in each stock?

1897.

1. Reduce $\dfrac{2\frac{1}{3}-1\frac{1}{4}}{2\frac{1}{3}+1\frac{1}{4}}\times15\frac{17}{86}+\dfrac{3\frac{1}{3}\times3\frac{1}{3}\times3\frac{1}{3}-1}{3\frac{1}{3}\times3\frac{1}{3}\times3\frac{1}{3}+1}$ of 1 cwt. 3 qrs. 7 lbs. to the decimal of $2\frac{1}{2}$ tons.

Find the Vulgar fraction equivalent to the recurring decimal $\cdot1\dot{3}\dot{3}$, without assuming any rule.

2. What do you understand by an *aliquot* part of a quantity? Is an area equal to $15\frac{1}{8}$ square yards an aliquot part of an acre?

Find by practice the income-tax of R1250. 10*a*. 8*p*. at the rate of 5*p*. per rupee.

3. What is meant by the *ratio* of one quantity to another? What is a *proportion*?

320 people dine together 4 days a week, but on the remaining 3 days some are absent; the consumption is thus reduced for the whole week in the ratio of 109 to 112. Find the number of absentees.

4. In what time will R3,546 amount to R7,683 at $3\frac{1}{2}$ per cent. simple interest?

5. A person has stock in the $3\frac{1}{2}$ per cent. Government Securities which yields R2856 a year. He sells out half of the stock at $109\frac{7}{8}$, and invests the proceeds on Howrah Mills shares at 153. What dividend ought the latter to pay that he may thereby increase his annual income by R330?

6. Extract the square root of 3·14159 to 4 decimal places.

1898.

1. What is the least number, which being divided by 48, 64, 72, 80, 120 and 140, leaves the remainders 38, 54, 62, 70, 110 and 130 respectively?

2. (*a*) Simplify $\dfrac{2\frac{3}{4}}{5\frac{1}{8}}$ of $\frac{3}{4}(\frac{7}{9}+\frac{1}{12}) \div \dfrac{5\frac{7}{8}}{7\frac{1}{4}}$ of $\dfrac{2s.\ 5d.}{3s.\ 11d.}$.

(*b*) What decimal of 2*l*. 3*s*. 4*d*. is ·0625 of $2\dot{6}$ of 1*l*. 6*s*. 8*d*.

3. Extract the square root of 54756; also of $(4·02)^3$ to 4 places of decimals.

4. What sum will amount to R300 in $3\frac{1}{4}$ years at $6\frac{1}{4}$ per cent. per annum simple interest?

5. A grocer buys 480 mds. of sugar for R6135 payable at the end of 3 months, and on the same day sells them at R12. 11*a*. per maund ready money; what per cent. does he gain or lose by the transaction, reckoning interest at 9 per cent per annum?

6. One-third of a certain capital is invested in the $3\frac{1}{2}$ per cent. Government Securities at 105, one-fourth in the 3 per cent. Government Securities at $97\frac{1}{2}$, and the remainder in the $4\frac{1}{2}$ per cent. Calcutta Municipal Debentures at $112\frac{1}{2}$. If the total annual income is R830, what is the capital?

1899.

1. Find the greatest number which will divide 1028, 1629, and 2130 leaving the remainders 3, 4 and 5 respectively.

2. (*a*) Simplify $\dfrac{\frac{3}{4}+\frac{7}{8}}{\frac{5}{9}+\frac{9}{10}}$ of $\dfrac{13s.\ 5d.}{9s.\ 10d.} + \frac{4}{5}(\frac{3}{7}+\frac{5}{9})$ of $\dfrac{3 \text{ tons } 8 \text{ cwt.}}{4 \text{ tons } 3 \text{ cwt.}}$

(*b*) Prove that $·2\dot{3}\dot{4} = \frac{232}{990}$ without assuming the rule of converting a recurring decimal into a vulgar fraction.

3. Find by Practice or otherwise the value of 7 tons 2 cwt. 2 qrs. at R3. 2*a*. per maund, assuming that 1 ton is equal to $27\frac{1}{4}$ mds.

4. Extract the square root of 51076 and of ·051076.

5. A grocer mixed 20 mds of one kind of rice at R4 a maund with a certain quantity of second kind of rice at R3. 8*a*. a maund, and selling the mixed rice at R3. 12*a*. a maund, gained R10 on the whole. Find how many maunds of the second kind of rice he mixed, and the gain per cent. on his outlay.

6. Find the discount on R218 due 6 months hence at 3 per cent. per annum simple interest.

1900.

1. What do you understand by the Greatest Common Measure and the Least Common Multiple of two or more whole numbers ?

Nine bells begin to strike simultaneously, and strike at intervals of 1, 2, 3, 4, 5, 6, 7, 8, 9 seconds respectively. After what interval of time will they next strike simultaneously ?

2. (*a*) Simplify : $\dfrac{16\frac{1}{12}-3\frac{1}{2} \text{ of } 2\frac{1}{3}}{\frac{3}{7} \text{ of } 5\frac{1}{2}+3\frac{3}{4}} \times \dfrac{2\frac{5}{16} \text{ of } 4\frac{3}{4}+\frac{3}{8} \text{ of } 13\frac{3}{8}}{5\frac{3}{4}-4\frac{1}{2} \text{ of } \frac{5}{9}}-\frac{67}{117}+1\frac{1}{2}$.

(*b*) Reduce $\cdot 04\dot{1}\dot{6}$ to its equivalent vulgar fraction in its lowest terms, and explain the reason for the process you employ.

3. Find the value of $(1\cdot25)^3+2\cdot25\times(1\cdot25)^2+3\cdot75\times(\cdot75)^2+(\cdot75)^3$, without reducing the decimals to vulgar fractions.

4. The length, the breadth, and the height of a room are 25 ft. 7 in., 20 ft. 5 in. and 14 ft. respectively. Its walls are papered at 3*s*. 6*d*. a sq. yd., and its ceiling painted at 1*s*. 2*d*. a sq. ft. Find the total cost.

5. The subscriptions to a certain memorial fund amounted to R976. 9*a*. and each person subscribed as many annas as there were subscribers altogether. Find the number of subscribers.

6. Explain clearly what you mean by saying that the $3\frac{1}{2}$ per cent. Government Securities are at 101.

A person invests R19,700 in the $3\frac{1}{2}$ per cent. Government Securities at $98\frac{1}{2}$, and when they rise to $101\frac{1}{2}$, he sells out and invests the proceeds in the $4\frac{1}{2}$ per cent. Calcutta Municipal Debentures at $114\frac{1}{16}$. Find the change in his income.

1901.

1. (*a*) Simplify $\dfrac{306}{323} \div \dfrac{204}{221} \text{ of } \dfrac{22\frac{6}{11}}{32\frac{9}{11}} - \cdot 58\dot{3} \times \cdot\dot{1}4285\dot{7}$,

expressing your answer as a decimal.

(*b*) Reduce £3. 15*s*. 4*d*. to the decimal of R100 (£1 = R15).

2. (*a*) What is meant by an *aliquot part* of a number ? Is $2\frac{3}{4}$ yds. an aliquot part of a mile ?

(*b*) Find by Practice, or otherwise, the value of 25 tons 15 cwt. 3 qrs. $17\frac{1}{2}$ lbs. at £2. 13*s*. 4*d*. per ton.

3. If the fourpenny loaf weighs 3 lb. 9 oz. when wheat is at 9*s*. 4*d*. per bushel, what ought the sixpenny loaf to weigh when wheat is at 11*s*. 1*d*. per bushel ?

4. (*a*) Define *Interest*. What do you understand by the expression *Rate per cent. per annum* ?

(*b*) At what rate per cent. per annum simple interest will £200 amount to £236. 13*s*. 4*d*. in 4 years 7 months ?

5. Extract the square root of 7468·4164.

6. A man invests one-third of his capital in the $3\frac{1}{2}$ per cent. Government Securities at $96\frac{1}{8}$, and remaining two-thirds in the $4\frac{1}{2}$ per cent. Calcutta Municipal Debentures at $105\frac{1}{2}$. If the difference of the two annual incomes be R1997, find his capital.

1902.

1. (*a*) How can you ascertain whether a given vulgar fraction can be reduced to a terminating or a recurring decimal, without actually converting it into a decimal? What kind of decimal will the fraction $\frac{161}{17920}$ Produce?

(*b*) Simplify :—

$$1-\cfrac{2}{3+\cfrac{6}{5-\cfrac{6}{7+\frac{8}{9}}}}+2{\cdot}08\dot{3} \text{ of } \frac{2 \text{ cwt. } 2 \text{ qrs } 21 \text{ lbs.}}{10 \text{ cwt. } 2 \text{ qrs. } 11 \text{ lbs.}},$$

and reduce the result to the decimal of 1·1.

2. The area of a rectangular field whose breadth is 500 yds. is 100 acres. Find the cost of cultivating it at R3. 2*a*. 8*p*. per 100 sq. yds. and also the cost of fencing it round at R2. 8*a*. per yard.

3. If 12 men and 15 boys can as a piece of work in 30 days, working $7\frac{1}{2}$ hours a day, how many boys must assist 21 men to do a piece of work twice as great in 25 days, working 9 hours a day? (3 men are equivalent to 5 boys.)

4. Extract the square root of $5\frac{1}{16}$ and 76·195441.

5. (*a*) Define *Discount*.

(*b*) Find the discount on £700 due 3 years 4 months hence at 5 per cent. per annum simple interest.

6. Which is the better investment, the $3\frac{1}{2}$ per cent. Government Securities at $95\frac{3}{8}$ or the 4 per cent. Calcutta Municipal Debentures at $101\frac{1}{2}$? What will be the difference in the annual income by investing R22127 in each of them?

1903.

1. (*a*) Simplify $\dfrac{{\cdot}67\times{\cdot}67\times{\cdot}67-{\cdot}001}{{\cdot}67\times{\cdot}67\times{\cdot}67+{\cdot}01}+\dfrac{{\cdot}5\dot{7}}{1+\dfrac{1}{3\frac{1}{14}}}$.

(*b*) What decimal of a mile is a yard?

2. (*a*) What is meant by the aliquot part of a number? Is an acre an aliquot part of a square mile?

(*b*) Find by practice, or otherwise, the price of 25 tons 12 cwt. 3 qrs. $12\frac{3}{4}$lb. at £6. 13*s*. 4*d*. per ton.

3. Three taps *A*, *B* and *C* can fill a cistern in 5, 6 and $7\frac{1}{2}$ minutes respectively. They are all turned on at once; but after one minute, *A* is turned off. How much longer will *B* and *C* take to fill the cistern?

4. (*a*) Define the square root of a number.

(*b*) Extract the square root of $10\frac{1}{36}$; and of $\frac{22}{7}$ to four places of decimals.

5. A man buys wine at 5*s.* a gallon; he mixes it with water, and by selling the mixture at 4*s.* a gallon gains $12\frac{1}{2}$ per cent. on his outlay. How much of water did each gallon of the mixture contain?

6. (*a*) Define *Present Worth.*

(*b*) A tradesman marks his goods with two prices, one for ready-money and the other for 3 month's credit, allowing interest at $4\frac{1}{2}$ per cent. per annum. If the credit price be marked at R50. 9*a.*, what ought to be the cash price?

1904.

1. Define the G. C. M. and the L. C. M. of two or more numbers.

(*a*) Find the greatest number of six digits which is exactly divisible by 27, 45, 60, 72 and 96.

2. Write down the local value of each of the figures in the number ·010203.

(*a*) Simplify

$$\frac{(\cdot01)^3+(\cdot02)^3+(\cdot03)^3}{(\cdot001+\cdot002+\cdot003)^2}-\cdot0208\dot{3}\div\frac{£2.\ 3s.}{£25.\ 16s.}\text{ of }\frac{1}{2}.$$

3. *A* can do a piece of work in 25 days, *B* in 20 days, and *C* in 24 days. The three work together for 2 days, and then *A* and *B* leave; but *C* continues, and after $8\frac{3}{8}$ days is rejoined by *A*, who brings *D* along with him, and these three finish the remainder of the work in 3 days. In what time would *D* alone have done the whole work?

4 The area of a square cricket field is 9 ac. 3 ro. 8·16 po.; find the length of a side.

5. Define *Discount.*

(*a*) The difference between the interest and the discount on a certain sum for 3 years 4 months at 5 per cent. per annum is £16. 13*s.* 4*d* Find the sum.

6. A person invests a certain sum in the $3\frac{1}{2}$ per cent. Government Securities when they are at $97\frac{3}{8}$; had he waited till they had fallen to $97\frac{1}{8}$, he would have had Rs. 400 more of Government Securities. How much money did he invest, $\frac{1}{8}$ per cent. being charged as brokerage in both cases?

1905.

1. When is one number said to be a *measure* of another? What is a *Prime Number*?

A man bought two heaps of mangoes, one for R10. 5*a.* and the other for R18. 0*a.* 9*p* If the price of each mango be the same, and not less than two and not more than three annas, find the total number of mangoes he bought.

2. (i) What is the meaning of $\frac{2}{5}$ and of $\frac{1}{2}$ of $\frac{3}{8}$?

(ii) Simplify :—

$$\left(5\frac{5}{8}-\frac{5}{12}\right)\text{ of }\left(\frac{\frac{5}{8}}{3\frac{1}{3}}\div\frac{7}{8}\text{ of }\frac{4}{5}\right)\div\frac{5}{7}\text{ of }\frac{3\text{ tons }3\text{ cwt.}}{9\text{ cwt.}}$$

3. Extract the square root of 19·951 and of $\frac{5}{8}$. Correct to three places of decimals.

4. Find the cost of having a pathway 6 ft. wide, round and immediately outside a flower garden, 21 yds. long and 10 yds. broad, at $5\frac{1}{4}$ pies per sq yd.

5. Find the price of 35 mds. $13\frac{3}{4}$ srs. of rice at R3. 2*a.* per maund. If it is sold at the rate of R3. $3\frac{1}{2}$*a.* per maund, what is the profit per cent. ?

6. I pay Rs. 45000 to a Bank for a Bill of Exchange payable in London. The rate of exchange is 1*s.* 4*d.* for the rupee and the Bank charges me 2 per cent. on the amount payable in England. How much will my agent in London receive ?

1906.

1. (1) When is one number said to be a *multiple* of another ? How can you ascertain by inspection whether a given number is a multiple of 3 ?

(2) What is the greatest number consisting of five digits which can be added to 8321 ; so that the sum may be exactly divisible by 15, 20, 24, 27, 32 and 36 ?

2 (1) What is the meaning of $\frac{3}{4}$ of $\frac{1}{2}$? Give an illustration.

(2) Simplify .—

(1) $12\times(\frac{2}{20}-\frac{1}{24}-\frac{1}{56}-\frac{1}{174}-\frac{1}{232})+\frac{323}{361}\div\frac{2\frac{5}{8}}{2\frac{5}{7}}$ of $\frac{11s\ 4d.}{12s.\ 3d.}$.

(2) $\frac{1·59\times15·9-·41\times4·1}{15·9-4·1}$.

3. The cost of matting a room 16 ft. broad and 12 ft. high at 3*a.* per sq. yd. is R7. 9*a.* 4*p* What will be the cost of papering its walls at the same rate, allowing for six doors, each 6 ft. by 3 ft. ?

4. Extract the square root of $·02\dot{7}$ and of $\frac{3}{8}$ correct to four places of decimals.

5. A book sent from England costs me (including R1. 2*a.* postage) R12. 1*a.* But my bookseller allows me a discount of 2*d.* in the shilling on the published price. What is the published price in English money, the rate of exchange being 1*s.* 4*d.* for the rupee ?

6. Define *Present Worth*

A man bought a horse for 30 guineas and sold him immediately for £36. 1*s.* payable at the end of 6 months. If interest be reckoned at 6 per cent. per annum, find his gain per cent. upon the transaction.

1907.

1. What do you understand by the G. C. M. and the L. C. M. of two or more integers ? What is a prime number ? Find the least number which is exactly divisible by 12, 34, 56 and 78.

2. Simplify

(1) $\dfrac{\cdot 2 \times \cdot 2 \times \cdot 2 + \cdot 02 \times \cdot 02 \times \cdot 02}{\cdot 6 \times \cdot 6 \times \cdot 6 + \cdot 06 \times \cdot 06 \times \cdot 06} \div \dfrac{2\frac{1}{3} - 1\cdot 1\dot{6}}{2\cdot\dot{3} + 1\frac{1}{6}}$

(2) $\dfrac{\text{R}2.\ 5a.\ 6p.}{\text{R}3.\ 12a.} - \dfrac{1 \text{ hr. } 16 \text{ m. } 45 \text{ sec.}}{2 \text{ hr. } 7 \text{ m. } 55 \text{ sec.}}$.

3. Find the price of 8 mds. 16 srs. 2 chks. of rice at R5. 9*a.* per maund.

4. How many paving stones, each of them 1 foot long and 9 in. wide, will be required for paving a street 30 ft. wide, surrounding the outside of a square grass plot, the area of the grass plot being 10 acres ?

5. If 8 men or 15 women can earn Rs.120 in 30 days, how much can 21 men and 24 women earn in 45 days ?

6. The debts of a bankrupt amount to £2134. 10*s.* 6*d.* and his assets consist of property worth £916. 5*s.* 4*d* and an undiscounted Bill of £513 due 4 months hence, Simple Interest at 4 per cent. How much in the pound can he pay to his creditors ?

1908.

1. (1) When can a vulgar fraction be converted into a terminating decimal ? What kind of decimal will the fraction $\frac{119}{130}$ produce ?

(2) Simplify :—

$$\frac{2\frac{1}{4} + \frac{3}{4} \text{ of } 2\frac{2}{3} - 1\frac{1}{4}}{3\cdot 6 + \cdot 15 \times 4 \div \cdot 24 - 2\cdot 1} \text{ of } \frac{20\cdot 1 \text{ of } 1 \text{ md. } 30\frac{1}{4} \text{ srs.}}{17\cdot 5625 \text{ of } 2 \text{ mds. } 20\frac{1}{2} \text{ srs.}} \text{ of R}2.\ 8a.$$

2. Find by Practice, or otherwise, the value of 5 acres 3 roods 7 poles $5\frac{1}{2}$ sq. yds. of land at £161. 6*s.* 8*d.* per acre.

3. (1) The hands of a clock coincide after every 66 minutes of correct time. How much is the clock fast or slow in 24 hours ?

(2) A race-course is 440 yards long. *A* and *B* run a race and *A* wins by 5 yards : *B* and *C* run over the same course and *B* wins by 4 yards : *C* and *D* run over it and *D* wins by 16 yards : If *A* and *D* run over it, which would win, and by how much ?

4. (1) What number multiplied by itself will produce $4\frac{285}{529}$?

(2) Extract the square root of $\frac{3}{7}$ correct to four places of decimals.

5. A trader allows a discount of 5 per cent. to his customers. What price should he mark on an article, the cost price of which is R712. 8*a.*, so as to make a clear profit of $33\frac{1}{3}$ per cent. on his outlay ?

6. A person invests R44100 in the $3\frac{1}{2}$ per cent. Government Securities at 98, and when they rise to $98\frac{1}{2}$, he sells out and invests the proceeds in the 5 per cent. Calcutta Municipal Debentures at $110\frac{1}{8}$. Find the alteration in his income.

MADRAS ENTRANCE PAPERS.

SELECTED QUESTIONS 1857-88.

1. A tank is 300 yards long and 150 yards broad; with what velocity per second must water flow into it through an aperture 2 feet broad and $1\frac{1}{2}$ feet deep, that the level may be raised 1 foot in 9 hours?

2. Find the interest of £250 for $3\frac{1}{2}$ yrs. at $4\frac{1}{2}$ P. C. Simple Interest.

3. A cubical tank, 24 ft. long, 18 ft. 6 in. wide and 12 ft. 4 in. deep is filled with water. Find the weight of water supposing that a cubic foot weighs 1000 oz. How long will it take to discharge itself at the rate of 15 gallons a minute, assuming that a pint of water weighs 1 lb.?

4. A rectangular field is $\frac{3}{10}$ths of a mile long and $\frac{2}{15}$ths of a mile wide; find the length of a line joining two opposite angles.

5. Find the number of degrees, minutes and seconds in an arc of a circle which is equal in length to its radius, the ratio of the diameter to the circumference being 1 : 3·14159.

6. A person having R8,500 in 4 per cent. Government bonds sells out when they are at $8\frac{1}{4}$ per cent. discount, and with the amount thus realised purchases 5 per cent. bonds, which are at $6\frac{5}{8}$ per cent. premium: what does he gain or lose in annual income by the change?

7. In long Multiplication the general product is the sum of the several partial products. Illustrate this in the example, 2359×576, and write down separately, the several partial products with their factors.

8. The receipts on the Madras Railway for a certain week in January 1859, when there were 96 mi. open, were R9554 3*a.* 10*p.*; for the corresponding week in 1858 when there were 81 mi. open, they were R8,554. 6*a.* Compare the average receipts *per mile* for the two years.

9. State the rule for division of Vulgar Fractions, and prove it, taking an example.

10. The French unit of linear measure is a *metre*, equal to 39·371 English inches: the square formed on a line of 10 metres (called an *are*) is the French unit of surface. Find the equivalent in English square measure, of a hectare (100 ares).

11. Multiply 76489 by 743, and explain fully the various steps of the process. In what cases does multiplication increase, leave unaltered, diminish, the multiplicand?

12. Find the least common multiple of 2191, 1252 and 1878. Illustrate the proof of your rule by this example.

13. If a cloth, 4 yards long and 15 inches wide, cost R3. 5*a.*: how much should you give for a cloth 19 yards long, 12 inches wide, and every square inch of which is worth $\frac{3}{8}$ths of the value of a sq. ft. of the former?

14. A bankrupt is indebted to *A*, *B*, *C* and *D*—*A*'s debt is twice *B*'s ; *B*'s three times *C*'s ; *C*'s half *D*'s. How much should each receive of assets to the amount of R45,680 ?

15. Divide 764·0468 by ·0007. Give the rule for the position of the decimal point in your quotient and show that the rule is correct.

16. If the daily wages of a labourer rise from four and three quarters to six annas, what percentage of the increase in the price of food and other commodities will cause his position to be unaltered ?

17. A gentleman buys a house for R24,650 and spends 23 per cent. in additions and improvements. At what monthly rental will he secure 8 per cent. per annum on his whole investment ?

18. Explain the decimal system of numeration. Write in words 14006, 3179040601, and 17·0461.

19. "Multiplication is a shortened form of addition." Of all additions or of some ? And if only of some, of what kind ?

Do the two statements *twice two are four and four times five are twenty* rest upon the same ground ? Could you shew without reference to the multiplication table, that five times five must exceed four times six by one ?

20. State and prove the rule for the division of vulgar fractions.

21. If, when the exchange is at 1*s.* $11\frac{1}{2}$*d.* per rupee, you wish to remit R4891. 4*a.* 3*p* to London ; what should be the amount of your bills in English money ?

22. A steam-ship whose speed averages 14 mi. an hour, reaches a certain port in 12 days ; how many days afterwards will a sailing vessel arrive which started at the same time and sailed on an average 8 mi. an hour ?

23. A train has been travelling 20 miles an hour : the steam power is doubled, whilst from various causes the resistance of the train is increased by one-half. (The original steam power is three times the resistance). At what rate will the train now travel ?

24. Shew that no number can be a perfect square which has an odd number of decimals after the point

25. A sailing vessel reached Madras from Calcutta in 6 days ; a steamer whose speed is to that of the sailing vessel as 3 : 2 starts at the same time, but meets with detentions that average 6 hours daily. Which will reach Madras first ? And by how much ?

26. A man rides at the rate of 11 miles an hour, but stops 5 minutes to change horses at the end of every 7th mile ; how long will he take to go a distance of 94 miles ?

27. How are Vulgar Fractions compared in regard to magnitude ? Of the fractions $\frac{7}{25}$, $\frac{113}{365}$, which is the greater and what is the difference ?

28. Express in a decimal form $3+\frac{4}{15}+\frac{6}{1000}+\frac{17}{10000}$.

29. A train starts from *A* at 12 o'clock and runs towards *C*, which is 100 mi. distant, at the rate of 30 mi. an hour ; at the same time the

mail cart starts for C from B, which is half way between A and C, and runs at 10 mi. an hour; at what distance from C will it be overtaken by the train ?

30. A person goes into a bookseller's shop with a certain sum of money, and after buying 20 books at R2-4-0 each, finds that $\frac{3}{8}$ of his money remain. How much had he when he entered the shop ?

31. A room is 16 ft. 5 in. long and 19 ft. 7 in. broad, and the cost of painting the walls at 7*a*. 6*p*. per square yard is R43-3-0. Required the height of the room.

32. A merchant buys goods for £568. 4*s*. and sells half of them at a gain of 1*d*. in the shilling of the cost price, one-third of them at a gain of 2*d*. in the shilling, and the remainder at a gain of £15. 15*s*. 8*d*. How much per cent. does he gain on the whole transaction ?

33. A person after paying an income tax of one anna in the rupee, devotes $\frac{1}{10}$ of the remainder of his income to purposes of charity, and finds that he has R5175 left; what is his income ?

34. A person paid a tax of 10 per cent. on his income, and had R15,000 per annum remaining. What was his income ?

35. If 10 compositors, who can set 3 letters in 5 seconds, finish 27 pages, in an hour and a half, how many compositors, who can set 5 letters in 6 seconds, will complete 50 pages in an hour ?

36. A can do a piece of work in 3 days, B can do 3 times as much in 8 days, and C 5 times as much in 12 days In what time will they do it together, supposing them to work at the rate of 9 hours a day ?

37. One inch of rain falls on an acre of ground. How much will it weigh, reckoning the weight of one cubic foot as 1000 ounces ?

38. A person bought a horse for R750 and kept it 15 months It cost during that time in gram, R190-10-6; in servants' wages, R135; and in other expenses, R35-14-6. He sold it for R625, what was the average monthly cost of the horse ?

39. A merchant clears 20 per cent. on a gross income of R50,000. How much per cent. must he clear if he receives the same amount from a gross income of R40,000 ?

40. A ship-captain owns $\frac{3}{8}$ of his vessel. In virtue of his command he receives $\frac{1}{25}$ of the profits, and of the remainder his share as proprietor. What proportion of the whole does he receive ?

41. A person with a monthly income of R264 spends as much in four months as he earns in three. After twelve years he divides his savings amongst his three children in such a manner that the eldest has twice as much as the second, and thrice as much as the youngest. How much did each receive ?

42. A man buys 16 lbs. of tea at R2. 2*a*. per lb.; 12 lbs. at R2. 5*a*. 4*p*. per lb.; and 24 lbs. at R2. 6*a*. 10*p*. per lb. At what price per lb. must he sell the mixture so as to gain R35. 12*a*. on the whole ?

43. If it is high water at noon on a certain day, find after how many days it will again be high water at noon, supposing the time of high water to be three quarters of an hour later every day ?

44. A crow wishing to quench its thirst came to a vessel which contained 21 cubic inches of water. The crow being unable to reach the water picked up several small stones, each three-quarters of a cubic inch in size, and let them drop into the vessel until the water came to the top of the vessel If the size of the vessel was such that it would exactly hold 73 cub. in. of water, find the number of stones dropped in by the crow.

45. The difference in the values of the two shares into which a certain property is divided is £48·575, and one share is ·51 of the whole. Find the value of the property and of each share.

46. Find, to within a foot, the length of the fence enclosing a square field whose area is $3\frac{1}{2}$ acres.

47. A person sets out to walk 26 miles; for a quarter of the distance he goes at the rate of 5 miles an hour, for half the remaining distance at 4 miles an hour, and 3 miles an hour for the other half. State the exact time occupied in the journey.

48. The Fort-Barracks are lighted with gas from 100 burners. Find the cost of lighting them per night of 10 hours, at the rate of R$5\frac{1}{2}$ for 1000 cubic feet of gas, assuming that for the first 3 hours each burner consumes 1 cubic inch per second, and during the remainder of the night the light is so reduced that the consumption of gas by each burner is only $\frac{3}{4}$ths of that quantity per second.

49. If 2 Malabar miles are equal to 1 kros, and 7 Malabar miles equal to 10 English miles, how many kros are there in 25 English miles?

50. A contractor bought 2250 pharas of unslaked lime at Madras at the rate of 45 Rupees for 100 pharas. On slaking it every phara gave 3 cubic feet of lime, but of this $\frac{1}{15}$th was unserviceable; the carriage of the remainder to the place where it was required (distant 18 miles) cost 4 annas per 100 cubic feet per mile. At what rate per cubic foot must he sell it there, in order to gain 90 Rupees on his outlay?

51. A rectangular field, whose diagonal measures 825 feet, has one of its sides ·75 of the length of the other. Find the length of each side in yards and the area in acres.

52. A creditor received on a debt of R3,600 a dividend of 9*a.* 10*p.* in the rupee: and a further dividend of 6*a* 8*p.* upon the remainder. What did he receive altogether and what fraction was it of the entire debt?

53. *A* and *B* each lends £250 for three years; *A* lends at $4\frac{3}{4}$ per cent. Simple Interest, and *B* at $4\frac{1}{2}$ per cent. per annum, Compound Interest. Find the difference in the amount of interest they receive.

54. A contractor agrees to supply $10\frac{1}{2}$ lacs of bricks for a particular work. His bricks cost him $3\frac{1}{2}$ rupees per 1,000 to make, and of these $12\frac{1}{2}$ per cent. are rejected. How many bricks must he make in order to fulfil his contract, and what price per 1,000 must he put on those supplied, in order to gain 25 per cent. on his outlay?

55. The distance by Railway from Madras to Salem is $206\frac{3}{4}$ miles. A passenger train travelling 20 miles an hour leaves Madras at 7 A.M.; and a special train at 10 A.M. the same day. At what rate must the latter

travel, so as just to overtake the former at Jollarpett Junction (132 mi. from Madras), and at what hour must a goods train leave Salem for Madras travelling 15 miles an hour, so as to reach Jollarpett at the same time as the other trains?

56. A work can be completed in 36 days by 30 men working 6 hours a day; in what time would 18 men and 60 women working 9 hours a day complete it; supposing that 3 men can do as much as 5 women; and that in the longer days a man does only $\frac{4}{5}$ per hour of what he does per hour in the shorter days?

57. *A* has shares in an estate to the amount of $\cdot 15 + \cdot 3\dot{6}$ of it. *B* has shares in the same estate to the amount of $\cdot 4\dot{7}\dot{2}$ of it; find the difference in value between the properties of *A* and *B*, when $\cdot 0\dot{5}\dot{6}$ of the estate is worth £373·3.

58. A reduction in the income-tax diminishes a tax which is R15 when the tax is 8 pies in the rupee by R3-12-0; what is the diminished rate of the tax in the rupee?

59. 23 cwt 3 qr. 7 lbs. are bought at £2. 10*s*. 8*d*. per cwt. and 72 cwt. 2 qr. 8 lbs at £2. 7*s*. 10*d*. per cwt. Find by Practice, the amount expended and give the average price per lb.

60 A person borrows £500 at 5 per cent. per annum, and subsequently £400 at $3\frac{1}{2}$ per cent.; if the amount of both sums 6 months after the latter was borrowed is £957, find the time for which interest is paid on the former sum.

61 A cask of $144\frac{2}{5}$ gallons is bought for £50 and kept 10 years, during which $\frac{7}{11}$ of a gallon evaporated yearly; at what rate per gallon must the contents be sold so as to clear 20 per cent. on the amount of the original outlay, at 4 per cent. per annum Simple Interest?

62. Water flows into a rectangular cistern whose dimensions are 12 ft. 1 in. long, 11 ft. 8 in. wide, and 5 ft 4 in. deep, through a pipe of 10 sq. in. aperture at the rate of $2\frac{1}{2}$ ft. per second, and flows out through an orifice at the rate of 2 ft. $5\cdot\dot{8}$ in. per second; if the cistern is filled in two hours, find the size of the orifice.

63. A lump composed of gold and silver measures 6 cubic inches and weighs 100 oz.; if a cubic inch of gold weighs 20 oz. and an equal bulk of silver 12 oz., find the weight of gold in the mixture.

64. A train which travels at the uniform rate of 30·8 ft. a second leaves Madras at 7 A. M.; at what distance from Madras will it meet a train which leaves Arconum for Madras at 7-20 A. M. and travels one-third faster than it does, the distance from Madras to Arconum being 42 miles?

65. A person buys a piece of land at £25 an acre, and by selling it in allotments finds that the value has increased by one-half, so that after reserving 20 acres for himself, he clears £200 on his purchase money by the sale of the remainder. How many acres were there?

66. Find how much rice a family requires monthly, when a reduction in the price from 7 to 10 measures for the rupee reduces the total monthly expenses from R$31\frac{1}{2}$ to R30.

67. A person going from Pondichery to Ootacamond travels 20 mi. by steamer, 330 mi. by rail and 30 mi. by horse-transit. The journey occupies 30 hrs. 50 min. and the rate of the train is 3 times that of the horse-transit and $1\frac{1}{2}$ times that of the steamer. Find the rate of the train.

68. A person bought 10 Bank of Madras shares at R1540 each and for 5 years got interest on his investment at the rate of $5\frac{1}{2}$ per cent. He then sold his shares at a loss of $22\frac{1}{2}$ per cent. How much did he make by the transaction and what rate per cent. per annum had he for his money?

69. A person borrows two equal sums at the same time at 5 and 4 per cent. respectively, and finds that if he repays the former sum with interest on a certain date 6 months before the latter, he will have to pay in each case the same amount, *viz.*, £1100. Find the amount borrowed and the time for which interest is paid.

70. A dealer buys 10 horses at R400 each, 8 horses at R500 each, and 4 horses at R600 each. He keeps the horses for 6 months, during which each cost R15 a month, and then sells them clearing $12\frac{1}{2}$ per cent. on his original outlay after paying all his expenses. Find the selling price.

71. A stream of water, 8 yds. broad at the surface and 6 yds. at the bottom and 2 yds deep, flows at the rate of $1\frac{1}{3}$ mi. an hour, into a tank 220 yds. long and 56 yds. broad, which holds 74,250 tons of water. Find the depth of the tank and the time in which it will be filled, a cubic foot of water weighing 1000 oz.

72. Two trains, running at the rates of 25 and 20 miles an hour respectively on parallel rails in opposite directions are observed to pass each other in 8 seconds; and when they are running in the same direction at the same rates as before, a person sitting in the faster train observes that he passes the other in $31\frac{1}{2}$ seconds; find the lengths of the trains.

73. A wine merchant mixes together one pipe (126 gallons) of wine at £80, one at £90 and one at £100, and sells one-third of the mixture at 13*s.* 4*d.* a gallon: at what price per gallon must he sell the remainder so as to gain £$34\frac{1}{2}$ by the transaction?

74. *A* barters sugar with *B*, for rice which is worth $1\frac{7}{8}$ annas a measure but in weighing his sugar uses a false maund weight. *B* discovers this, and to make the exchange fair, raises the price of his rice to $2\frac{1}{2}$ annas a measure. Find the real weight of the false maund which *A* uses.

75. A certain sum put out at compound interest amounts in two years to £270·4 and in three years to £281·216. Find the sum and the rate per cent.

76. A person pays an income-tax of 4*d.* in the £ during the first half of the year and of 3*d.* in the £ during the second half, and finds that owing to an increase in his income he pays the same amount of tax for the second as for the first half of the year. If his gross income for the year is £700, find his net income.

77. A cistern measuring 13 feet in length, 5 feet in breadth and 4 feet in depth has a tap which, not being properly opened, discharges 54 gallons an hour less than it would otherwise do, and empties the cistern in $7\frac{1}{2}$ instead of 6 hours. How many cubic inches are there in a gallon ?

78. Gold costs £3. 17*s.* $10\frac{1}{2}$*d.* per oz., and silver 5*s.* 6*d.* ; in what proportion must these metals be mixed, that a lb. of the mixture may be worth £32. 5*s.* ?

79. A person walks from *A* to *B* at the rate of 3 miles an hour, and after transacting some business which occupies him an hour, returns to *A* by the tramway at the rate of 5 miles an hour. He then finds he has been absent 2 hours 20 minutes. Find the distance from *A* to *B*.

80. Define a decimal ; and show how its value is affected by affixing and perfixing cyphers.

81. If 210 coolies in 7 days of 10 hrs. each dig a channel 1 mile long, 6 ft. broad, and 2 ft. deep ; in how many days of 7 hrs. each should 35 coolies dig a channel 660 ft. long, $7\frac{1}{3}$ ft. broad and $2\frac{1}{4}$ ft. deep ? And how many cubic feet does each cooly dig in an hour ?

82. The expenses of a family when rice is 12 sr. for a rupee are 50 rupees a month : when rice is 14 sr. for a rupee the expenses are 48 rupees a month (other expenses remaining unalterable) ; what will they be when rice is at 16 sr. per rupee ?

83. What are the prime factors in 45090045, and what is the smallest whole number by which it must be multiplied in order to become a perfect square ?

84. The cost of carpeting a room is £7. 4*s.*, and of papering the same room, with paper at $2\frac{1}{2}$*d.* per sq. foot. £10. 12*s.* 6*d.* The length of the room is 18 ft., and if the width had been 4 ft. less the cost of the carpet would have been £1. 16*s.* 0*d.* less. Find the height of the room.

85. Find the sum for which the difference between the simple and compound interest, at 5 P. C. per annum for 3 yrs, is £12. 4*s.* 0*d.*

86. What length of wire will go round the edges of a cube, the surface of which contains 187 yards 54 inches ?

. What is the least number of such cubes which will contain an exact number of cubes whose edges are 1 foot 3 inches ?

87. A merchant's average rate of profit for five years was 5 per cent. on his capital, and for the first four years his average profit was 4 per cent. What was his rate of profit in the fifth year ?

88. If 2 men and 5 women can do a piece of work in 8 days of 9 hours each ; how long will it take 3 men and 6 women to do a piece of work twice as great, working 8 hours a day : the work of a man being double that of a woman ?

89. A merchant buys cloth at such a price that by selling it at R2. 3*a.* per yard he will gain 5 per cent. on his outlay. What percentage will he lose if the cloth be sold at R1. 13*a.* per yard ?

90. A sum of R18240 is remitted to England at the rate of exchange of 1 shilling and $8\frac{3}{4}$ pence per Rupee and is invested in the 3 per cent. consols at 95. Find the yearly income in pounds sterling.

91. A merchant buys in Madras 210 bags of rice at R10-12-0 per bag of 164 pounds. He sends them by rail 320 miles at $6\frac{1}{2}$ pies per ton per mile, but during the journey $7\frac{3}{4}$ pounds are stolen from each bag. Find at how many measures per rupee he must sell the remainder in order to clear R95-15-0 by the transaction. (One measure $= 3\frac{1}{8}$ pounds.)

92. The materials of an old building were sold for R1,500 upon condition that they should be removed within 30 days under a penalty of R10 per day for every day beyond 30 days. The purchaser employed 40 men at $3\frac{1}{2}$ annas per day to do the work, and after selling the materials for R2365, he cleared R190 by his bargain. Find the number of days the men were at work

93. *A* and *B* enter into partnership; *A* supplies the whole of the capital amounting to R45,000 upon condition that the profits are to be equally divided, and that *B* pays *A* interest on half the capital at 10 per cent per annum, but receives R120 a month for carrying on the concern. Find their total yearly profits when *B*'s share is equal to $\frac{1}{2}$ of *A*'s share.

94 Find the difference between the *true* discount on R259·2 due two years hence and the interest on the same sum for two years, allowing in both cases Simple Interest at 4 per cent. per annum

95. A room, 21 feet long by $13\frac{1}{2}$ feet wide, is surrounded by walls $1\frac{1}{2}$ feet thick and 14 feet high. There are two doors each $4\frac{1}{2}$ feet by 6 feet, and one window 3 feet by $4\frac{1}{2}$ feet. Find (1) the cost of building the walls at the rate of R5. 1*a*. 0*p*. per cubic yard and (2) the number of bricks each measuring 9 in. × 4 in. × $2\frac{1}{4}$ in. required for the work.

96. If 38 men working 6 hours a day can do a piece of work in 12 days, find in what time 57 men working 8 hours a day can do a piece of work twice as great, supposing 2 men of the first set to do as much work in 1 hour as 3 men of the second set can do in $1\frac{1}{2}$ hours.

97. A person's net income from 5 per cent. Government paper is R1225 after paying income tax at the rate of 2 per cent. Find the number of shares of R1000 each owned by him.

98. The circumference of a circle being equal to $3\frac{1}{7}$ times its diameter; find the diameter of an engine wheel which makes three revolutions a second when the engine is moving at 40 miles an hour?

99. If 24 men build a wall $2\frac{1}{2}$ miles long, 2 feet broad and 6 feet high, in 146 days of 10 hours each, what length of wall $2\frac{1}{2}$ feet broad and 5 feet high, will 15 men build in 365 days, working 8 hours a day?

100. A person sold 86 measures of rice for R13-7-0, thus gaining 25 per cent.: and 154 measures at a profit of 10 per cent. Supposing he had sold the whole at a profit of 16 per cent., how much more would he have gained?

101. The length of a room is $32\frac{1}{2}$ feet. The cost of painting the walls at R1. 14. 0 per sq. yd. is R308. 2. 0; and the cost of carpeting the room at R2. 4. 0 per sq. yd. is R150 5. 0. Find the height and width of the room.

102. If R32,000 put out at compound interest, amount in 2 years to R34,279$\frac{1}{8}$, what is the rate per cent. ?

103. A person leaves R6,780 to be divided among his 5 children and 4 brothers, so that after the legacy duty has been paid, each child's share shall be twice as great as each brother's share. The duty on a child's share is one per cent. and on a brother's share four per cent. Find what amounts they respectively receive.

104. A room measuring 42 ft. 6 in. by 22 ft. 9 in. inside, with walls 2 ft. 3 in thick, is surrounded by a verandah 10 ft. 6 in. wide. Find the cost of paving this verandah with tiles measuring 4$\frac{1}{2}$ in. by 3 in. and costing R3. 2*a*. 0*p*. per hundred.

105. A bankrupt has book debts equal in amount to his liabilities; but on R8,640 of such debts he can recover only 8$\frac{1}{3}$*a*. in the rupee, and on R6,300 only 5$\frac{1}{4}$*a*. in the rupee. After allowing R1,054. 11*a* 0*p*. for the expenses of bankruptcy, he finds he can pay his creditors 12*a*. in the rupee. Find the total amount of his debts.

106 A sum of money in 10 years at 3$\frac{1}{2}$ per cent. Simple Interest amounts to R727. 0*a* 6*p*. In how many years would it amount to R840. 2*a*. 0*p*. at 4 per cent. ?

107. Find the cost in rupees of one mile of railway, which consists of two rails each weighing 40 lbs. per yard on wooden sleepers weighing 70 lbs. each placed 2 ft. 8 in. apart. The rails cost in England £6. 13*s*. per ton and the sleepers 2*s*. 4$\frac{1}{2}$*d*. each. The rate of freight is £1. 5*s*. per ton and landing charges amount to R2. 8*a*. per ton. Rate of exchange 1*s*. 8*d*. per rupee.

108 For what sum should a cargo worth R26,315 be insured at 7$\frac{2}{3}$ per cent., so that the owner may recover in case of loss the value both of the cargo and the sum paid for insurance ?

109 Two trains measuring 330 feet and 264 feet respectively, run on parallel lines of rail. When travelling in opposite directions they are observed to pass each other in 9 seconds, but when they are running in the same direction at the same rates as before, the faster train passes the other in 27$\frac{1}{2}$ seconds. Find the speed of the two trains in miles per hour.

110. The wheels of a cart are 13 ft. 6 in. in circumference. One breaks down and is replaced by a new one, which is rather small. To test it, the owner makes a chalk mark on each wheel where it touches the ground and tells his man to drive over a piece of level road, and to count the turns made by each wheel until the chalk marks next touch the ground at the same time. The man obeys : but when he returns to his master, can only recollect that one wheel made one more turn than the other. His master, however, measures the distance traversed by the cart, 360 yds., and thence finds the circumference of the new wheel. What is it ?

111. A bank advances R1,500 to a person on agreement that interest at the rate of 9 per cent. per annum shall be paid half yearly for its use. The person fails to make any interest payment, and at the end of eighteen months, the Bank obtains judgment against him for the principal and compound interest at the rate and on the terms agreed to. Find to the nearest pie the amount he has to pay.

112. The roof of a verandah is supported by 16 teak beams, each 9 ft. long, 3 in. broad, and 5 in. deep. If the weight of a cubic inch of teak is $\frac{18}{25}$ of that of a cubic inch of water, and if a cubic foot of water weighs 1,000 oz., find the weight in lbs. of the timber in the verandah.

113. A cistern, whose capacity is 43,092 gallons, is to be filled with water by a pipe which conveys 23 gallons 1 qt. per minute. On account of a leakage the cistern is only just filled in $31\frac{1}{3}$ hours. What is the average amount of leakage per hour?

114. If 40 men and 50 boys can do a piece of work in 6 days, working 6 hours a day, in how many days will 8 men and 20 boys do a piece of work half as large again, working 7 hours a day, assuming that a man does as much work in 3 hours as a boy in 5 hours?

115. *A* and *B* start on a journey at the same time. *B* travels at $\frac{4}{7}$ths of *A*'s rate, and arrives 3 hours 15 minutes after him. In what time did each complete the whole journey?

116. If an investment of £75 becomes £78. 15*s*. 0*d*. in 8 mo., what sum invested at the same rate of interest will become £201. 17*s*. 6*d*. in 10 mo?

117. *A* and *B* started on a race and ran a certain distance exactly together. Then *B* began to fail and gave up the race when he had run 56 yards further, *A* having gone during the same time 320 yards. The average of the entire distance run by the two men was 1,188 yards. What distance had they run together?

118. A tea-merchant has a rectangular space for storing tea. It is $15\frac{3}{4}$ ft. long, $10\frac{1}{2}$ ft. broad and $9\frac{1}{3}$ ft. high. He wishes to fill this space with packets of a cubical shape all of the same size. What is the largest size of such cubical packets that can be made to fill it exactly, and what would be the number of such packets?

119. *A* starts in business at the beginning of the year with R3,000. On March 1st, he takes a partner *B* with R4,000. And on June 1st, he receives another partner *C* with R5,000. The profits at the end of the year amount to R180. What share of the profits should each partner receive? And what is the rate per cent. per month of the profits on the capital invested?

120. A tradesman has been accustomed to give his customer three months' credit but wishes to introduce the ready money system into his business. For how much ready cash should he sell an article that he has hitherto sold for £8-2-0, the rate of interest charged being 5 per cent. per annum?

121. What rate per cent. will be received for money invested in $3\frac{1}{2}$ per cent. stock at 84?

122. Find the cost of building the walls of a rectangular room, 20 ft. long, 16 ft. broad, and 10 ft. high, with a door 7 ft. by 4 ft. and a window 5 ft. by 3 ft.. at $2\frac{1}{2}$ annas per cubic foot, the walls being 2 ft. thick.

123. If the rupee is worth 1*s*. $6\frac{3}{4}$*d*., express R6-5-4 as a fraction of £1; and find the least number of rupees equal in value to an integral number of pounds.

124. The carriage of $17\frac{1}{2}$ cwt. for 52 miles on a certain railway is 8*s*. 4*d*., find what will be the cost of carrying $4\frac{3}{4}$ cwt. for 300 miles on a railway on which the rate per mile is 9 per cent. lower.

125. A landlord pays 1 per cent. for collecting his rents and a tax of 7 pies in the rupee on what he receives after paying the collector. He has a clear rental of R1,831-8-0. Find his gross rental.

126 A grocer mixes four kinds of a tea which cost him 5*s*., 4*s*., 3*s*., 2*s*. per lb. respectively, in the proportions of 2, 3, 4, 7 respectively. Find at what rate he must sell the mixture so as to gain 25 per cent on the whole.

127. Define the terms *interest*, *discount*, and find in what time £533-6-8 will amount to £672 at $6\frac{1}{2}$ per cent. per annum Simple Interest.

128. A person invests £4,800 in 4 per cent. stock at 96, and after a year sells out at $92\frac{1}{2}$ and invests the proceeds together with the interest for the year in stock at $96\frac{1}{2}$. How much stock does he then purchase ?

129 Find to four places of decimals the square root of $\frac{137}{162}$; and calculate the cost of surrounding with a fence a square field of $22\frac{1}{2}$ acres at 3*d*. per yard.

130. The population of a country increases at the rate of 7 per cent. every 10 years. What was the population 20 years ago of a country whose present population is 4,007,150 ?

131. A clock which gains 3*m*. 56*s*. in 24 hours was set correctly at noon on the 1st of January 1884. Find to the nearest minute the next date at which it indicated correct time.

132. Twenty men are employed to make a tank 40 ft. long, 20 ft. broad and 6 ft deep They work for 30 days and have just completed one-third of the work, when it is resolved to increase the length of the tank by 10 ft., the breadth by 4 ft. and the depth by 2 ft. How many additional men must be employed in order that the work may be completed in 30 days more ?

133 The capital of a certain railway is £1,000,000 in 20,000 shares of £50 each, fully paid up. The gross annual receipts are £105,000, of which 48 per cent is absorbed in working expenses, £4,600 goes to the reserve fund, and the remainder to pay dividend. Find what annual income a person will obtain from the investment of £4,500 in the undertaking, the shares being at £62-10-0.

134. Ice is manufactured for 6 pies a pound and sold for 9 pies a pound. Two-thirds of the quantity made is kept for sale at the factory and the remainder is sent to branch shops. If the average loss from melting of the former be $12\frac{1}{2}$ per cent and that of the latter be 25 per cent., find the gain on every ton made.

135. The average width and depth of a river at its mouth are 240 yards and 6 feet respectively, the average rate of flow is 3 miles per hour and the amount of sediment per cubic foot of water discharged is $1\frac{1}{4}$ cubic inches. Find the amount of sediment deposited annually ; and the depth of the deposit, supposing it spread uniformly (*i. e.*, to the same depth throughout) over an area of 146 square miles.

136. When exchange is at the rate of 1*s*. $4\frac{1}{2}$*d*. per rupee, a person in Madras orders from a bookseller in England a parcel of books, the pub-

lished price of which is £5. The bookseller allows discount at the rate of 25 per cent. on the published price, but includes in his bill a charge of 13*s.* for packing, freight, &c. When the books arrive in India, a further sum of R2-8 has to be paid on account of landing charges and cost of delivery. If the books can be obtained from a bookseller in Madras at the rate of $9\frac{1}{2}$ annas per shilling of the published price, find how much the person loses by ordering from England.

137. A person holds forty R500-shares in a concern which pays dividend at the rate of 6 per cent. per annum. When the shares are at R675, he sells out and invests half the proceeds in 4 per cent. stock at 90. With the other half he buys a house, for which he receives an annual rental of R1,440 subject to a deduction of 3*a.* 9*p.* per rupee for repairs and taxes. Find the alteration in his annual income.

138. In a certain year a country produces 50,000,000 bushel of wheat. Of this quantity 40 per cent. is available for export at R3-2 per bushel. In the following year the acreage under wheat has increased 20 per cent., but the yield per acre is only seven-eighths of what it was in the previous year, while the quantity required in the country has increased 5 per cent. If at the same time the export price has fallen to R3 per bushel, find the increase in the value of the wheat available for export.

139. The population of a country is 33,264,000, and there are 99 males to 101 females; 2 out of every 11 boys and 1 out of every 33 girls of school-age are under instruction. If the boys of school-age form one-seventh of the male population and the girls of school-age from one-seventh of the female population, find the total number of pupils under instruction.

1889.

2. Simplify $\frac{(\frac{1}{3}+\frac{1}{7})+(\frac{1}{7}-\frac{1}{9})}{(\frac{1}{3}+\frac{1}{7})+\frac{1}{7}-\frac{1}{9}}+\frac{1\frac{7}{8}+\frac{3}{25}}{\frac{7}{12}-\frac{1}{7}}-\frac{\frac{1}{2}\text{ of }\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}$.

3. Multiply $41\cdot36\dot{5}1\dot{4}$ by $\cdot001\dot{9}$ expressing the result as a decimal; and find the value of $\cdot347\dot{2}$ of £1. 4*s.*—$\cdot03\dot{2}8\dot{8}$ of £2. 6*s.* 3*d.*

4. Find by any method the cost of 79 ca. 17 m. 5 v. 25 pal of salt at R21. 10*a.* 8*p.* per candy.

5. The cost of rice for a family of 2 adults and 3 children from Jan. 1st 1889, to Dec. 11th, 1889, both days inclusive, during which time rice was selling at 15·4 sr. per R, was R70. 7*a.* What will be the cost of rice for a family of 3 adults and 5 children from Dec. 19th, 1889, to May 11th, 1890, both days inclusive, assuming that the price of the rice will be 14·7 sr. per R, and assuming also that the quantity required per day by each adult is the same in both cases, and that in both cases, the quantity required by a child is $\frac{2}{3}$ of the quantity required by an adult?

6. On what sum due 1 yr. 4 mo. hence does the true discount amount to £100-18-9, Simple Interest being reckoned at $4\frac{3}{4}$°/ₒ per annum?

7. How much 3 per cent. stock must a person sell when the selling price is 91 in order that by investing the proceeds in the $4\frac{1}{2}$ per cents. at $113\frac{3}{4}$ he may derive from the investment an annual income of R9817. 8*a.* after paying income-tax at the rate of 5*p.* per rupee?

8. *A* and *B* can do a piece of work in 10 days, *B* and *C* in 15 days, and *C* and *A* in 20 days. They all work at it for 6 days; then *A* leaves and *B* and *C* go on for 4 days more. If *B* then leaves, how long will *C* take to complete the work?

9. In a certain year the total amount received by a railway company for the carriage of passengers was R2,751,000. Of this sum 6 per cent. was contributed by first class passengers, 5 per cent. by second class, and the remainder by third class. The fares were 18, 6, and $1\frac{3}{4}$ pies per mile for first, second and third class passengers respectively. Assuming that the average distance travelled by each third class passenger was 36 miles and the average distance travelled by each passenger of the other classes was 160 miles, find the total number of passengers carried during the year.

10. The length of a rectangular field is twice its breadth. If the rent of the field at £3 7*s.* 6*d.* an acre is £151. 17*s.* 6*d.*, find the cost of surrounding it with a fence at $4\frac{3}{4}$*d.* per yard.

11. Extract the cube root of 9 to 5 decimal places.

1890.

1. Reduce 2149908480 sq. in. to acres, &c. If this is the area of a rectangle the length of which is 5 mi. 7 fur. 5 p. 1 ft. 6 in., find its breadth.

2. Simplify $\frac{1835}{2202}+\frac{5468}{12303}+\frac{147}{441}-3\frac{1}{7}$ of $\frac{6\cdot25}{5\cdot5}$ of $\frac{\cdot04}{1\cdot\dot{2}8571\dot{4}}$

3. Find the value of 237 ca. 17 mds. 6 v. @ R4100. 1*a.* 4*p.* per candy.

4. 300 coolies are set to build a tank-bund. In 14 weeks they have done $\frac{7}{10}$ of the work, when rain stops the work for 4 weeks and washes away $\frac{2}{7}$ of what they have done. At the end of that time the work is resumed with only 250 coolies. In what time from the commencement will the work be finished?

5. Find the amount of R58,59,375 for 3 years at $4\frac{1}{6}$ per cent. per annum, reckoning Compound Interest.

6. Explain the difference between Discount and Interest. If the discount on £2830. 15*s.* $7\frac{1}{2}$*d.* be equal to the simple interest on £2784. 7*s.* 6*d.* for the same time, find the time, the rate of interest being 5 per cent. per annum.

7. A person invests £34,539 in the 3 per cents. at 87. After receiving one year's dividend he sells out at 89. He then invests the whole in Railway stock paying 5 per cent. at 115. What will the difference in his income be?

8. A cistern 10 ft. 6 in. long by 7 ft. 6 in. wide by 3 ft. 4 in. high is lined inside with lead, 7 lbs. of which cover a square foot. Find the weight of the lead and its cost at 53*s.* 4*d.* per cwt.

9. A cask contains 16 gal. of spirits, 2 gal. are drawn off and the cask filled up with water. 2 gal. are again drawn off and the cask filled up as before. This is done a third time. Compare the quantities of spirits and water remaining in the cask.

10. Find the square root of 379749833·583241.

1891.

2. Subtract 13 times R17. 6*a.* 11*p.* from 17 times R13. 6*a.* 11*p.*

3. R330. 3*a.* 7*p.* are to be divided among 193 persons, two of whom receive R2 each, and 10 R3 each. The others receive equal shares. Find the value of each share.

4. Find the value of $\dfrac{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}}{\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}} \times 3\frac{1}{2} \div \dfrac{\frac{1}{15}+\frac{1}{20}+\frac{1}{24}}{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}-\frac{1}{6}}$ and simplify (without reduction to vulgar fractions if you can)

$$2{\cdot}03+1{\cdot}3\dot{4}\dot{5}+27{\cdot}34+16{\cdot}\dot{2}31\dot{7}.$$

5. How long will it take to walk round a square field 14 acres 1 rood 24 poles in extent at the rate of 3 miles an hour ?

6. Find the cost of white-washing a room $22\frac{1}{2}$ ft. by 12 ft and 11 ft. high, at one anna per square yard, making allowance for four windows each 4 ft. × $2\frac{1}{2}$ ft and two doors each $8\frac{1}{2}$ ft. × 4 ft. Find also the cost of a carpet for the same room with 3 ft. border all round the carpet, costing R4 per square yard and the border R6 per square yard.

7. Find the compound interest on £3143. 6*s.* 8*d.* for 3 years at 3 per cent. per annum.

8. A cistern can be filled by three pipes in 30, 40, and 60 minutes respectively, and emptied by an escape pipe in half an hour. The three taps are turned on at noon, but the escape pipe is at the same time accidentally left open and not closed for a quarter of an hour. At what time will the cistern be full ?

9. I purchase 16 lbs. of tea at 1*s.* 7*d.* per lb., 14 at 2*s.* 2*d.* and 17 at 1*s.* 8*d.* Seven pounds of the mixture becoming spoiled have to be sold at a low price, but by selling half the remainder at 2*s.* 4*d.* per lb. and the other half at 2*s.* $7\frac{1}{2}$*d.*, I eventually make a profit of 25 per cent on the original outlay. At what price per pound was the spoiled tea sold ?

10. A person invests a sum of money in the 4 per cents. at 102. When they have risen to 104, he transfers R6000 stocks to another investment paying 5 per cent. of which the shares are at 120. When the 4 per cents. fall to par, he transfers the remainder to the 5 per cents. which are still at the same price and now finds his income R25 more per annum than it was at first. What was the sum originally invested ?

1892.

2. Simplify $\dfrac{\frac{5}{7}\text{ of }1\frac{3}{8}-\frac{5}{8}\text{ of }\frac{9}{7}}{1-\frac{1}{7}\times(\frac{5}{12}+\frac{1}{3})} \times \dfrac{\frac{1}{3}+\frac{1}{7}+(\frac{1}{7}-\frac{1}{6})}{(\frac{1}{3}+\frac{1}{7})\div(\frac{1}{7}-\frac{1}{9})}$.

3. Find the value of $\cdot041\dot{6}$ of £33. 7*s.* 6*d.*—$\cdot03\dot{4}\dot{5}$ of £32. 13*s.* $1\frac{1}{2}$*d* ; and express R371. 2*a.* 6*p.* as the decimal of a lakh of rupees.

4. Find by any method the cost of making a road 37 m. 6 f. 31 p. 3 yd. long at R1785. 3*a.* 4*p.* per mile.

5. Find the present value of £4822. 6*s.* $10\frac{1}{2}$*d.* due three years hence at 5 per cent. per annum Compound Interest.

6. Extract the sq. root of 13·697142031225 to 6 places of decimals.

7. The annual rainfall of a district is 49·7 inches. Assuming that the fall is distributed uniformly over the district, and that a cubic foot of water weighs 62·5 lbs., find the weight in tons of the rain that falls throughout the year on a square mile.

8. When exchange is 1*s.* $2\frac{5}{8}$*d.* per rupee a Madras bookseller sends to London publisher a bill for £104 in payment of books ordered. Freight and landing charges amount to R37. 8*a.* The publisher allows the book-seller discount at the rate of 35 per cent. on the published price and the latter sells the books at the rate of $10\frac{1}{2}$ annas per shilling of the published price. Find how much he gains on the transaction.

9. In the year 1891, the cost of rice for a family of 2 adults and 4 children was R86. 7*a.* 9*p.* In that year rice sold at 11·2 seers per rupee, and each child received two-fifths of the amount given to an adult. Assuming that in 1893 the price of rice will be 13·5 seers per rupee, what will be the cost of rice for the same family from January 5 to August 11 both days inclusive, if the allowance of each adult be increased by one-fourth and the allowance of each child be three-sevenths of that of an adult?

10. The capital of a railway company amounts to R1,890,00,000 of which one-fourth is 5 per cent. preference stock and one-third $4\frac{1}{2}$ per cent. preference stock. In a certain year the receipts are R1,81,50,000, and the working expenses amount to 55 per cent. of receipts. Of the next receipts R540,000 are added to the reserve fund, and the remainder, after paying dividend on the preference stock, is divided among the ordinary shareholders. What rate of interest will they receive?

11. In the ten years from 1871 to 1881 the population of a country increased at the rate of 9·5 per cent., and in the ten years from 1881 to 1891 the rate of increase was 10·5 per cent. If the population in 1891 was 31,023,759, find what it was in 1871.

1893.

2. Simplify $\dfrac{\frac{2}{3}-\frac{1}{5}\text{ of }\frac{7}{8}+\frac{8}{9}}{\frac{4}{9}+\frac{8}{7}-\frac{1}{3}+\frac{2}{5}}-\dfrac{\frac{6}{11}-\frac{4}{9}}{1-\frac{5}{9}+\frac{3}{5}}$.

3. Find the value of $2·0475\dot{2}$ of £2. 2*s.* 1*d.*—1·734375 of £2. 6*s.* 8*d.*

4. Find by any method the value of 59 ca. 14 m. 7v. 27 pal. of salt at R26. 10*a.* 8*p.* per candy.

5. In a certain year the produce of a tea-estate was sold in London at an average rate of $9\frac{1}{2}$*d.* per lb., and the amount realised was remitted at an average rate of exchange of 1*s.* $2\frac{1}{4}$*d.* per rupee. In the following year the average price realised was only $8\frac{3}{4}$*d.* per lb., but the quantity sold exceeded by $12\frac{1}{2}$ per cent. the quantity sold in the previous year and the average rate of exchange at which remittances were made fell to 1*s.* $1\frac{1}{8}$*d.* If in this year the total amount realised from sales in London was R105000, find how much was realised in the previous year.

6. A sum of money was invested for four years, interest payable annually. The rate of interest was 5 per cent per annum for the first two years.

and 4 per cent., per annum for the last two; and the amount at the end of four years was £1,164. 10s. $3\frac{3}{4}d$. What was the sum invested?

7. Ice is manufactured for $2\frac{1}{2}$ pies per lb., and sold at 6 pies per lb. Of the total quantity made one half is kept for sale at the factory, and the remainder sent to branch shops. The loss from melting is $12\frac{1}{2}$ per cent. in the case of the former and 25 per cent. in the latter; and the agents at the branch shops receive commission at the rate of 15 per cent. on the price of every pound sold by them. Find the profit on every ton of ice manufactured.

8. Two persons, *A* and *B*, set out together on a journey. They walked at the rate of 3 miles an hour; and after they had proceeded for three quarters of a mile, *B* returned, walking at the same rate, to the place of starting. Here he was detained three quarters of an hour. Setting out again he overtook *A*, who had been walking all the time, at the end of $2\frac{1}{2}$ hours from the second time of starting. At what rate did he walk?

9. A person sold 25 Bank of Madras shares and invested the proceeds in the Government $3\frac{1}{2}$ P. C. when they were at $3\frac{1}{4}$ premium. If his net annual income from the investment, after paying income-tax at the rate of 5*p*. in the R. be R876. 9*a*., find the price at which he sold each of his bank shares.

10. In the year 1891 the population of a country was 35640000 and there were 1025 females to every 1000 males. Of the total population 7·5 per cent. could read and write, but of the females only 1 per cent. could do so. Find what percentage of males could read and write.

11. Extract the square root of 81·13183159704101 to seven places of decimals.

BOMBAY ENTRANCE PAPERS.

SELECTED QUESTIONS 1859-87.

1. Define a fraction: and explain the effect on the value of a fraction of adding the same number to the numerator and the denominator. Why do you bring fractions to the same common denominator before adding them together?

2. Define a decimal, and reduce 14 minutes to the decimal of a day.

3. The top of a tank is a rectangle, whose sides are 9 ft. and 15 ft.; it is of the same horizontal section throughout its depth. What must be its depth in order that it may contain 12960 gallons of water, one gallon containing 277·274 cubic inches?

4 The sum of R6000 is to be divided among 24 men, 36 women and 72 children, so that the shares of 2 men shall be equal to those of 3 women, and each woman's share to the shares of 2 children What will be the share of each?

5. State the distinction between Direct Proportion and Inverse Proportion; and find how much land at 27*s*. per acre should be given in exchange for 480 acres at 35*s*. per acre.

6. In multiplication of decimals how do you determine the position of the decimal point in the product? State the reason of the rule.

7. *A* buys 200 shares in the G. I. P. Railways at R1,000 each and when they are paying 2 per cent., sells them at R460 each and invests the proceeds in the $4\frac{1}{2}$ per cent. Government loan at 92. Find the effect on his income.

8. What will 3650 rupees amount to in 4 years and 2 months at R3. 6*a*. 8*p*. per cent. per annum at Simple Interest? In what time would a sum of money double itself at the above rate?

9. If a cubic foot of marble weigh 2·716 times as much as a cubic foot of water, find the weight of a block of marble 9 ft. 6 in. long, 2 ft. 3 in. broad and 2 ft. thick, supposing a cub. ft. of water to weigh 1000 oz.

10 The surface of a cube is 346·56 sq. ft., what is the length of an edge?

11. How is it that the value of a decimal fraction is not altered by adding on the right hand any number of cyphers.?

12. What sum must *A* bequeath to *B* so that *B* may receive R10,000 clear, after deducting a legacy duty of 10 per cent.?

13. A bag contains a certain number of rupees, half as many again two-anna pieces, and 4 times as many pysas, and the value of the whole is R300; how many rupees, how many two-anna pieces and how many pysas are there?

14. A creditor receives upon a debt of ₨3,270 a dividend of 9*a.* 2*p.* in the rupee, and afterwards he receives a further dividend on the deficiency of 3*a.* 4*p.* in the rupee ; how much does he receive on the whole ?

15. Find the true present value of two sums of ₨100 payable at the end of one year and two years respectively, money making $7\frac{1}{2}$ per cent. per annum.

16. If mangoes be bought at the rate of seven for an anna, how must they be sold to gain 33 per cent. ?

17. Four French feet are equal to 1·3 metres, and 15 French feet are equal to 16 English feet ; how many metres are 27 English feet equivalent to ?

18. Explain the principle of the Decimal System of Numeration. What number expressed in the Decimal System, is identical with the number 4321, in which the base of the system of numeration is 12 ?

19. *A* barters some sugar with *B* for flour, which is worth 2*s.* 3*d.* per stone, but uses a false stone-weight of $13\frac{1}{2}$ lbs. ; what value should *B* set upon his flour, that the exchange may be fair ?

20. An annual tax of ₨2,255 is laid upon a district containing four villages,—*A*, *B*, *C*, *D*,—and the rate to be paid by each of the villages *A*, *B*, and *C* is to the rate to be paid by *D*, as 3 to 2 ; what are the annual payments due from the villages ?

21. Explain the following terms—*an improper fraction*, *a compound fraction*, *a mixed number.*

22. The area of the entire surface of a pond is 9 acres, 2 roods, 15 poles ; find to 3 places of decimals, the number of yards in the side of a square piece of ground of equal area.

23. A man sells a horse for ₨246 and loses $26\frac{1}{3}$ per cent. on what the horse cost him ; what was the original cost ?

24. Explain the difference between *interest* and *discount* ; and find the discount on £397-6-8 due 9 mo. hence, at 4 per cent. per annum.

25. If the carriage of 150 feet of wood, that weighs 3 stones per foot cost ₨30 for 40 miles, how much will the carriage of 54 feet of wood, that weighs 8 stones per foot, cost for 25 miles ?

26. Express in the scale of 8, the number seven hundred and eighty-four millions three-thousand and forty-two.

27. Demonstrate the rule for pointing the quotient in the division of Decimal Fractions.

28. The proportions used in making English gunpowder are saltpetre 75 parts, sulphur 10 parts and charcoal 15 parts. How many pounds weight of each material are there in 10 cwt. of gunpowder ?

29. *A*, *B* and *C* form a Joint Stock of ₨75000, of which ₨36000 are contributed by *A*, ₨30000 by *B* and the remainder by *C*. At the end of the year, the profit is found to be ₨16791. Required the shares of this which each is to receive, ₨800 a month being allowed as salary to *C* as acting partner.

30. If 12 iron bars, each 4 feet long, 3 inches broad and 2 inches thick, weigh 576 lbs., how much will 11 weigh, each 6 feet long, 4 inches broad and 3 inches thick ?

31. If a room is 28 feet long, 20 feet wide, 13 feet high, and the windows and doors take up half the walls, find the cost of papering at 12*a*. a sq. yard.

32. If I sell R500, 4 per cents. at 93, and buy $5\frac{1}{2}$ per cents. at 109, what is the change in my income ?

33. Divide a lakh of rupees between *A*, *B*, and *C* in the proportion of 2, 3, 4, and the same amount between *D*, *E* and *F*, in the proportion of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.

34. If I sell 40 shares of R250 each in the Oriental Bank at 121 per cent. premium, how many shares of R1000 each in the Madras Bank at 72 per cent. premium can I buy ? And how much will be left ?

35. A person travelled 120 miles by railway at 15 miles an hour, 120 by road at 8 miles an hour and 60 by bullock cart at 2 miles an hour : how long did he take ?

36. Define the terms :—Stocks, shares, consols. State some of the circumstances which affect their value in the market.

37. How much stock can be purchased by the transfer of R2000000 from the 4 per cents. at 90 to the $5\frac{1}{2}$ per cents. at 110 ; and what change would be effected in the income derived from the two investments ?

I bought cloth at 15 annas a yard, and lost 5 per cent in selling ; what was it sold for ?

38. If a person owe R100 payable in 2 mo. and R750 payable in 7 mo., what is the just time for the payment of the two debts ?

39. Give a demonstrative example, illustrative of the following axiom :—

If the divisor be increased a certain number of times, the quotient is diminished in the same degree ; but if the divisor be diminished, the quotient is increased.

40. Define *prime* and *composite* numbers. Resolve 54180 into prime factors.

41. Find the product by contracted multiplication of 72·49 and 19·87632 to three places of decimals.

42. Explain *direct* and *inverse* proportion.

43. 250 men are employed to work on a railway embankment, a mile and a half long which they are expected to finish in four weeks. But at the end of one week it is found that they have only finished 520 yards. How many more men must be engaged to finish it in the required time ?

44. What time must elapse between the time of placing R250 in the Government Savings Bank and taking out the amount just as it goes over R300, supposing interest to be at 5 per cent. per annum, Compound Interest ?

45. What is the cost of a marble slab, 6 ft. 3 in. long, 2 ft. 8 in. broad and 4 in. thick, at R7. 8*a*. per cubic foot ?

What is the weight of the slab, one cubic foot weighing 170 lbs. ?

46. If £442 amount to £530. 8*s*. in 5 years, what is the rate per cent. of Simple Interest ?

47. If 27 men take 15 days to mow 225 acres of grass, how long will 33 men take to mow 165 acres ?

48. A person has R100000 stock in Government 4 per cents. ; he sells out all his stock at 92½, and then re-invests the purchase money in Bank of Bombay Shares of R500 each, at R625 which pay 6 per cent. per annum ; find the alteration in his income.

49. If a family of 9 persons spend R4800 in 8 mo., how much will serve a family (living upon the same scale) of 24 persons for 16 mo. ?

50 A ship worth R9000 being entirely lost, of which one-fourth belonged to *A*, one-sixth to *B*, and the remainder to *C* ; what loss will each sustain, supposing R5400 of the ship were insured ?

51. How much stock in the 3 per cents. must I sell to pay off a debt of £550, the price of the stock being 94⅝, and the commission of ⅛ on £100 of stock being also taken into consideration ?

52. Define a fraction, and prove that the value of a fraction is not altered if we multiply both its numerator and denominator by the same whole number.

53. State and prove the rules for reducing terminating and circulating decimals into their equivalent vulgar fractions. *Ex*. ·03125 and ·72$\dot{9}$

54 If 10 horses and 98 sheep can be kept 9 days for £37 17*s*. 6*d*., what sum will keep 45 horses and 216 sheep for 40 days, supposing 5 horses to eat as much as 76 sheep ?

55. If the par of exchange be two English shillings for the Indian rupee, but if an Indian bill of exchange for R540. 12*a* be negotiated in London for £51. 10*s*. ; how much per cent. below par is the rate of exchange ?

56. Distinguish between interest and discount. The interest on a certain sum of money for three years is R825, and the discount for the same time is R645, Simple Interest being reckoned in both cases. Find the rate per cent. per annum and the sum.

57. A person desires to paper his room with postage stamps ; the room is 14 ft. 9 in. long, 9 ft. 3 in. broad and 10 ft. 6 in. high ; it contains two windows each 5½ ft. by 4 ft. and 3 doors each 6 ft. by 3 ft. ; a postage stamp is $\frac{15}{16}$ in. long and ⅞ in. broad. Find the number of postage stamps required to cover the room.

58. A person invests 1250 gold mohurs in the Government five per cent. rupee stock at 105. The stock is converted subsequently to 4½ per cents. at 95. Find the difference in his income, each gold mohur being considered equivalent to R17.

59. A certain number of persons agree to subscribe as many pies each as there are subscribers ; the whole subscription being R5797. 0*a*. 1*p*., how many subscribers were there ?

60. If the income-tax be 6 pies in the rupee for the first half of the year and 3 per cent. in the second, what is the gross income of a gentleman whose net annual receipts amount to R1454. 1*a.* ?

61. Five men do $\cdot 600\dot{6}$ of a piece of work in 2·12 hours ; how long will 6 boys take to finish it, it being known that 3 men and 7 boys have done the whole piece of work in 3 hours ?

62 If the difference between the simple and compound interest on a sum of money for 2 years at 5 per cent. be £5. 18*s.* $9\frac{3}{4}d.$, find the sum.

63 When the 3 per cents. were at 90, I found that by selling out and investing in the four per cents. at 95, I could improve my income by R243 What was the amount of my stock in the three per cents ?

64 A gardener plants an orchard with 5776 trees and arranges them so that the number of rows of trees equals the number of trees in each row. How many rows were there ?

65. How many seconds will a train 184 feet in length, travelling at the rate of 21 miles an hour, take in passing another train 223 feet long proceeding in the same direction at the rate of 16 miles an hour ?

66 In a subscription list one-half of the subscriptions are a guinea each, one-third a half guinea each, and the 5 shilling subscriptions which complete the list amount to £12 ; find the whole amount subscribed.

67. If the work done by a man, a woman and a child be in the ratio of 3, 2, 1, and there be in a factory 24 men, 20 women, and 16 children whose weekly wages amount to R204 ; what will be the yearly wages of 27 men, 40 women, and 15 children ?

68. The debts of a bankrupt amount to £2134 10*s.* 6*d.*, and his assets consist of property worth £916. 15*s.* 4*d.*, and an undiscounted bill of £513 due 4 months hence, Simple Interest being reckoned at 4 per cent. How much in the pound can he pay his creditors ?

69 A merchant buys 4,000 maunds of rice, one-fifth of which he sells at a gain of five per cent , one-fourth at a gain of ten per cent., one-half at a gain of twelve per cent., and the remainder at a gain of sixteen per cent. If he had sold the whole at a gain of eleven per cent , he would have made R728 more What was the cost of the rice per maund ?

70. The shares in a banking concern are R1000 each ; R426. $10\frac{2}{3}a.$ only are paid up, and the shares are quoted in the market at R460. The dividend is $R7\frac{1}{2}$ per share quarterly. A gentleman holds 100 original shares. Find what interest he makes per cent., and what he would make and how much per cent., if he sold out and invested in 4 per cent. Government stock at par.

71. *A* and *B* are the termini of a Railway 144 miles long. A fast train starts from *B* at 9 h. 0 m. ; another fast train, travelling at the same rate, starts from *A* at 10 h. 0 m. A slow train starts from *B* at 10 h. 20 m. ; the fast train from *A* meets the other fast train at 11 h. 30 m., and the slow train at 12 h. 32 m. ; find the rates at which the trains travelled.

72. Explain what is meant by the following words and give examples :—

Measure, Multiple, Greatest Common Measure, Least Common Multiple.

73. What are *continued fractions*, and when do you make use of them? Find three fractions approximating to $\frac{134}{202}$.

74. Find the product of 17·302 and ·579 to three places of decimals, by the rule of the contracted Multiplication.

75. What sum will discharge a debt of R7,200 due a year and a half hence, at 4 per cent. per annum?

76. Divide a guinea between *A*, *B*, *C*, *D*, so that *B*'s share is $\frac{1}{3}$ more than *A*'s, *C*'s $\frac{1}{3}$ more than *B*'s and *D*'s $\frac{1}{3}$ more than *C*'s.

77. How much stock can be purchased by the transfer of R20,000 stock from the 3 per cents. at 90 to the $3\frac{1}{2}$ per cents. at 96; and what change will be effected in income by it?

78. Define the arithmetical terms—*notation*, *numeration*, *unit*, *integer*, *fraction*, *abstract*, *concrete*. Can you (1) multiply concrete numbers together? (2) divide a concrete number by a concrete number? Give examples to illustrate the nature of such operations.

79. A lb. of tea and 3 lbs. of sugar cost R3, but if sugar rose 50 per cent. and tea 10 per cent., they would cost R3. 8*a*.; find the prices per lb. of tea and sugar.

80. *A*, *B* and *C* rent a field for R2,878. *A* puts in 12 horses for 5 months and 45 sheep for 3 months: *B* puts in 15 oxen for 6 months and 54 sheep for 2 months; *C* puts in 6 horses and 48 oxen, for 3 months. Now 4 horses and 3 sheep together eat as much as 5 oxen, and 1 horse and 2 oxen eat as much as 7 sheep: how much of the rent should *A*, *B*, *C*, pay respectively?

81. What sum of money will amount to 699*l*. 13*s*. 2·4*d*. in 2 years, reckoning compound interest for the first year at 4 per cent. and for the second $3\frac{1}{2}$ per cent. per annum?

82. A person finds that if he invest a certain sum in railway shares paying £6 per share when the £100 share is at £132, he will obtain £10. 16*s*. a year more for his money than if he invest in 3 per cent. consols at 93 What sum has he to invest?

83. A man near the sea-shore sees the flash of a gun fired from a vessel steaming directly towards him, and hears the report in 15″. He then walks towards the ship at the rate of 3 miles an hour, and sees a second flash 5 minutes after the first, and immediately stops; the report follows n 10·5″. Find the rate of the ship, the velocity of sound being 1,200 ft. per second.

84. A bankrupt has goods worth R9750, and had they realized their full value, his creditors would have received 13*a*. in the rupee: but $\frac{3}{8}$ths were sold at 17·5 per cent., and the remainder at 23·75 per cent. below their value. What sum did the goods fetch, and what dividend was paid?

85 What sum will amount to £1,591. 13*s*. 2·16*d*. in 3 years at Compound Interest; the interest for the first, second and third years being 3, 2 and 1 per cent. respectively?

86. Find the true discount on £2,750 due two years hence at $4\frac{1}{4}$ P. C.

87. If 4 men earn as much in a day as 7 women, and one woman as much as 2 boys, and if 6 men, 10 women and 14 boys working together for 8 days earn £22, what will be the earnings of 8 men and 6 women working together for 10 days ?

88 A person having a certain sum of money to invest, finds that an investment in a railway stock bearing five per cent. interest at $117\frac{1}{2}$, will yield him £29 more annually than an investment in the 3 per cents, at $92\frac{1}{4}$. How much money has he to invest ?

89. Explain the terms *measure*, *common measure* and *greatest common measure*, and prove that every common measure of dividend and divisor is a measure of the remainder ?

90. If by selling wine at R6 per gallon I loose 25 per cent., at what price must I sell it to gain 25 per cent. ?

91. A person borrows £130 on the 5th of March, and pays back £132. 10*s*. 6*d*, on the 18th October ; find the rate of interest charged.

92. If three soldiers or ten coolies can dig 155 cubic feet of earth in 5 days, how many coolies must be employed to assist 7 soldiers in removing 600 cubic feet of earth so as to get it done in 4 days.

93. In what time will R2,250 amount to R2,565 at 7 per cent. per annum ?

94. A merchant sells a lakh of rupees out of the 4 per cents. at 16 discount, and invests the proceeds, while exchange is at 2*s*. 1*d*., in the three per cent. consols at 96. What income does he derive therefrom ?

95. If the income-tax be 7*d*. in the pound in the first half of the year and $3\frac{1}{2}$*d*. in the second, what is the net income of a gentleman whose gross annual receipts are £1,542. 10*s*. 6*d*.

96. A passenger train going 41 miles an hour and 431 feet long, overtakes a goods train on a parallel line of rails. The goods train is going 28 miles an hour, and is 713 feet long. How long does the passenger train take in passing the other ?

97. A person invests R48,000 in the 4 per cents. at 80, and at the end of each year invests the dividend, which become due, in the same stock ; supposing the funds to remain at 80 for 3 years, find his dividend at the end of the third year.

98. Define *discount*. If the discount on R2,261. 5*a*. 4*p*. due at the end of a year and a half be R128, what is the rate of interest ?

99. A railway passenger counts the telegraph posts on the line as he passes them. If they are 58 yards apart and the train is going 48 miles per hour, how many will he pass per minute ?

100. Three men can do as much work as five boys ; the wages of three boys are equal to those of two men. A work on which 40 boys and 15 men are employed takes 8 weeks and costs £360 ; and how long would it take, if 20 boys and 20 men were employed ; and how much would it cost ?

101. What sum will amount to £5431. 15*s*. $11\frac{1}{4}$*d*. in 6 years at $4\frac{1}{4}$ per cent. Simple Interest ?

102. The sides of two squares contain 77 yards 1 foot 9 inches and 7 yards 2 feet 4 inches respectively ; find the side of a square whose area is equal to the sum of the areas of the two squares.

103. In a bicycle race of two miles over a circular course of 1 furlong, the winner in his last round overtook the second at a point in his fifteenth round Their paces were as 159 to 149. At what distance was this point from the winning post ?

104. Find the expenses of an excursion which includes 5782 miles of railway at $\frac{3}{4}$*d.* per mile, 517 miles of carriage at $10\frac{1}{2}$*d.* per mile, 57 days of hotel-keep at 14*s.* 3*d.* per day, allowing 5 guineas for extras.

105 A merchant buys 1260 maunds of corn, one-fifth of which he sells at a gain of 5 per cent., one-third at a gain of 8 per cent , and the remainder at a gain of 12 per cent. If he had sold the whole at a gain of 10 per cent , he would have obtained £22. 13*s.* more. What was the cost price per maund ?

106. A room, 10 ft. 6 in. high, 22 ft. long, and 14 ft. broad, is painted up to one-third of the height and the remaining two-thirds papered. The painting is charged at $7\frac{1}{2}$*d* per sq. yd , the paper costs 5*s.* 2*d.* per sq. yard., and the work of papering is charged at 2*d.* per sq. yard. How much will the whole cost amount to ?

107. A person sells out £3850 four per cent, stock at 104 and invests the proceeds in another stock at 143. If the dividend on this be $5\frac{3}{4}$ per cent., what will be the change in his income ?

108. What must be the rate of interest in order that the discount on £387. 7*s.* $7\frac{1}{3}$*d.* payable at the end of 3 years may be £41. 10*s.* $1\frac{1}{3}$*d* ?

109 A man contracts to perform a piece of work in 30 days and immediately employs 15 men on it . at the end of 24 days the work is only half done. How many boys should be given to assist them that the contract may be fulfilled, each boy working two-fifths as much as each man ?

110. A person buys 80 tons of coal, and after selling them again at 1*s.* 6*d.* per sack finds that he has gained £4 : had he sold them for 1*s.* 4*d.* per sack he would have lost £6. Find the weight of each sack and the cost price per ton.

111. A field of 7 acres is sown with wheat, barley, and maize, the areas of the crops being respectively as $2\frac{1}{2} : 3\frac{1}{2} : 4\frac{1}{2}$. If the values of an acre of each be also respectively in the same ratios, and an acre of wheat be worth £7, what is the worth of all the crops in the field ?

112. If the three per cents. are at $92\frac{3}{4}$ and the four per cents. at $123\frac{1}{4}$, in which should one invest ? And how much is one investing when the difference in income is a shilling ?

1887.

1. Explain carefully the meaning of *prime number*, *factor*, *divisor*, *measure*, *multiple*.

Resolve 5005 into its prime factors.

Add together as decimals $8\cdot1\dot{3}\dot{8}$, $14\cdot6\dot{5}65\dot{1}$, $\cdot205\dot{0}896\dot{3}$

2. The circumference of the forewheel of a carriage is $6\frac{7}{8}$ feet and that of the hindwheel is $12\frac{5}{6}$ feet. How many feet must the carriage pass over before the wheels shall have made a complete number of revolutions ?

3. A vessel is filled with a liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup ?

4. (i) The surface of a cube is $308\cdot1\dot{6}$ square feet. Find the length of its edge.

(ii) Extract the cube root of $45\cdot\dot{6}9\dot{8}$ to four places of decimals ?

5. If the price of gold be £3. 10*s.* $10\frac{1}{2}d.$ an ounce and a cubic inch of gold weigh 10 ounces, what is the price of the gold that would be required to gild a dome whose surface is 5,000 square feet, the thickness of the gold gilding being 0002 of an inch ?

6. A person invests in 4 per cent. Government paper so as to receive 4 per cent. clear when the income-tax is 5 pies in the rupee. What percentage will be received, if the tax be increased to 7*p.* in the rupee ?

1888.

1. Simplify $\dfrac{\cdot\dot{1}4285\dot{7} \times \cdot\dot{0}7692\dot{3}}{\cdot\dot{0}1098\dot{9}} + \dfrac{2\cdot7\dot{5} \times 11\cdot25}{6\cdot2}$.

2. If 9 lb. of rice cost as much as 4 lb. of sugar, and 14 lb. of sugar are worth as much as $1\frac{1}{2}$ lb. of tea, and 2 lb of tea are worth 5 lb. of coffee, find the cost of 11 lb. of coffee if $2\frac{1}{2}$ lb of rice cost $6\frac{1}{4}d.$

3. If R165. 14*a.* and $1\frac{7}{11}p.$ be the discount of a debt of R2820, Simple Interest being at the rate of $3\frac{1}{4}$ per cent., how many months before due was the debt paid ?

4. The price of gold is £3. 17*s.* $10\frac{1}{2}d.$ per oz.; a composition of gold and silver weighing 18 lb. is worth £637. 7*s.*, but if the proportions of gold and silver were interchanged, it would be worth only £259. 1*s.* Find the proportion of gold and silver in the composition, and the price of silver per ounce.

5. By selling 4 dozen mangoes for 13 rupees, it was found that $\frac{3}{10}$ths of the outlay was gained ; what ought the retail price per mango to have been in order to have gained 60 per cent. ?

1889.

1. One clerk has $24\cdot\dot{4}2857\dot{1}$ and a second clerk has $38\frac{4}{7}$ sheets to copy ; they call in a third clerk and agree to divide the work equally among the three and to pay the third clerk at the rate of $\cdot2430\dot{5}$ shillings per sheet. How much will he receive from each of them ?

2. If the manufacturer makes a profit of 20 per cent., the wholesale dealer a profit of 25 per cent., and the shopkeeper a profit of 40 per cent., what was the cost of the manufacturer of an article bought at a shop, for 17*s.* 6*d.* ?

3. If 15 men eat 28 shillings worth of bread in 14 days when wheat is at 52 shillings per quarter ; what must be the price of wheat per quarter that 18 shillings worth may provide bread for 13 men for 5 days ?

4. Find the value of $\sqrt{902525080 1\cdot7424} - \sqrt[3]{347740371\ 686161}$.

5. If the discount on £678. 8*s*. which is due at the end of a year and a half be £38. 8*s*., what is the rate per cent. of Simple Interest ?

1890.

1. Simplify :— $$\frac{5\frac{1}{2} \text{ of } \cdot\dot{2} \text{ of } 2\cdot57142\dot{8} - 1 \div (\frac{1}{5} + \cdot5)}{1 - \frac{3}{11} \text{ of } \left\{ \cdot5 + \frac{1}{3} \text{ of } \dfrac{05}{\cdot\dot{1}4285\dot{7} \text{ of } 1\frac{1}{20}} \right\}}$$

2. A rectangular cistern, whose length is equal to its breadth, is $5\frac{5}{7}$ feet deep and contains 5 tons of water. If a cubic foot of water weighs 1000 ounces, find the dimensions of the cistern.

3 *A*, *B* and *C* can walk at the rate of 3, 4, 5 miles an hour ; they start from Poona at 1, 2, 3 o'clock respectively ; when *B* catches *A*, *B* sends him back with a message to *C* ; when will *C* get the message ?

4. If I borrow money at 3 per cent. per annum, interest payable yearly and lend it immediately at 5 per cent. per annum, interest payable half-yearly (receiving compound interest for the second half year), and gain thereby at the end of the year R660 ; what was the sum of money which I borrowed ?

5. A person buys tea at 6 annas per seer and also some at 4 annas per seer. In what proportions must he mix them so that by selling the mixture at $5\frac{1}{4}$ annas per seer he may gain 20 per cent. on each seer sold ?

1892.

1. Simplify : (i) $$\frac{\frac{7}{13} \text{ of } \frac{91}{126} + \frac{5}{6} \text{ of } \frac{13}{35}}{5\frac{2}{7} \text{ of } \frac{132}{153} - \frac{299}{323} \text{ of } \frac{19}{23}}.$$

(ii) $$\frac{3\cdot6\dot{4}28571 - (\cdot009923 + \cdot0102 - \cdot000123)\dfrac{\cdot145}{\cdot0056}}{\sqrt{34\ 5744} - \sqrt[3]{9\ 663597}}.$$

2. Two passengers have together 5 cwt. of luggage and are charged for the excess above the weights allowed 5*s*. 2*d*. and 9*s*. 10*d*. respectively : but if the luggage had all belonged to one of them he would have been charged 19*s* 2*d* How much luggage is each passenger allowed to carry free of charge, and how much luggage had each passenger ?

3. Two clocks *A* and *B*, whose rates are uniform, at noon yesterday indicated 11 hr. 55 min. A. M. and 0 hr 2 m. P. M. respectively. *A* indicated the correct time at 9 P. M. yesterday and *B* at 6 A. M. this morning. When did *A* and *B* last agree and what time did they then indicate ?

4. A person borrows two equal sums of money at the same time at 5 per cent and $3\frac{3}{4}$ per cent. simple interest respectively, and finds that if he repays the former sum with interest on a certain date a year before the latter, he will have to pay in each case the same amount, *viz:* R736. Find the amounts borrowed.

1893.

1. What decimal of a rupee is ·964 pie ? Find the value of R·97625.

Simplify :— $$\frac{\frac{5}{14} - \frac{3}{7} \text{ of } \frac{1}{2}}{\frac{5}{16} + \frac{7}{12} \text{ of } 3\frac{1}{4} - (\frac{7}{3} \text{ of } \frac{37}{21} - \frac{1}{3})} + \frac{\frac{1}{5} \text{ of } \frac{1}{2} + \frac{3}{7} \text{ of } 5}{9\frac{1}{3} \cdot 1\frac{2}{3}}.$$

2. How long will two examiners, working 8 hours a day, take to look over the answers to this paper, if four examiners, working 5 hours a day can do it in 8 days?

3. On a river, *B* is intermediate to and equidistant from *A* and *C*; a boat can go from *A* to *B* and back, in 5 hours 15 minutes, and from *A* to *C* in 7 hours; how long would it take to go from *C* to *A*?

4. What income will a retired officer obtain in England, from one lakh of rupees, Indian Government $4\frac{1}{2}$ per cent. bonds, when for drawing and remitting it, his agents in India charge him 3 per cent., and exchange is at 1*s*. $2\frac{1}{4}$*d*. for the rupee?

5 Three equal glasses are filled with a mixture of spirits and water, the proportion of spirits to water in each glass being as follows; in the first glass as 2 : 3, in the second 3 : 4, and in the third 4 : 5. The contents of the three glasses are poured into a single vessel: what is the proportion of spirits to water in it?

1894.

1. Reduce to their simplest forms :—

(*i*) $\dfrac{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}{\frac{1}{2}\text{ of }\frac{1}{3}\text{ of }\frac{1}{6}}$; (*ii*) $\dfrac{2}{3+\dfrac{4}{5-\frac{6}{7}}}$.

2. Find, by Practice, the value of 9 cwt. 3 qr. 24 lb. at £3. 5*s*. 8*d*. per cwt.

3. If 40 men, 60 women or 80 children can do a work in 6 mo, in what time will 10 men, 10 women, and 10 children do $\frac{1}{3}$ of the work?

4. A person invested £1,000 in the 3 per cents. at $90\frac{5}{8}$; but the price rising to $91\frac{1}{4}$ he sold out, and invested the proceeds in the $3\frac{1}{2}$ per cents. at $97\frac{1}{3}$; find the increase in his income.

5. A cistern can be filled by two pipes, *A* and *B* in 12 minutes and 14 minutes, respectively, and can be emptied by a third, *C*, in 8 minutes. If all the taps be turned on at the same moment, what part of the cistern will remain unfilled at the end of 7 minutes?

6 Two clocks point to 2 o'clock at the same instant on the afternoon of 25th April; one loses 7 seconds, and the other gains 8 seconds, in 24 hours; when will one be half an hour before the other, and what time will each clock then shew?

1895.

1. When the number representing the year is a multiple of four, it is a leap year, consisting of 366 days, except when this number is a multiple of 100, in which case it is an ordinary year consisting of 365 days: but when the number is a multiple of 400, is again a leap year; on this supposition calculate the number of days from the 1st January 1495 to 31st. December 1894, both days inclusive.

2. A school of boys and girls consists of 453 children; the number representing the boys is ·52 of the number of the girls. How many boys were there?

3. Two-thirds of a certain number of poor persons received 1*s.* 6*d.* each, and the rest 2*s.* 6*d* each ; the whole sum spent being £2. 15*s.* ; how many poor persons were there ?

4. If 3 men and 5 women do a piece of work in 8 days, which 2 men and 7 children can do in 12 days, find how long 13 men, 14 children and 15 women will take to do it.

5. *A* sells a horse to *B* for R4860, thereby losing 19 per cent. ; *B* sells it to *C* at a price which would have given *A* 17 per cent. profit. Find *B*'s gain.

6. The compound interest on one rupee is one quarter of a rupee at the end of three years ; find the rate per cent. per annum, correct to two places of decimals ; and calculate exactly the compound interest at the end of 9 years.

THE PUNJAB UNIVERSITY PAPERS.

SELECTED QUESTIONS, 1875-90.

1. If five pums each having a length of stroke of 3 feet, working 15 hours a day for 5 days empty the water out of a mine ; how many pumps with a length of stroke of $2\frac{1}{2}$ feet working 10 hours a day for 12 days, will be required to empty the same mine ; the strokes of the former pumps being performed four times as fast as those of the other ?

2. A piece of land measuring 48 ghumas 3 kanals and 17 marlas of which 39 ghumas 4 kanals and 17 marlas are cultivated and the rest uncultivated is sold at the rate of R75 a ghuma for cultivated and R35 a ghuma for uncultivated land. What is the price of the whole ?

3. The revenue of a village containing 15,756 acres of cultivated land is assessed at 13 annas an acre. What will the local rate of $6\frac{1}{4}$ per cent. on the land revenue payable by the village amount to ?

4. A bania purchase 1,526 maunds of grain at 36 seers for a rupee. He sells one-half at 26 seers the rupee ; at what rate must he sell the remainder so as to clear 50 per cent. on the transaction ?

5. A man hires a workman on this condition that for every day he worked he should get one rupee, but that for every day he was absent he should be fined 12 annas. When 356 days were past, the workman was to receive R118. How many days had he worked ?

6. If a pound of pure silver be worth 62*s.*, the shillings containing 222 parts of pure silver in 240, what will be the value in shillings of a rupee weighing 180 gr., the rupee containing 979 parts of pure silver in 1,000 ?

7. A room whose height is 11 ft. and length twice its breadth takes 143 yards of paper 2 feet wide for its four walls ; how much carpet will it require ?

8. One cubic inch of water weighs 253·17 grains; while one inch of air weighs 31 grain find the number of inches of water (to three places of decimals) that would be equivalent to one cubic foot of air?

9 A rectangular field measures 6 acres and 960 yards; its length is 3 times its breadth; find the distance between the diagonal angles

10. A person withdrew 5,000 rupees from a bank which paid him interest at $5\frac{1}{2}$ per cent., and invested the money in the 6 per cent. Municipal Debentures at $103\frac{1}{2}$. Find the change in his income.

11. A railway train having travelled at $\frac{1}{5}$ of its proper speed reaches its journey's end $2\frac{1}{2}$ hours behind time; in what time should the journey have been done?

12. Five hundred boys are distributed in 3 houses, the smallest house contains $\frac{7}{25}$ of the whole number and the largest contains $\frac{19}{14}$ of the smallest; what is the number in each?

13 A person realises R185500 by selling his $3\frac{1}{2}$ per cent. stock at $92\frac{3}{4}$. He invests one-fifth of the realised money in the 4 per cents. at 96 and the remainder in the 3 per cents. at 90. What is the difference in his income by this transaction?

14. The assets of a bankrupt consist of R9,560. 4*a*. a bank-share of R1200 quoted at $107\frac{5}{8}$ and an undiscounted bill of R3225, due four months hence at 4 per cent. per annum Simple Interest; his liabilities amount to R25014. How much in the rupee can he pay his creditors?

15. A legacy of £1901. 5*s*. is to be distributed amongst a number of persons, in such a way that each shall receive as many shillings as there are persons; what will be the portion of each?

16. What is the smallest number of square yards which can be measured either by roods or square chains?

17. Four per cents. are offered at R98, five per cents. at $R120\frac{20}{29}$; which is the better investment? How much is the investment, when the difference of income is R30?

18. A clever housekeeper went out shopping and found that 2 cocoanuts were selling for the same price as 144 plums; she bought half a dozen cocoanuts, exchanged one of them for 5 melons, and a couple of melons for 5 oranges: she then gave 3 oranges for 42 limes, and finally secured a couple of plums for 5 limes. Has she gained or lost in buying the plums?

19. A merchant having 100 maunds of grain sold 50 maunds at R9 per maund, and thereby gained $7\frac{1}{2}$ per cent. At what rate should he sell the remainder so that he may gain 10 per cent. on the whole?

20 A merchant in trade successively admits three partners at the end of 3 months, 5 months, and 6 months respectively from the opening of the business. The capitals embarked by them were R400, R450, R480 and R495 respectively. After 6 months more, the profit was found to be R1,000. Divide this rateably between the partners.

21. What sum of money invested in the 4 per cents. at par would realise the same income at R10,000 invested in the $4\frac{1}{2}$ P. C. at 102?

22. Four men working together all day, can finish a piece of work in 11 days; but one of them having other engagements can work only half time, another only quarter time. How long will it take the men to complete the work?

23. A merchant sells his goods worth R500 directly for R600 giving three months' credit. Find his profits per cent., interest being calculated at 12 per cent. per annum.

24. A farmer buys 57 sheep for R120 payable at the end of 12 months and sells them directly at R1. 12*a*. ready money; what does he lose by the transaction; supposing the interest of money to be 5 per cent.?

25. A man bequeathed $\frac{5}{12}$ of his property to one son, 30 per cent. of the remainder to another, and the surplus to his widow. The difference of his son's legacies was £784. How much did the widow receive?

26. A ship with 1200 men on board had sufficient provisions to last 17 weeks. The survivors of a wreck having been taken aboard, the provisions were consumed in 15 days. How many men were taken aboard?

27. At what price must a person invest in the 4 per cent. Govt. Promissory Note, so that after paying income-tax at the rate of 5 pies in the rupee, he may receive $4\frac{1}{2}$ per cent. on his investment?

28 *A* and *B* travel together 120 miles by rail. *A* takes a return ticket for which he has to pay one fare and a half. Coming back they find that *A* has travelled cheaper than *B* by 4*a*. 2*p*. for every 100 miles. Find the fare per mile.

1891.

1. Simplify

$$(1)\ \frac{\frac{1}{7}+\frac{1}{3}+\dfrac{\frac{2}{5}}{1-\frac{1}{35}}}{1-\frac{1}{7}\text{ of }\left(\frac{1}{3}+\dfrac{\frac{2}{5}}{1-\frac{2}{5}}\right)}. \qquad (2)\ \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$$

2. Express $2{\cdot}75$ oz $+ {\cdot}075$ cwt. as decimal of $2{\cdot}2\dot{5}$ of ${\cdot}\dot{2}\dot{7}$ of a ton.

3. A sum of money invested at 5 per cent per annum, Simple Interest, amounts in 6 yrs. to R1,326; in what time will it amount to R1,530?

4. What is discount? Distinguish between True and Commercial Discount.

The interest on a certain sum at 5 per cent. per annum for a certain time is £50, and the discount at the same rate for the same time is £40. Find the sum and time.

5. Nine gallons are drawn from a cask full of wine: it is then filled with water. Nine gallons of the mixture are drawn, and the cask is again filled with water. The quantity of wine now left in the cask is to that of the water in it as 16 : 9. How much does the cask hold?

1892.

1. Find by how much the square root of

$$9+\cfrac{1}{1+\cfrac{1}{7+\frac{1}{6}}}$$ differs from $\frac{355}{113}$; which of these comes nearest to $3+\frac{1}{10}\sqrt{2}$?

2. Find the value of $\left(\frac{\cdot0019}{3\cdot1\dot{6}} \text{ of } \frac{4\cdot\dot{4}}{\cdot0005}\right) \div \left(\frac{8\cdot\dot{8}}{7} \text{ of } \frac{4}{5\cdot625}\right)$.

3. A stream which flows at a uniform rate of 1·109 miles an hour is 20 yards wide, the depth at a certain ferry being 6 ft.; how many gallons pass the ferry *in a minute*? (Each gallon contains about $277\frac{1}{4}$ cub. in.)

4. A person invests £14,970 in the purchase of 3 per cents. at 90 and $3\frac{1}{2}$ per cents at 97. His total income being £500, how much of each stock did he buy?

5. A spirit merchant buys 80 gallons of whisky at 18*s*. per gallon and 180 gallons more at 15*s*. per gallon, and mixes them. At what price must he sell the mixture to gain $8\frac{1}{3}$ per cent. upon his outlay?

1893.

2. Multiply 319·9657 by ·04286.

3. Find the value of $\frac{\sqrt{(2-\sqrt{2})}}{\sqrt{(2+\sqrt{2})}}$ correct to 5 places of decimals.

4. Calculate the income-tax on R666. 10*a*. 8*p*. at 5*p*. per rupee.

5. A local train which travels at the rate of 24 mi. an hour, leaves Lahore at 20 min. past 8 and reaches Amritsar at 5 min. past 10 the same morning. It stops at Mianmir for 10 min. and at each of three other stations for 5 min. Find the distance between Lahore and Amritsar.

1894.

1. Convert $\frac{1}{7}$ and $\frac{3}{7}$ into circulating decimals and point out the relation between the figures in their periods.

2. The sides of a rectangular are as 3 : 4 and the area is 1452 square feet. Find its length and breadth.

3. Exchange R7080 for English money at 1*s*. $3\frac{31}{32}d$. per rupee.

4. What is discount? How is it commonly calculated? If a sum of R1,000 becomes due three months hence, what is its present value as commonly calculated, and what as correctly calculated, interest being reckoned at 5 per cent.?

5. Find the square root of 101 correct to five places of decimals.

1895.

1. Divide $\frac{48\frac{4}{7}}{1085\frac{7}{10}}$ by $\frac{7\frac{3}{11}}{174\frac{3}{7}}$, and reduce the quotient to a recurring decimal.

2. The imperial gallon contains 277·27 cubic inches and a cubic foot of water at its maximum density weighs 62·42 lb. ; find the weight of a pint of water correctly to 2 places of decimals.

3. The capital of a firm consists of £713. 3*s.*: £964. 17*s.*; £2391. 3*s.* subscribed by three partners ; divide £2231 among them in proportion to their several capitals.

4. Find the square root of 5 correctly to seven places of decimals.

5. The area of a rectangular field is $\frac{3}{8}$ of an acre ; and its length is twice its breadth ; determine the lengths of its sides approximately.

1896.

1. Make out a bill for the following articles supplied by Messrs. Mool Chand and Co. to Lala Gajar Mal :—

10 lbs. of tea at R1. 2*a.* per lb.
6 seers of sugar at R2. 2*a.* per bag of 5 seers.
4 tins of coffee at R1. 1*a.* per tin.
8 silk handkerchiefs at R3. 8*a.* per dozen.
3 maunds 37 seers of Portland cement at 8 seers per R.
A child's perambulator, price R30.

Subtract 18 per cent. discount for cash.

2. Reduce to its lowest terms :—$\frac{1}{15}$ of $\frac{1}{3}$ of $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}}$.

3. A cubic foot of copper weighs 560 lbs. It is rolled into a square bar 40 ft. long. An exact cube is cut from the bar. What is its weight to four decimals of a pound ?

4. The area of a country is 32,300 000 acres. It consists of three kinds of land, the areas of which are in proportion to the numbers 2, 3 and $\frac{2}{3}$. How many acres are there of each kind of land ?

5. If 3 per cent. stock is at 98¼, how much money must be invested in the stock to yield an annual income of R120 ?

1897.

1. Define a fraction, and prove that the value of a fraction is not altered by multiplying both its numerator and denominator by the same whole number. Deduce from this principle a rule for the addition of fractions.

2. The sum of £177 is to be divided among 15 men, 20 women, and 30 children, in such a manner that a man and a child may together receive as much as two women, and all the women may together receive £60. What will they respectively receive ?

3. Find the value of $\dfrac{\sqrt{(2+\sqrt{2})}}{\sqrt{(2-\sqrt{2})}}$, correct to 7 places of decimals.

4. A garrison of 800 men has provisions sufficient for 10 weeks. How long would they last if the garrison were reduced to 560 men ?

5. Find the Least Common Multiple of $4\frac{1}{2}$, $5\frac{2}{5}$, $6\frac{2}{3}$, and $\frac{5}{7}$.

ALLAHABAD UNIVERSITY PAPERS.

SELECTED QUESTIONS, 1889-90.

1. How long will it take to walk along the four sides of a square field which contains 16 acres 401 square yards, at 3 miles an hour?

2. A and B complete a piece of work in 8 days, B and C do the same in 12 days, and A, B and C finish it in 6 days. In how many days will A and C complete the work?

3. A who travels $3\frac{1}{2}$ miles an hour starts $2\frac{1}{2}$ hours before B who goes the same road at $4\frac{1}{2}$ miles an hour; where will he overtake A?

4. The weight of a cubic inch of water is 253·17 grains, that of a cubic inch of air is ·31 grains: find to 3 places of decimals how many cubic inches of water are equal in weight to one cubic foot of air.

5. On measuring a distance of 32 yds. with a rod of a certain length it was found that the rod was contained 41 times with $\frac{1}{2}$ an inch over. How many inches will there be over in measuring 44 yds. with the same rod?

1891.

1. Define "Notation," "Numeration" and prove that "three times four" = "four times three."

2. Reduce to a single fraction $\dfrac{919\frac{2}{17}}{7\cdot 9\dot{5}\dot{4}} + \dfrac{4\cdot\dot{1}0\dot{0}}{442\frac{10}{37}} + \frac{7}{11}$ of ·07344.

3. The wine in a pipe when full is worth £19. 9*s.* 9*d.* How much has leaked away, if what is left is worth £9. 16*s.* $7\frac{5}{224}$*d.*?

4. In discounting a bill, what you mean by "The Banker's profit"? If the simple interest on £923-18*s*-$1\frac{1}{2}$*d* amounts to £17-9*s.*-$3\frac{3}{4}$*d.* exactly in 138 days, what is the rate of interest per cent. per annum?

5. Extract the square root of 99,980,001; and of $60\frac{01}{100}$.

1892.

1. How is a fraction affected by adding the same number to the numerator and denominator?

Prove that $\dfrac{3+4}{4+5}$ is greater than $\dfrac{3}{4}$ and less than $\dfrac{4}{5}$.

2. (*a*) Divide $\frac{1}{3}\left[3+\frac{1}{3}\left\{3+\frac{1}{3}\left(3+1\frac{1}{2}\right)\right\}\right]$ by ·125.

(*b*) Reduce $\frac{1741}{1532}$ and $\frac{1520}{5232}$ to their lowest terms and express their difference as a decimal.

3. Forty men finish a piece of work in 40 days; if 5 men leave the work after every tenth day, in what time will the whole work be completed?

4. Find the difference between the Simple Interest and Discount of £330 in 4 years at $2\frac{1}{2}$ per cent. per annum.

5. Extract the square root of $\frac{1000\cdot20001}{1000}$.

1893.

1. Two recurring decimals are added together; prove that the number of digits in the period of the result, can not exceed the product of the numbers of the digits in the original periods.

2. Find the value of $\cdot\dot{5}\dot{4}$ of $\cdot30\dot{7}\dot{2}$ of 1 mile 5 fur. 30 poles.

3. Multiply R2. 1*a*. by $\frac{\frac{4}{5}+\frac{6}{7}+\frac{13}{15}+\frac{10}{12}}{\frac{1}{5}+\frac{5}{7}+\frac{7}{15}+\frac{13}{21}}$.

4. Find by Practice the cost of 10 cwt. 3 qr. 23 lb. 8 oz. £1. 5*s*. 8*d*. per cwt.

5. A sum of money was divided amongst 5 people: 4 of them received respectively ·15, $\frac{3}{40}$, ·1, $\frac{4}{9}$ of the whole, while the 5th received £105. 3*s*. 6*d*. What was the sum divided?

6. An oz. of standard gold, $\frac{1}{12}$ of which is alloy, is worth £3-17-10½; how many sovereigns would be coined from 36 lb. 8 oz. of pure gold?

7. Find the square root of 6246·057024 and of $71\frac{154}{225}$.

1894.

1. (*a*) A multiplication sum having been worked is partially rubbed out: the figures that remain are the entire multiplicand 999 and the last three digits 193 in the product. Restore the complete work.

(*b*) Simplify $\frac{1}{1\frac{1}{20}}\times\frac{1+\cdot0025\times\cdot05}{1\cdot0025-\cdot05}-\frac{45}{8}\times\cdot3\dot{5}$.

2. (*a*) What decimal of R100 must be added to $\frac{141}{1400}$ of R5. 10. 8. that the sum may be 10 annas?

(*b*) Extract the square root of 25 6.

3. Two trains start at the same time from Mirzapur and Delhi and proceed towards each other at the rates of 16 and 21 miles per hour respectively. When they meet it is found that one train has travelled 60 miles more than the other. Find the distance between the two stations.

4. Two years and six months ago, I borrowed a sum which with simple interest at 6 p. c. per annum now amounts to R638. 4. 0. Find the sum.

1895.

1. (*a*) Explain what is meant by the following terms:—

Prime factors; common measure; common multiple; lowest common multiple.

(*b*) A courtyard, 452 ft. long and 404 ft. wide, is to be paved with square stones all of one size. What is the largest size which can be used?

2. (*a*) Simplify $\frac{5\cdot75}{4\cdot25}$ of $\frac{1}{4}+\frac{8}{7}+\frac{4}{9}\times\frac{3}{5}-\frac{1}{2}$.

(*b*) Find the square root of 3·1415926 to four places of decimals.

3. The *difference* between the Interest for 4 months, and the Discount, on a certain sum due in 4 months at 4 per cent., is one rupee. What is the sum ?

4. A merchant sells silk of two qualities which cost him R5. 5*a*. 4*p*. and R4. 4*a*. 4*p*. per yard, respectively. The selling price of the latter is $\frac{2}{3}$rds. that of the former, but the quantity sold is double and the merchant gains 25 p. c. on the whole. Calculate the selling price per yard of each.

5. A policeman goes after a thief who has 100 yards' start ; if the policeman run a mile in 6 minutes, and the thief a mile in ten minutes, how far will the thief have gone before he is overtaken ?

1896.

1. Simplify :—

(*a*) $5-5\times\frac{2+1\frac{1}{5}(2+1\frac{1}{5})}{1\frac{1}{5}+2(2+1\frac{1}{5})}$.

(*b*) $\frac{\cdot125\times(\cdot175 \text{ of } \cdot\dot{2}8571\dot{4})}{\cdot00025}$.

2. (*a*) Express $\frac{2}{3}$ of 7*s*. 6*d*. + 1·25 of 5*s*. − ·$5\dot{4}\dot{5}$ of 9*s*. 9*d*. as a decimal fraction of £10.

(*b*) Extract the square root of 40000 400001.

3. What is an 'aliquot' part of a quantity ?

Find, by Practice, the time of building a wall 27 yd. long, 1 yd. thick and 6 ft. high, of which one c. yd. is built in 3 hr. 18 min. 45 sec.

4. How far shall I ride with a friend who leaves Allahabad at 9 A. M., and will drive to Karchana which is 10 miles from Allahabad in one hour, that I may, by walking back at the rate of 4 miles an hour, reach home at 11-30 A. M. ?

5 *A* owes *B* R1,435 due at the end of 4 months R630 due at the end of 8 months, R860, due at the end of a year. *B* wants his money forthwith. What ought *A* to pay him reckoning interest at $7\frac{1}{2}$ per cent. ?

SELECTIONS FROM HIGHER EXAMINATION PAPERS.

1. A person has 24180 rupees to invest ; the $5\frac{1}{2}$ per cent. Govt. Loan, being at 108, and the 6 per cent. Municipal Loan of 1000 rupees, being at 1020, find how he must divide his capital between the Government and Municipal Loans, that he may obtain the same income from each.

2. The length of the Eastern Bengal Railway being 110 miles and the capital employed in its construction 1500000*l*, what must be the gross annual traffic receipts per mile in order that a dividend of 5 per cent. may be paid to the share-holders after allowing 45 per cent of the gross receipts for current expenditure ?

3 A merchant bought a fifty-gallon cask of wine for R741. Supposing it to have lost $3\frac{1}{4}$ gallons, at what price per dozen bottles (nine bottles holding a gallon) should he sell it in order to gain 15 per cent. upon the whole original cost ?

4. The debt of a bankrupt amount to R21345 and 4*a*. and his assets consist of property worth R9167. 10*a*. 8*p*. and an undiscounted bill of R5130 due 4 mo. hence, Simple Interest being reckoned at 4 p. c. per annum. How much in the rupee can he pay his creditors ?

5. The income of a Company would justify a dividend of 4 per cent. if there were no preference shares But as R2000000 of the stock consists of such shares which are guaranteed 5 per cent per annum, the dividend for the ordinary shares is $3\frac{1}{2}$ per cent. Find the whole stock.

6. A coal merchant had 150 tons of coal of which he sold 50 tons at R27 per ton, and found that he was only gaining $7\frac{1}{2}$ p c At what rate must he sell the remainder, so that he may gain 10 per cent. on the whole ?

7. A broker charges one-eighth per cent. commission on the money invested. He is handed over R14800 to invest in a certain stock at $92\frac{5}{8}$ and succeeds in obtaining the stock at $92\frac{1}{2}$, keeping the balance to himself ; what is his total profit ?

8. A merchant buys 5000 mds. of rice, one-fifth of which he sells at a profit of 5 per cent., one-fourth at a profit of 10 per cent. and the remainder at a profit of 16 per cent. If he had sold the whole at a profit of 15 per cent. he would have made R438. 12*a* more. What was the cost of rice per maund ?

9. I invest in the Government $5\frac{1}{2}$ per cent. loan, which is at 11 per cent. premium. Supposing that the loan is to be paid off at par in 6 years, calculate what interest I get for my money

10. A man buys 27 sheep for R90, and sells 12 at a loss of 3 per cent.; at how much price must he sell the remainder, in order that he may gain $2\frac{1}{2}$ per cent. on the whole purchase ?

11. In a field of cabbages the distance between the rows of cabbages is 2 feet, the distance between the cabbages in row is 9 inches . how many cabbages are there in an acre ?

12. A share-holder in a commercial company receives one year a dividend of 5 per cent. on his shares. Next year he receives a dividend of $7\frac{1}{2}$ per cent., and finds that he is R412. 8*a*. richer. Find the amount of his shares.

13. If the price of the 4 per cents. just before the payment of a half-yearly dividend be 93, what ought to have been the price three months previously, supposing no change in the value of money to have taken place during that interval ?

14. If 21 horses and 217 sheep can be kept 10 days for the same sum as it would cost to keep 9 horses and 60 sheep for 27 days, find how many sheep eat as much as 3 horses.

15. A certain reef of quartz when crushed yields ·0011 per cent. of gold. If the working expenses amount to 62·5 per cent. of the gross receipts, and the profit on each 100 tons crushed amounts to £52. 10*s*.; find the number of grains in sovereigns. (The lb. Av. contains 7,000 grs. Troy).

16. A man who has sold tea at R2. 8*a*. per seer making a profit of 25 per cent, lowers his price so as to gain only 2*a*. per seer; in what ratio must his monthly sale increase that he may make twice as much as before?

17. By selling out 4 per cent stock at 96 and investing the proceeds in railway stock which pays dividends of 7 per cent. per annum, a man finds that he can increase his income by one half; what is the price of the railway stock?

18. Two men undertake to do a piece of work for R7. 8*a*. One could do it alone in 5 days, the other in 6 days. With the assistance of a boy they finish it in 2 days. How should the money be divided?

19. Two watches *A* and *B* whose rates are uniform, at noon yesterday indicated 11-55′ and 12-2′ respectively. *A* indicated the correct time at 9 P. M yesterday, and *B* at 6 A M this morning. When did *A* and *B* last agree, and what time did they then indicate? When will they agree next?

20. *A* and *B* start at the same time on a journey. *A* walks at the rate of 4 miles an hour and *B* of 3 miles an hour. When *A* has gone half way, *B* gets a ride, and goes at twice the rate of *A*, until he has ridden a distance equal to $\frac{2}{15}$ of the whole journey beyond the spot at which he passes *A*, *B* then walks the remainder of the journey, *A* having walked it all. Will *A* or *B* arrive first? And what fraction of the whole journey will the other still have to travel?

21. A man has £1583. 17*s*. 11*d*. in 3 per cent. stock and £982. 12*s*. 6*d*. in $3\frac{1}{2}$ per cent stock; he transfers a certain sum from the former to the latter when the stocks are at 91 and 98 respectively and thus makes an income derived from each the same. How much has he finally in 3 per cent. stock.

22. *A* owes *B* £500, in liquidation of which debt he gives him a bill of £300 due 10 years hence, another bill due 4 years hence and £133. 6*s*. 8*d*. in cash What is the value of the latter bill, interest being at the rate of 5 per cent. per annum, and allowing true discount?

23 *A*, *B*, *C*, *D*, working together can perform a piece of work in 8 days. *A* and *B* or *B* and *D* together take twice as long as *A*, *B*, *C*, *D*, together to perform the same work. *A* works during the whole of the day, *B* during three-fourths, *C* during a half and *D* during $\frac{1}{4}$th of the day. In how many days will the work be finished?

24. A person has a certain sum to invest; he divides it equally between the $3\frac{1}{2}$ per cents. at 84 and the 4 per cents. at 88 and finds that the latter stock gives him an annual income of £3. 7*s*. 6*d*. more than the former; what is the sum?

25. Two clocks of which one gains and the other loses one minute in an hour, strike one o'clock together; shew that the interval between their respectively striking 2 will be $2\frac{1}{3599}$ minutes by a correct clock.

26. A square area is bounded by 36 yds. of wire fencing. If the enclosed area be increased by 40 sq. yds., still retaining the form of a square, how many additional yards of fencing will be required?

27. A merchant buys China Tea at 3*s*. 6*d*. per lb. To improve the flavour he adds 2 oz. of Assam Tea to every lb. of China Tea and finds

that the mixture costs him 4*s.* per lb. How much per lb. did he give for the Assam ?

28. A book sent from England costs me (including 1*s.* 6*d.* postage) 16*s.* 1*d* But my book-seller allows me two pence in the shilling discount on the published price. What is the published price ?

29. A room 35 ft. long by 18 ft. broad is enclosed by walls 18 in. thick, and all round the outside there is a verandah 9 ft. wide. What will be the cost of paving this verandah at the rate of 8*a.* per sq. yard ?

30. In sending 100 cheroots to England, I paid freight $\frac{4}{7}$rds of their prime cost ; landing charges $\frac{1}{3}$rd of their cost including freight, and duty $2\frac{1}{2}$ times their cost including freight and landing charges. Altogether the cheroots, duty paid, in London cost me £7. What did I give for them at Madras ?

31. For two-thirds of the distance of a ghaut the rise is 1 foot in 24 (measured along the road), and for the remaining third 1 in 16. The top of the ghaut is 1400 feet above the bottom ; what is the length ?

32. At an examination $\frac{1}{5}$th of a class gains $\frac{7}{8}$ths of the maximum number of marks, $\frac{1}{10}$th gains $\frac{3}{4}$ths, $\frac{2}{5}$ths gains $\frac{1}{2}$, $\frac{1}{4}$th gains $\frac{1}{4}$, and the rest $\frac{1}{8}$. The average number of marks gained by the whole class is 166 ; what is the maximum ?

33. A clock in which the hour-hand has been displaced shows the time to be 16 minutes past 3, and the two hands are together : the time is between 3 and 4 o'clock. Find by how many minute-divisions the hour-hand has been displaced.

34. A man who can walk down a ghaut at the rate of $4\frac{1}{3}$ and up it at the rate of $3\frac{1}{4}$ miles an hour, descends and returns to his starting point after walking for 2 hrs. and 4 min. How far did he walk ?

35. An express train owing to a defect in the engine goes at $\frac{5}{6}$ths of its proper speed, and arrives at 6-49 P. M. instead of 5·55 P. M. ; at what hour did it start ?

36. *A* and *B* own a ship in shares which are as 2 : 3. They dispose of parts of their shares to *C*, so that *A*, *B*, *C*, hold the ship in equal shares. What is the ratio of the payments that *C* must make to *A* and *B* ?

37. Find the diagonal of a square whose side is one-fifth of a mile.

38. A land-owner pays his agent 5 per cent. on the gross rental of his estate, and after paying an income-tax of 9*d.* in the £ on the remainder, has £2456. 12*s.* 5*d.* left. What was the gross rental ?

39. A tradesman, selling goods for a certain price to be paid six months hence, offers to give one-tenth more of the same goods for the same price in ready money : what is the rate of discount ?

40. The hands of a clock which gains uniformly at the rate of 15″ a day were set at sunset on the evening of the first of the month at 6 o'clock. The true time of sunrise on the 3rd was known to be a quarter to six, but the clock indicated a quarter past six. Find the error made in setting the clock on the 1st.

41. How much per cent. must be added to the cost price of goods that a profit of 20 per cent. may be made, after throwing off a discount of 10 per cent. from the labelled price?

42. In an examination A obtains 10 per cent less than the minimum number of marks required for passing, B obtains $11\frac{1}{9}$ per cent. less than A, and C $41\frac{3}{17}$ per cent. less than the number of marks obtained by A and B together. Does C pass or fail?

43 If the cost of making bread be one rupee per bushel of wheat, what is the price of wheat when the two-anna loaf is twice as large as it is when wheat is R5 a bushel?

44. What sum must a person invest in the 3 per cents. at 90, in order that by selling out £1000 *stock* when they have risen to $93\frac{1}{2}$, and the remainder when they have fallen to $84\frac{1}{4}$, and investing the whole proceeds in the 4 per cents. at par he may increase his annual income by £9. 5*s.*?

45. A and B engage in trade, their capitals being as 3 : 2. At the end of 3 months A takes out a sum equivalent to one-third of B's capital and at the end of another 3 months B puts in a sum equivalent to what A took out If A's profits at the end of the year are £110 more than B's, find the amount of the profits of each

46. A person puts £1197 out at 30 per cent. per annum interest, and spends at the end of each year £300 *more* than the annual interest on £1197, and thus at the end of a certain time has nothing left. If he had spent £300 *less* than the annual interest, how much would he have had at the end of the same time?

47. A certain number of men and women subscribe a sum of money, the number of women being four times the number of men. Each man subscribes as many annas as there are men altogether, and each woman as many pies as there are women altogether The total amount subscribed being R756, find the number of men and women.

48 There are two compound metals, one consisting of a mixture of copper and gold, and the other of a mixture of copper and silver. The values of 1 oz. of gold, silver and copper are £5. 5*s.* and 5*d.* respectively. Find how much copper must be mixed with the first mixture, so that the value of a given quantity of the first may be 15 times the value of an equal weight of the second, the latter containing 80 p. c. of pure silver.

49 A well is fed by a spring which flows continuously and uniformly into it When there are 10000 cubic feet of water in the well, 7 men can empty it in 20 days; and when there are 15000 cub. ft. of water in the well 5 men can empty it in 50 days. How many cubic feet of water flow into the well in one day?

50. A merchant sells 49 quarters of wheat at a profit of 7 per cent, and a certain number of quarters at a profit of 11 per cents. The price of a quarter of wheat being £3 12*s.* 6*d.* he would have lost £2 10*s.* 9*d.* if he had sold the whole at a profit of 9 per cent. Find the total number of quarters of wheat sold by him.

51. A man has a certain amount of 5 per cent. stock. He sells out one-third of it at 104 and invests the proceeds in the 4 per cents. at 98.

He sells out from the 4 per cents. when they have risen 2 per cent. and then repurchases the same amount of 5 per cent. stock at 102 as he sold out originally. His gain being £202, find the amount of 5 per cent. stock originally held by him.

52. If £31250 put at compound interest amount in three years to £43904, what is the rate per cent.?

53. Two clocks begin striking the hour of noon together on a certain day, the interval between every two strokes being 1″ and 2″ respectively. They gain 1″ and 2″ respectively in every 24 hours. Shew after what length of time they will end striking the hour of noon together, both shewing the correct time at the fall of the last stroke.

54 The only three creditors of an insolvent, whose assets amounting to £200 can only pay 5*d*. in the £, agree among themselves to take dividends in the proportion of the number of £ *s*. and *d*. respectively contained in the amounts due to them. The dividends thus taken are in the proportion of 12 : 7 : 6. What are the amounts of their debts?

55. The shorter of two roads between *A* and *B* is 15 miles and goes over a hill the summit of which is 3 miles from *B*, and the longer is 36 miles on level ground. A thief runs away from *A* by the shorter road at the rate of 3 miles per hour up-hill and 4 miles per hour down-hill. Three-fourths of an hour afterwards, a Constable, whose speed up-hill is 4 miles per hour and down-hill 5 miles, starts in pursuit of the thief but takes the wrong road. The thief, an hour after he turns into the longer road at *B*, sees the Constable $1\frac{1}{3}$ miles ahead of him, and at once turns back and retraces his steps increasing his speed by $1\frac{1}{14}$ miles per hour on the level ground, his speed up and down the hill being the same as before. If the speed of the Constable to the original speed of the thief be as 4 : 3 on the level ground, find where the Constable will overtake the thief.

56. The sum of £2100 is due in 4 years, but it is paid by instalments as follows:—£275 at the end of 2 years, £460 at the end of the 3rd year, £500 at the end of the 4th year, and £600 at the end of the 5th year. What amount should be paid at the end of the 6th year, in order to clear off the balance, Simple Interest being reckoned at 5 P C. per annum.

57. A reservoir is to be emptied, the rate of the discharge of the contens being diminished by 100 gallons every hour. The first half will be emptied in 3 hours, the second in 4 hours. How many gallons does the reservoir contain?

58. In a certain meadow there is a crop of 525 stone of grass which grows uniformly. If 11 oxen turned in would consume all the grass in 48 days, but 6 oxen would require 98 days, what weight of grass would each ox eat in a day?

59. *A* and *B* exchange goods; *A* gives 13 cwt. of hops, the retail price of which is 56*s*., but in barter he rates them at £3. *B* gives 10 barrels of beer, the retail price of which is 1*s*. a gallon, but the value of which he raises in proportion to the increased price of the hops. How much must *B* give in money?

60. A person, who pays 5*d*. in the £ income-tax, finds that a rise of interest from 6 to $6\frac{1}{4}$ per-cent. increases his income by £23. 10*s*. 0*d*. What is his capital?

61. If 36 oxen in four weeks eat up the grass on a field of 12 acres and what grows upon it during the time; and 21 oxen eat up the same in 9 weeks; how many oxen will it maintain for 18 weeks supposing the grass to grow uniformly during the time?

62. Forty-eight horses eat the grass of 35 acres of one field in 11 days of 8 hours each: in what time would 20 horses eat the grass of another field of 52 acres, where there is at first twice as much grass per acre as in the former field, the growth of the grass being neglected?

63 There are three cubical boxes; the edge of the first is 12 in., that of the second 20 in., and that of the third 30 in. Find the length of the edge of a cubical box which shall contain as much as all three.

64 A merchant buys 1260 quarters of corn, $\frac{1}{4}$ of which he sells at a gain of 5 per cent, $\frac{1}{3}$ at a gain of 8 per cent., and the remainder at a gain of 12 per cent.; if he had sold the whole at a gain of 10 per cent., he would have gained £23. 2*s.* more. Find the cost price per quarter

CAMBRIDGE UNIVERSITY PAPERS.

SELECTED QUESTIONS 1881-92.

1. A lawn-tennis ground is half as long again as it is wide. The cost of levelling at 9*d.* per square yard is £176 8*s.* Find the cost of enclosing it with an iron railing at 7*s.* 6*d* per yard.

2. A sum of £650 is due from *A* to *B* on a certain day, and £495. 12*s.* 6*d* is also due 7 mo. later Shew that, reckoning interest at 5 P. C. if *A* pay both debts at the end of 3 mo. neither of them will lose.

3. Brussels carpet is 2 ft. wide, costs 6*s.* 6*d.* a yard, and will last 5 years. Kidderminister carpet is $2\frac{1}{2}$ ft. wide, costs 5*s.* a yard, and will last 3 years. Which is the cheapest, not reckoning interest on your outlay?

4 Two men and 3 boys can level and turf 352 yds. of a cricket ground in 4 days and 3 men and 2 boys can complete 276 yds. in 3 days. Compare the rates of working of a man and a boy.

5 A man has £7220 stock in the 3 per cent. Consols; when they are at $102\frac{5}{8}$ he sells and invests in the New $2\frac{1}{2}$ per cents. at $90\frac{1}{8}$. Find the change in his income (Brokerage is $\frac{1}{8}$ per cent.)

6. A steel rod, 1 ft. long and 1 inch square, weighs $3\frac{1}{3}$ lb., and will just support 50 tons. What is the greatest length of steel wire which, when hung up by one end, will just not break by its own weight?

7. Calculate the value of $\sqrt{4+2\sqrt{3}}$ to 2 places of decimals.

8. The width of a lawn-tennis ground is two thirds of its length. The cost of levelling at 8*d.* per square yard is £115. 4*s.* Find the cost of enclosing it with an iron railing at 6*s.* 8*d.* per yard.

9. A sum of £560 is due from *A* to *B* on a certain day. and £756 is also due 7 months later. Shew that, reckoning interest at 5 per cent, if *A* pay both debts at the end of 4 months, neither of them will lose.

10. An Axminister carpet 15 ft. wide and 20 ft. long costs £30, and will last 8 yrs Velvet pile carpet is 2 ft. 6 in. wide, costs 11*s.* 3*d* per yd. and will last 6 yrs. Which is the cheapest not reckoning interest on the outlay?

11. Three men and four boys can level and turf 372 yards of a cricket ground in 3 days, and four men and three boys can complete 640 yards in 5 days. Compare the rates of working of a man and a boy.

12 A man has £2740 stock in the New $2\frac{1}{2}$ per cent.: when they are at $90\frac{1}{8}$ he sells, and invests in 3 per cent. Consols at $102\frac{3}{4}$. Find the change in his income. (Brokerage is $\frac{1}{8}$ per cent.)

13. A steel wire 32000 ft. long, if hung up by one end will just support its own weight without breaking. A cubic inch of steel weighs $4\frac{2}{5}$ ounces. Find the greatest weight which a rod, 1 inch square, is capable of supporting.

14 A man who has sold tea at half-a-crown per lb making a profit of 25 P. C., lowers his price so as to gain only 2*d.* per lb. In what ratio must his weekly sale increase that he may make twice as much as before?

15. The amount of sunshine recorded in Jersey last year, in the month of April, was 33 per cent. of the possible amount, and the average length of the night in that month is 10 hrs. 30 min. Find how many hours of sunshine there were in the month.

16 The length of a side of the base of the Great Pyramid, which is square, is 500 Egyptian cubits ·

(i) Find the area covered by it in acres, roods &c. knowing that an Egyptian cubit equal to 18·24 English inches.

(ii) Find the height of the Pyramid in cubits, having given that the height of a model of the Pyramid, the area of whose base is 8 sq. ft. 73 sq. in., is 22·225 inches.

17. Which is the more profitable stock to invest in 3 per cent. at $83\frac{1}{2}$ or $3\frac{1}{2}$ per cent. at 97?

18. Two men undertake to do a piece of work for one guinea. One could do it alone in 6 days, the other in 7 days. With the assistance of a boy they finish it in 3 days. How should the money be divided?

19. Two men undertake to do a piece of work for $1\frac{1}{4}$ guineas. One could do it alone in 7 days, the other in 8 days With the assistance of a boy they finish it in 3 days. How should the money be divided?

20 A grocer buys sugar at £1. 5*s.* 4*d.* per cwt. and sells it at 5*d.* per lb, taking off 5 per cent. for cash payment. What percentage of profit does he make?

21. Find the cost of putting a fence round a square field, whose area is four acres, at 1*s.* 6*d.* per yard.

22. A Railway which used to pay 5 P. C duty on all its passenger traffic has now to pay nothing on its third class traffic, and only 2 P. C. on the remainder. If the whole amount of duty payable is now $\frac{1}{10}$th of what it was before, find what fraction the third class traffic is of the whole.

23. By selling out 3 per cent. consols at $102\frac{3}{4}$ and investing the proceeds in a railway stock at 137, a man finds that he can double his income. What is the annual dividend on the railway stock ?

24. The Flying Dutchman runs from London to Exeter, a distance of $193\frac{1}{8}$ mi. in $4\frac{1}{4}$ hrs. making 1 stoppage of 10 min, 2 of 5 min. each, and 1 of 3 min. on the way. What is its average speed when in motion ?

25. A grocer sells sugar at 4*d*. per lb, and takes off 5 per cent. for cash payment. Find what it costs him per cwt. in order that he may make a profit of 60 per cent.

26. A Railway which used to pay 5 P. C. duty on all its passenger traffic has now to pay nothing on its third class traffic, and only 2 P. C. on the remainder. If the third class traffic is $\frac{3}{8}$ths of the whole, find in what ratio the whole amount of duty payable has been reduced.

27. By selling out 3 per cent. consols at $102\frac{7}{8}$ and investing the proceeds in a railway stock which pays dividend of 7 per cent. per annum, a man finds that he can double his income. What is the price of the railway stock ?

28. A man has a sum of money due to him at the end of 4 years' time. What fraction of the sum could he take off for payment at once ; simple interest being reckoned at 5 per cent. ?

29. Find the cost of papering the walls of a room 20 ft. 7 in. long, 14 ft. 5 in. broad and 12 ft. high, paper 2 ft. 6 in. wide at $4\frac{1}{2}$*d*. per yard.

30. If five men can mow a square meadow in 4 days, in what time can 9 men mow a square meadow half as long again as the former ?

31. A dealer bought two horses giving one-third as much again for one as for the other. He sold the more expensive horse at a gain of 15 per cent. and the other at a loss of 5 per cent., receiving £149 in all for the two. What was the price paid for each ?

32. Find the change in income due to transferring £5670 $4\frac{1}{2}$ per cent. stock at $83\frac{1}{8}$ to $5\frac{1}{2}$ per cent. stock at $94\frac{3}{8}$, a brokerage of $\frac{1}{8}$ per cent. being charged on each transaction.

33. A dealer bought two horses giving one-twelfth as much again for one as for the other. He sold the more expensive horse at a gain of 10 per cent. and the other at a gain of 15 per cent., receiving £146. 10*s*. in all for the two. What was the price paid for each ?

34. A man transfers his capital from 4 per cent stock to $5\frac{1}{2}$ per cent. stock when the former is at $100\frac{1}{4}$ and the latter at $109\frac{7}{8}$ and finds that by the change his income is increased by £12. If a brokerage of $\frac{1}{8}$ per cent. be charged on each transaction, find how much 4 per cent. stock he held.

35. Divide £330 among four persons so that the first may have twice as much as the second, the second twice as much as the third, and the first and the third together as much as the second and fourth together.

36. The rental of an estate is £750 a year, but repairs and other outgoings amount to 15 per cent. of the whole rent. What sum must be invested in the 3 per cents. at 99 to produce the same net income ?

37. Divide £520 among four persons so that the first may have twice as much as the second, the second twice as much as the third, and the first and third together as much as the second and fourth together.

38. The rental of a farm is £450 a year, but repairs and other outgoings amount to 18 per cent. of the whole rent. What sum must be invested in the 3 per cents. at 99 to produce the same net income.

39. A merchant buys goods at 24 guineas a cwt., and by retailing them at 5*s*. 3*d*. the lb. makes 10 per cent more profit than if he had sold the whole for £240. What weight did he buy?

40 It is found that it takes 10 min. to fill a certain cistern by means of one tap and 15 by means of another, and only 5 min. when both taps are used. Show that this indicates that the cistern has a leak which would empty it in half an hour.

1893. (A).

1. A man pays £116. 2*s*. 9*d*. income tax; if the rate be $7\frac{1}{2}$*d*. in the pound, what is the man's income?

2. Find the G. C. M. of 1901235 and 4320960.

3. Arrange the following fractions in order of magnitude beginning with the least $\frac{3}{5}$, $\frac{4}{7}$, $\frac{15}{24}$, $\frac{18}{28}$.

4. Find the difference between $\frac{9}{3}$ of $\frac{3}{5}+\frac{1}{4}$ and $\frac{2}{3}$ of $(\frac{2}{5}+\frac{1}{4})$.

Simplify (i) $\dfrac{3\frac{5}{8}+2\frac{5}{12}-1\frac{5}{24}}{5\frac{4}{5}+2\frac{9}{10}-1\frac{9}{20}}$; (ii) $5\frac{7}{8}\times 1\frac{3}{17}-3\frac{3}{8}\div 2\frac{4}{7}-\frac{13}{16}$.

5. Find the value of $2\frac{1}{3}$ of $4\frac{1}{5}$ of £1. 15*s*. − $5\frac{1}{7}$ of $9\frac{5}{8}$ of 5*s*. 3*d*.

6. Multiply ·1357 by ·0026, and divide your result by ·013.

Reduce to a vulgar fraction $\cdot 452\dot{0}2\dot{7}$.

7. How long will it take a man to walk round a square field whose area is 160 acres at the rate of 4 miles an hour?

8. 5000 men have provisions for 107 da. and after 17 da. 500 of them go away; how long will the remaining provisions last those who are left.

9. What sum amounts to £477. 15*s*. in. 2 yrs. at 5 % compound interest?

10. If a cubic ft. of water weigh 1000 oz., find in tons, etc the weight of water contained in a reservoir $\frac{3}{4}$ mi. long, 1 fur. broad and $20\frac{1}{4}$ ft. deep.

11. I transfer £4950 stock from 3 per cents. at $97\frac{5}{8}$ to Railway 4 per cent. debentures at $89\frac{7}{8}$; find the change in my income (brokerage $\frac{1}{8}$ per cent. being charged on each transaction).

(B).

1. A man pays £126. 17*s*. 2*d*. income tax; if the rate be $6\frac{1}{2}$*d*. in the pound, what is the man's income?

2. Find the G. C. M. of 1901235 and 4320960.

3 Arrange the following fractions in order of magnitude beginning with the least $\frac{3}{8}$, $\frac{4}{9}$, $\frac{13}{63}$, $\frac{17}{56}$.

4. Find the sum of $\frac{3}{5}$ of $\frac{2}{3}+\frac{1}{6}$ and $\frac{3}{5}$ of $(\frac{2}{3}+\frac{1}{6})$.

Simplify (i) $\dfrac{2\frac{1}{5}\times 1\frac{3}{4}}{\frac{3}{7}\times 1\frac{2}{17}}+\dfrac{1\frac{2}{5}\times 1\frac{1}{6}}{\frac{2}{13}\times 1\frac{6}{17}}$; (ii) $\frac{1}{6}\times\dfrac{1}{\frac{37}{40}}-\frac{2}{13}$ of $1\frac{2}{37}+\dfrac{\frac{2}{3}}{\frac{37}{17}}$.

5. Find the value of $\frac{7}{8}$ of $11\frac{1}{5}$ of £3. 10*s.* $-4\frac{1}{2}$ of $5\frac{1}{2}$ of 10*s.* 6*d.*

6. Multiply ·06351 by ·039, and divide your result by ·00013. Reduce to a vulgar fraction $\cdot 90\dot{4}0\dot{5}$

7. How long would it take a man to walk round a square field of 90 acres at the rate of 5 miles an hour.

8. 4000 men have provisions for 100 days, and after 10 days they are joined by 500 more men, how long will the provisions now last ?

9. What sum amounts to £308. 14*s.* in 3 yrs. at 5 % compound interest ?

10 If a c. foot of water weigh 1000 oz., find in tons, etc., the weight of water contained in a reservoir $\frac{1}{4}$ mi. long 3 fur. broad and $18\frac{1}{3}$ ft. deep.

11. How much income shall I get by investing £5012 in 5 per cent. stock at $89\frac{1}{2}$? How much stock do I hold by such an investment ?

1894. (A).

1. Divide 347892385 by 34768 and express in words the sum of the quotient and remainder.

2. If the length of a man's step be $30\frac{1}{2}$ in., how many steps does he take in walking 11 mi. 3 fur. $69\frac{1}{3}$ yd. ?

3 A tradesman's receipts of money in a week were—Monday, £2-3-4 ; Tuesday, £1-18-7 ; Wednesday, £1-12-9 ; Thursday, £1-15 ; Friday, £1-16-5 ; Saturday, £5-9-5 What was the average daily receipt ?

4. Simplify $\dfrac{2\frac{1}{2}+3\frac{2}{5}-5\frac{1}{10}}{\frac{1}{60}+\frac{1}{40}\text{ of }2\frac{1}{2}\text{ of }3\frac{1}{3}\text{ of }5\frac{1}{8}}$.

5. If 8 men and 16 boys working together can do 10 times as much work per hour as a man and a boy together ; compare the work of a man with that of a boy.

6. Add together $\dfrac{7\cdot 6}{1\cdot 52}$ and $\dfrac{\cdot 00148}{\cdot 111}$; and divide 3·9 by ·0026.

7. Find by practice the value of 1234 articles at £3. 19*s.* $4\frac{1}{2}$*d.* each.

8. Find to the nearest penny the compound interest on £4895. 10*s.* for 4 years at $3\frac{1}{4}$ per cent. per annum.

9. A tradesman who is selling off, states that he will make a reduction of 10 per cent. from the prices marked ; at what price will he mark goods for which he wishes to receive 8*s.* 3*d.*

10. What will it cost to cover with lead the sides and bottom of a cistern whose length, breadth and depth are respectively 3 ft. 11 in., 1 ft. 9 in. and 2 ft. 8 in. at 15*s.* $2\frac{1}{4}$*d.* per sq yd.?

11. A person invests £6000 in $2\frac{3}{4}$ per cent. Stock at 99 and pays income-tax at 8*d.* in the pound ; on the stock rising to 101 he sells out and

invests the proceeds in $2\frac{1}{2}$ per cent. Stock at 71, free of income tax. Find to the nearest penny the net change in his income.

(B).

1. Divide 347894579 by 23456 and express in words the sum of the quotient and remainder.

2 If the length of a man's step be 31 in., how many steps does he take in walking 5 mi. 6 fur $89\frac{1}{3}$ yds ?

3. A tradesman's receipts of money as one week were :—Monday, £18-9-7 ; Tuesday, £15-4-8 ; Wednesday, £11-7-6 ; Thursday, £37-19-4 ; Friday, £14-8-3 ; Saturday, £7-14-2. What was the average daily receipt ?

4. Simplify $\dfrac{5\frac{5}{8}+\frac{2}{3}}{5\frac{1}{9}+\frac{6}{5}}+\dfrac{\frac{4}{5}}{6\frac{4}{6}-\frac{8}{9}}$.

5. If 7 men and 2 boys working together can do six times as much work per hour as a boy and a man together, compare the work of a boy with that of a man.

6. Add together $\dfrac{5\cdot7}{1\cdot52}$ and $\dfrac{\cdot0111}{74}$ and divide ·00371 by 1·28.

7. Find by practice the cost of 1342 articles at £4. 17*s.* $9\frac{1}{2}$*d.* each.

8. Find to the nearest penny the compound interest on £3789. 15*s.* for 4 years at $2\frac{3}{4}$ per cent. per annum.

9. A tradesman who is selling off, states that he will make a reduction of 20 per cent. from the marked price : at what price will he mark goods for which he wishes to receive 11*s.* 8*d.* ?

10. What will it cost to cover with zinc the sides and bottom of a cistern whose length, breadth and depth are 7 ft. 10 in., 3 ft. 6 in. and 5 ft. 4 in., respectively at 5*s.* $0\frac{3}{4}$*d.* per sq. yd ?

11. A person invests £8000 in $2\frac{3}{4}$ per cent. stock at 96 and pays income-tax at 8*d* in the pound ; on the stock rising to 99 he sells out and invests the proceeds in $2\frac{1}{2}$ per cent. stock at 73, free of income tax. Calculate to the nearest penny the net change in his income.

1895 (A).

3. Simplify $\dfrac{\frac{1}{3}+\frac{1}{5}}{\frac{1}{3}-\frac{1}{7}}$ of $\dfrac{\frac{1}{4}+\frac{1}{6}}{\frac{1}{4}-\frac{1}{6}}$ of $\dfrac{\frac{1}{2}+\frac{1}{7}}{\frac{1}{2}-\frac{1}{8}}$ of $2\frac{14}{365}$.

4. Multiply together 1·3. ·13, and ·013 ; and find the sum of 2·575, $3\cdot\dot{4}\dot{6}$, $2\cdot\dot{0}7$, $1\cdot8\dot{7}1$.

5. Find the cost of 5 cwt. 22 lbs. 10 oz. at 12*s.* 8*d.* per lb.

6 In 1881 the number of lunatics in Scotland was 8404, being one for every 444 of the population, and the proportion of drunkards was one for every 528 of the population. Determine the number of drunkards in Scotland in 1881.

7. What would be the cost of the material of a closed cubical box, 4 in. in thickness and whose cubic content is 27 c. ft, at 6*s.* 9*d.* the c. ft.?

8. Find the compound interest on £16000 for 4 ys. at $2\frac{1}{2}$ P. C.

9. Two men *A* and *B* working alone can finish a piece of work in 3 and $4\frac{1}{2}$ hours respectively. If they work at it for an hour alternately, *A* beginning, and starting work at 10 A. M, when will the work be finished.

10. What income would be derived from investing £8415 in $3\frac{3}{4}$ per cent. stock at $137\frac{1}{2}$?

11. If a bankrupt pay 7*s.* 10*d.* in the £, how much will a creditor receive to whom he owes £865. 2*s* 6*d.*?

(B).

3. Simplify $\dfrac{\frac{1}{3}-\frac{1}{6}}{\frac{1}{5}+\frac{1}{7}}$ of $\dfrac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}+\frac{1}{6}}$ of $\dfrac{\frac{1}{2}-\frac{1}{7}}{\frac{1}{2}+\frac{1}{8}}$ of $16\frac{2}{3}$.

4. Multiply together 1·2, ·12 and ·012; and find the sum of $3{\cdot}1\dot{8}\dot{5}$, 1·162, $\cdot\dot{9}\dot{8}$, $2{\cdot}8\dot{7}$, $1{\cdot}3\dot{2}\dot{0}$.

5. Find the cost of 3 cwt. 18 lbs. 5 oz. at 15*s.* 4*d.* per lb.

6. In 1881 the number of drunkards in Scotland was 7067, being one for every 528 of the population, and the proportion of lunatics was one for every 444 of the population. Determine the number of lunatics in Scotland in 1881.

7. What would be the cost of the material of a closed cubical box, 3 in. in thickness and whose cubical content is 8 c. ft., at 13*s* 4*d.* a c. ft.?

8. Find the Compound Interest on £4000 for 4 years at $2\frac{1}{2}$ per cent.

9. Two men *A* and *B* working alone can finish a piece of work in 6 and $7\frac{1}{2}$ hours respectively. If they work at it for an hour alternately, *A* beginning and starting work at 8 A. M., when will the work be finished?

10. What income would be derived from investing £14025 in $2\frac{3}{4}$ per cent. stock at $103\frac{1}{8}$?

11. If a bankrupt pay 8*s.* 9*d.* in the £, how much will a creditor receive to whom he owes £638. 5*s.*?

1896. (A).

1. In 1893 the number of scholars on the Register of the Elementary Schools in Cambridgeshire was 34226, the number of schools being 214; what was the average number in a school?

2. The front wheel of a bicycle makes 660 revolutions in a mile; find how many revolutions will be made by the hind wheel, of which the circumference is greater by 3 in.

3. Simplify $\dfrac{11\frac{2}{5}\times4\frac{5}{7}+2\frac{3}{4}}{3\frac{1}{5}+4\frac{4}{7}}-\dfrac{2\frac{1}{6}-1\frac{3}{4}}{3\frac{1}{6}+2\frac{1}{2}}$.

4. Divide 3·52 by 16·249 to 3 places of decimals Find the value of $\cdot 4\dot{3}\dot{6}$ of 6*s*. 5*d*. + 3·52 of 3*s*. $6\frac{1}{2}$*d*.

5. Find the cost of 15 sq. yds. 7 ft. 44 in. at £2. 7*s*. 3*d*. per sq. yd.

6. Find the simple interest on £2465 for $1\frac{2}{3}$ yrs. at $2\frac{3}{4}$ P. C per annum.

7. By selling oranges at 10*d*. per dozen a woman loses 10 P. C. on her outlay. What would she gain or lose per cent. if she sold them at 10 for 1*s*.

8. At what time between 7 and 8 o'clock are the hands of a clock opposite to each other ?

9. What is the price of $3\frac{1}{2}$ per cent. stock when an annual income of £472. 6*s*. 6*d*. is obtained by investing £12853. 19*s*. 9*d*. ?

10. A train passes a post in 10 sec., and passes a man walking 4 mi. an hour in 12 sec. Find the length of the train.

(B).

1. In 1892 the number of scholars on the register of the Elementary Schools in Cambridgeshire was 33358, the number of schools being 218; what was the average number in a school ?

2. The hind wheel of a bicycle makes 640 revolutions in a mile ; find how many revolutions will be made by the front wheel, of which the circumference is less by 3 in.

3. Simplify $\dfrac{9\frac{1}{6}\times 3\frac{3}{8}\div 2\frac{3}{4}}{2\frac{3}{5}+3\frac{4}{7}} - \dfrac{2\frac{1}{2}-2\frac{1}{4}}{1\frac{4}{5}+2\frac{5}{8}}$.

4. Divide 2·35 by 14·327 to 3 places of decimals. Find the value of $\cdot 3\dot{4}\dot{5}$ of 8*s*. 3*d*. + 2·48 of 4*s*. $4\frac{1}{2}$*d*.

5. Find the cost of 20 sq. yds. 5 ft. $31\frac{1}{2}$ in. at £3. 12*s*. 9*d*. per sq. yd.

6 Find the Simple Interest on £2735 for $1\frac{3}{4}$ ys. at $3\frac{2}{3}$ P. C. per annum.

7. By selling oranges at 10 for 1*s*. a woman gains 10 P. C. on her outlay. What would she gain or lose per cent. it she sold them at 10*d*. per dozen ?

8. At what time between 3 and 4 o'clock are the hands of a clock opposite to each other ?

9. What is the price of $3\frac{1}{2}$ per cent. stock when an annual income of £456. 7*s*. 6*d*. is obtained by investing £12915. 8*s*. 3*d*. ?

10. A train passes a post in 8 sec., and passes a man walking 5 mi. an hour in 10[illegible] sec. Find the length of the train.

MADRAS UNIVERSITY.

1894.

1. Simplify :—$\dfrac{3\frac{3}{17}+\frac{4}{11}-2\frac{2}{11}\times 1\frac{1}{8}}{4\frac{4}{11}-1\frac{1}{7}\text{ of }1\frac{1}{17}+\frac{4}{7}}\times\dfrac{\frac{3}{4}-\frac{1}{2}}{\frac{2}{3}+\frac{3}{4}}$.

2. Divide 2·01171875 by ·0000614375; and express $\frac{18063}{199800}$ as a recurring decimal.

3. If a cubic yard of brick weighs a ton and a half, find the weight of a million bricks each measuring 15 in. by $4\frac{1}{2}$ in. by $2\frac{1}{4}$ in.

4. Find the value of ·013125 of Rs. 29-2-8.

5. Find by any method the cost of 21 tons 13 cwt. 3 qrs. 26 lbs. of coal at R40-13-4 per ton.

6. A person sent £100 to the bank on the 1st of January, 1890, £200 on the 1st of January, 1891, and £250 on the 1st of January, 1892. If the bank allowed him compound interest at the rate of 5 per cent. per annum, find how much he had at his credit on the 1st of January, 1893

7. A merchant wishing to clear out his old stock sells one lot of goods at a reduction of 10 per cent., another at a reduction of 25 per cent, and a third at a reduction of 50 per cent. on the usual prices. He realises R540, R645, and R375 for these lots respectively, and finds that on the whole he has a loss of $2\frac{1}{2}$ per cent. on the price paid by him for the goods. What would he have gained per cent. if he had sold the goods at the usual rate?

8. It takes $7\frac{1}{3}$ yards of cloth to make a suit of clothes. In Madras the cost of cloth is R4-8 a yard, and the cost of making amounts to 15 per cent. of the cost of the cloth. In London the cost of the same kind of cloth is 5s. a yard, and the cost of making amounts to 40 per cent of the cost of the cloth, while the cost of carriage from London to Madras amounts to 5 per cent of the price of the made-up clothes. If exchange is at 1*s.* $3\frac{3}{4}$*d.* per rupee, find how much a person living in Madras will save by ordinary a suit of clothes locally instead of sending to London for it.

9. A person buys a square field containing $22\frac{1}{2}$ acres for Rs. 10,000, and surrounds it with a fence costing R2-13-4 a yard. At what annual rental per acre must he let the field so as to get a return of $6\frac{1}{4}$ per cent. on his outlay?

10. In 1890 the gross earning of a certain railway amounted to 63 lakhs of rupees, of which the working expenses absorbed 58 per cent. In the same year the gross earnings of another railway amounted to 88·2 lakhs, of which the working expenses absorbed 52 per cent. If the capital outlay on the first railway was $\frac{7}{12}$ of the capital outlay on the second, and if the net earnings of the former amounted to 3 6 per cent. of the capital outlay, find what percentage of the capital outlay the net earnings of the second amounted to.

1895.

2. Simplify :—

$$\frac{\frac{7}{8}\text{ of }\frac{4}{5}+\frac{3}{4}\text{ of }\frac{9}{10}-\frac{5}{6}}{\frac{5}{6}+\frac{1}{2}(\frac{3}{4}-\frac{1}{2})-\frac{3}{5}\text{ of }1\frac{2}{3}}\div\frac{2\frac{1}{4}-\frac{3}{7}\text{ of }2\frac{3}{7}}{1\frac{3}{4}\div\frac{1}{4}-3}.$$

3. Find the value of $\cdot 04\dot{1}96\dot{2} + \cdot 9\dot{3}\dot{6}$, expressing the result as a decimal.

4. Find the value of 97 miles 5 fur. 170 yds. of wire at R34-6-0 per mile.

5. When iron was at £2-14-2 per ton and the rupee at 1*s*. $7\frac{1}{2}$*d*., the cost of laying a railway with iron rails weighing 50 lbs. per yard was Rs. 2,78,250. Find what will be the cost of relaying it with steel rails weighing 75 lbs. per yard, when the price of steel is £3-17-6 per ton and the rupee is at 1*s*. $1\frac{1}{4}$*d*.

6. The reservoir from which a certain city draws its water supply has a surface area of $2\frac{1}{2}$ square mile. If the city has a population of 450,000, and if the average daily supply is at the rate of 20 gallons to each inhabitant, find what must be the average depth of the reservoir so that when full it may contain a year's supply. (1 gallon = 277 274 cubic inches and a year = 365 days.)

7. Find what sum will amount to £669-10-$3\frac{3}{4}$ in two years at $3\frac{1}{2}$ per cent. compound interest, payable annually.

8. The capital of a certain railway is 275 lakhs of rupees, in shares of Rs. 500 each, fully paid up. The gross receipts in a certain year amounted to $22\frac{1}{2}$ lakhs, and the working expenses amounted to 48 4 per cent. of the gross receipts Of the net receipts the sum of Rs. 1,31,400 was placed in the reserve fund, and the remainder went to pay dividend. Find the amount of dividend received by a person holding 1,500 shares, after deducting income-tax at the rate 5 pies per rupee.

9. Rupee silver is an alloy consisting of 11 parts of silver to one part of copper; and the weight of a rupee is 180 grains. If the price of silver is 2*s*. 6*d*. per ounce troy, the price of copper $3\frac{3}{4}$*d* per lb. troy, and if the rate of exchange is 1*s*. $1\frac{3}{8}$*d*. per rupee, find the total value in rupees of the silver and copper required for coining a lakh of rupees.

10. A merchant pays a lakh of rupees for a season's goods. He marks the goods 25 per cent. over prime cost, and from what he sells at this rate realises Rs. 1,12,500. At the end of the season he sells the remaining goods at reduced rates—one half at a reduction of 25 per cent. on the former prices, one-quarter at a reduction of 50 per cent, and the remainder at a quarter of the former prices. If the expenses of the business amount to 12 per cent. of the sale-receipts, what is his rate of profit on the transactions of the season?

11. Find the dimensions of a cubical cistern having the same capacity, as a tank 31 ft. 6 in long, and 1 ft. 9 in. deep.

1896.

2. Simplify :—

$$\frac{3\frac{7}{10} - 2\frac{1}{8} \times \frac{1}{2} + 2\frac{4}{5} \text{ of } (2\frac{1}{2} - 1\frac{6}{7})}{2\frac{1}{5} + 3\frac{1}{3} - \frac{6}{21} \text{ of } 10\frac{1}{2}} - \frac{1\frac{1}{6} \text{ of } 2\frac{3}{11}}{5\frac{6}{11} - 1\frac{2}{3}}$$

3. Prove that any number of pies can be expensed as thousandths of a rupee by multiplying the number by 5 and adding to the product $\frac{1}{24}$ of itself. Apply this rule to express 6*a*. 9*p*. as the decimal of a rupee, and verify the correctness of your result by obtaining it in another way. Find the value of $\cdot 0012\dot{3}7\dot{6}$ of a lakh of rupees.

4. Find by any method the cost of constructing a railway 329 miles 5 fur. 176 yds. long at R77,386. 13*a*. 4*p*. per mile.

5. A contractor undertook to finish a certain piece of work in 150 days. He employed 20 men, 30 women, and 75 children, but at the end of 60 days finding that only one-fourth of the work was done, he dismissed all the women and 50 of the children, and employed more men. The work was then finished 5 days before the stipulated time. Assuming that 3 men could do as much as 5 women, and 2 women as much as 3 children, find how many additional men were employed.

6. Find the present value of R1115. 13*a*. 9*p*. due 2 years hence at $3\frac{1}{2}$ per cent. per annum compound interest.

7. How much $3\frac{1}{2}$ per cent. stock at $109\frac{1}{2}$ must a person sell in order that by investing the proceeds in 3 per cent stock at $103\frac{1}{8}$ he may derive from the investment an annual income of R6,825-8, after paying income tax at the rate of 5 pies in the rupee ?

8. A grocer imports sugar at 15*s*. 4*d*. per cwt., the cost of which he remits when exchange is at the rate of 15 $2\frac{2}{8}$*d*. per rupee. Freight and landing charges amount to R38 6*a* per ton, and import duty at the rate of 10*a*. 6*p*. per cwt. has also to be paid. At what rate per maund of 25 lbs. must the grocer sell the sugar so as to gain 12 per cent. on his total outlay ?

9. In a certain year the total value of the exports from the Presidency of Madras showed an increase of 12 5 per cent. as compared with the total value of the exports for the previous year. Of the various items of export, coffee, which in the first of these two years represented 13·59 per cent. of the total value of the exports, showed an increase of 7 5 per cent. What percentage did coffee represent of the total value of the exports in the second of the two years ?

10. In a certain year the quantity of wheat raised in a country was 54,000,000 bushels Of this one-third was available for exports at R2. 4*a*. per bushel. In the following year the acreage under wheat showed an increase of 20 per cent., but the yield per acre was only three-fourths of what it was in the previous year, and of the total quantity of wheat raised only one-fourth was available for export. If the value of wheat exported in this year was $101\frac{1}{4}$ lakhs of rupees less than in the previous year, what was the export price of wheat per bushel ?

11. Extract the square root of 4985 067295890281 to six places of decimals.

1897.

2. Simplify :—

$$\frac{\frac{6}{7} \text{ of } 2\frac{5}{8} - \frac{3}{7} \text{ of } 3\frac{1}{3}}{\frac{5}{7} \text{ of } 2\frac{3}{8} \text{ of } 1\frac{3}{7} - 1\frac{4}{5}} + \frac{4\frac{1}{4} - 3\frac{1}{3}}{4\frac{1}{4} \times \frac{3}{8}}.$$

3. Find the value of ·875 of R4. 5*a*. 4*p*. + ·859375 of R1. 5*a*. 4*p*. − $\cdot 5\dot{7}4\dot{0}$ of R3. 1*a*. 6*p*.

4. Find by any method the value of 21 tons 17 cwt. 2 qrs. 23 lbs. of coffee at R1547. 4*a*. 8*p*. per ton.

5 When the price of gram is 11 measures per rupee, the cost of gram for 24 ponies for 31 days is R182. 2*a*. Assuming that, 5 horses

require as much as 8 ponies, find the cost of gram for 25 horses for 6 weeks when the price is $10\frac{1}{2}$ measures per rupee.

6. The interest on a certain sum of money for 3 months at 5 per cent. per annum exceeds the true discount on the same sum due 3 months hence at the same rate of interest by *11a. 3p.* Find the sum.

7. A person holding sixty rupees 500 shares in a concern which paid dividend at the rate of 5 per cent. per annum, sold out when the shares were at R625 and invested half the proceeds in $3\frac{1}{2}$ per cent. Covernment paper at 105 With the other half he bought a house, for which he received an annual rental of R1920, subject to a deduction of *4a. 3p.* per rupee for repairs and taxes. Find the alteration in his annual income

8 50 men, 100 women, and 150 children, working for a certain time on a tank bund, earn together R1181. 4*a*. If the wages of a man, a woman and a child be 4*a*. 2*a*, 6*p*., and 1*a*. 6*p*, per day respectively, find how much is earned on the whole by each man, woman, and child.

9. A merchant buys a quantity of tea at an average rate of 12*a* 6*p*. per lb. He assorts the tea into three kinds, which he sells at R1. 2*a*, 14*a*, and 9*a*. per lb. respectively. If in the process of assortment $2\frac{1}{2}$ per cent. of the tea is lost, and if of what remains 36 per cent. is of the dearest kind and 24 per cent. of the cheapest, find the merchant's gain per cent. on the transaction.

10. A merchant in Madras owes 12270 marks to a merchant in Hamburg. If exchange on Hamburg is at the rate of 1·32 marks per rupee, while exchange on London is at the rate of 1*s*. $3\frac{3}{8}$*d*. per rupee and the exchange between London and Hamburg is 20·45 marks per pound, sterling, find, to the nearest pie, how much the merchant will gain by remitting through London instead of direct.

11. The length of a field containing 21 ac. 3 ro 25 sq. po. $3\frac{3}{4}$ sq. yds. is twice its breadth. Find the length of the field.

1898

1. The following table shows the expenditure from the provincial, local, and municipal funds on grants in aid of education in the Madras Presidency for ten years ending 1896-97 :

Year.	Expenditure from Provincial Funds.	Expenditure from Local Funds.	Expenditure from Municipal Funds.
	Rs.	Rs.	Rs.
1887-88	4,89,405	1,97,346	90,146
1888-89	4,17,759	2,05,533	95,435
1889-90	5,02,538	2,02,257	88,751
1890-91	6,40,341	2,18,520	99,190
1891-92	6,33,029	2,24,358	96,401
1892-93	6,15,727	2,32,505	1,01,852
1893-94	6,17,663	2,31,936	1,05,348
1894-95	5,94,349	2,25,947	93,665
1895-96	6,19,780	2,32,460	99,124
1896-97	6,23,218	2,39,285	1,07,280

Write down (*a*) the total expenditure from provincial funds during these ten years, (*b*) the total expenditure from municipal funds, (*c*) the total expenditure from provincial, local, and municipal funds in each of the years 1890-91, 1896-97.

2. Simplify $\dfrac{8\frac{6}{7}-7\frac{5}{6}+6\frac{4}{5}-5\frac{3}{4}}{(4\frac{1}{3}+3\frac{1}{4})+(3\frac{1}{2}+2\frac{1}{3})}-\dfrac{\frac{4}{7}-\frac{3}{5}}{\frac{2}{5}\times\frac{3}{4}}$.

3. Find the value of $\cdot 11\dot{4}8\dot{1}$ of R9. 13*a*. 6*p*.; and express the difference between $\frac{15}{16}$ of R37. 13*a*. 4*p*. and $\frac{23}{24}$ of R37. 2*a*. as the decimal of R50.

4. Find the cost of having a rectangular area 35 ft. 9 in. long and 23 ft. 6 in. broad, at the rate of R5. 13*a*. 6*p*. per square yard.

5. Two watches, one of which gained at the rate of $1^m\,54^s.6$ and the other lost at the rate of $1^m\,55^s.8$ daily, were set correctly at noon on the 1st of January 1896. When did the watches next indicate the same time, and what time did each of them indicate?

6. Find what sum will amount to R7,364, 10*a*. 9*p*. in two years at $3\frac{1}{2}$ per cent. per annum compound interest.

7. A person invested R16,500 in $3\frac{1}{2}$ per cent. Government paper at $96\frac{1}{4}$ and an equal sum in bank shares of the nominal value of R500 each. If, when the bank is paying dividend at the rate of R50 per share, his annual income from the bank shares exceeds his annual income from the Government paper by R87. 8*a*., find what he paid for each of his bank shares.

8. A merchant at Madras imports 600 tons of English Coal, the price of which at the pit's mouth is 12*s*. 6*d*. per ton. The cost of carriage and freight to Madras amounts to 16*s*. $6\frac{1}{2}d$. per ton, and landing charges amount to R3. 5*a*. 4*p*. per ton. If the merchant remits the price of the Coal and the cost of carriage and freight to Madras when exchange is at the rate of 1*s*. $3\frac{15}{16}d$. per rupee, at what rate per maund of $82\frac{2}{7}$ lbs. avoirdupois must he sell the coal in Madras in order that he may gain R3,255 on the transaction?

9. In 1896 the working expense of a certain railway amounted to 50·8 per cent. of the gross earnings, and the net earnings to 4·41 per cent. of the total expenditure. In the same years the working expense of another railway amount to 54·9 per cent. of the gross earnings, and the net earnings to 5·25 per cent. of the total capital expenditure. If the total capital expenditure on the former railway was 1222·5 lakhs of rupees, and if the gross earnings of the latter were four-fifths of the gross earnings of the former, what was the total capital expenditure on the latter?

10. In a certain year 2·5 per cent. of the articles given out for delivery from post offices in the Presidency of Madras were returned undelivered. Next year there was an increase of 7·5 per cent. in the number of articles given out for delivery and an increase of 10·5 per cent. in the number of articles returned undelivered. If in this year the number of such articles was 1,957,176, find how many articles were given out for delivery in each year.

11. Extract the cube root of 1754·099916 to two places of decimals. [The remainder must be written down].

PUNJAB UNIVERSITY.

1898.

1. State in words the value of the figures 2000690125, and multiply ·056931 by 1879·6958.

2. On what day of the week will December 25 fall next year ?

3. Find the cost of papering the walls of a room 22 ft. long, 18 ft. wide and 20 ft. high, with rolls of paper 21 inches wide, at R2. 10*a.* per roll of 12 linear yards.

4. Simplify

$$\frac{\frac{3}{11}\left(\frac{2}{19} \text{ of } 2\frac{5}{7}+\frac{1}{3} \text{ of } 1\frac{2}{7}\right)}{\frac{2}{13}\times 1\frac{3}{8}\times 1\frac{1}{7}-\frac{1}{11}}+\frac{\frac{1}{3}}{\frac{2}{9} \text{ ot } 3-\frac{1}{17} \text{ of } 5\frac{4}{5}}.$$

5. A person holding £10.000 in the 3 per eents. sells out at 93⅝, and invests the proceeds in the 4 per cent. Stock at 101⅞. Find the change in his income, allowing ⅛ per cent. commission in each transaction.

1899.

1. Express $\cdot 763\dot{9}$ as a non-recurring decimal.

(*a*) Simplify $\cdot 00\dot{6}2\dot{1}+\cdot 1089\dot{1}+81\cdot\dot{0}56\dot{3}+21\cdot\dot{0}\dot{2}$ *without reducing the terms to vulgar fraction.*

2. The length of :a hall is three times the breadth. The cost of whitewashing the ceiling at 5½*d.* per square yard is £4. 12*s.* $7\cdot\dot{1}d.$ and the cost of papering the walls at 1*s* 9*d.* per square yard is £35. Find the height of the wall.

3. Shew that the difference between the interest and the true discount on a given sum at a given rate for a given time is equal to the interest of the discount.

4. A man has £5. 17*s.* consisting of sovereigns, half-crowns, and shillings, in the proportion of 2, 3, 11. How many has he of each coin ?

5. Which is the better investment, the 3½ per cents. at 102, or the 3 per cents. at 97 ?

1900.

1. Find the square root of 4001204·090601.

2. Find the present worth of R10000 due 8 years hence at 4½ per cent.

3. A rectangular court yard the sides of which are as 5 : 11. costs R144. 6*a.* for having at 10*a.* 6*p.* per square yd. Find the lengths of its sides.

4. Shew that *this* year the 23rd of March, and 23rd of November fall on the same day of the week.

5. Shew that Compound Interest reckoned quarterly at R1. 3*a*. $7\frac{1}{2}$*p*. per cent. is nearly equal to Interest reckoned yearly at 5 per cent.

1901.

1. Eight bells which toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds respectively, begin tolling all simultaneously with the clock striking. How many hours must elapse before they all toll simultaneously again with the clock striking? (N. B.—The clock is supposed to strike at the hour only).

2. Find the true discount on a bill for £721. 13*s*. 8*d*. paid 73 days before due, the rate of interest being $3\frac{1}{2}$ per cent. per annum.

3. Divide each of the numbers 4061250 and 2572125 by 125 and express the ratio of the quotients correctly to three places of decimals.

4. A man buys eggs at 1*s*. 3*d*. per dozen and sells them at 11*s*. 8*d*. per hundred. Find his gain per cent.

5. There are four vessels of equal capacity; $\frac{1}{2}$ of the first is filled with spirit, $\frac{1}{3}$ of the second, $\frac{1}{4}$ of the third, and $\frac{1}{5}$ of the last. The first is then filled with water and from this mixture the second is filled up, again from the second mixture, the third is filled up, and in like manner the fourth from the third. What proportion of spirit to water is there in the fourth vessel?

1902.

1. Define a prime number. Find the prime factors of 555555.

2. A railway truck is 29 ft. 4 in. in length; how many such trucks will be required to fill up the entire length of the line between Lahore and Amritsar, a distance of 32 miles?

3. The difference between the simple and compound interest on a sum of money for 2 years at 5 per cent. per annum is R12. Find the sum.

4. If 3 fowls aad 3 pigeons cost R2. 3*a*. 6*p*., and 5 fowls and 2 pigeons cost R2. 12*a*., find what must be paid for 4 fowls and 3 pigeons.

5. A person sold 60 yards of cloth for R28. 2*a*. gaining thereby the cost price of 9 yards. Find his gain per cent.

1901.

1. (*a*) What is the greatest length which is contained a whole number of times exactly in both $25\frac{15}{28}$ feet and $21\frac{9}{20}$ feet ?

(*b*) Find the value of $\frac{\cdot 49}{2\cdot 1}$ of $\frac{(3\frac{1}{3}-2\frac{1}{2})\div\frac{5}{8}\text{ of }\frac{3}{8}}{2\frac{2}{3}\div(\frac{1}{3}+\frac{1}{4})}$ of £46.

2. (*a*) Express the difference between $\cdot 9\dot{4}2857\dot{1}$ and $\cdot\dot{8}5714\dot{2}$ as a vulgar fraction in its lowest terms.

(*b*) Extract the square root of $\frac{\cdot 0253\times\cdot 365}{8\cdot 03}$ to 5 places of decimals.

3. In a two mile race *A* wins, *B* being 22 yards behind, and *C*, 106 yards behind *B*. By how much would *B*, beat *C* in a three mile race ?

4. What sum at a compound interest will amount to R650 at the end of the first year and to R676 at the end of the second year ?

5. How much $3\frac{1}{2}$ per cent. Government Securities at $95\frac{1}{4}$ must be sold out in order to purchase enough 5 per cent. Calcutta Municipal Debentures at $119\frac{3}{4}$ to produce an annual income of R665 ? (a brokerage of $\frac{1}{4}$ per cent. being charged on each transaction).

1902.

1. Find the *G. C. M.* and *L. C. M.* of 49·383 and ·142569.

2. Simplify :— $\frac{1\cdot 5}{\cdot 075}\times\frac{3\frac{1}{4}}{1\frac{1}{2}}+\frac{1\cdot 875}{2\cdot 1}\times\frac{3\cdot 5}{2\cdot 75}-\cdot 1\dot{6}$.

3. Find by Practice the value of $246\frac{3}{8}$ maunds of sugar at R13. 5*a*. 4*p*. per maund.

4. *A* and *B* have between them 132 horses ; ·25 of *A*'s $=\cdot\dot{1}4285\dot{7}$ of *B*'s. How many had each of them ?

5. Six men and five boys can do a piece of work in 7 days : they work at it till they have completed $\frac{3}{4}$ of it ; then two of the men leave and two more boys come. How long will the work be in hand, if a boy does half as much work as a man ?

6 If I lend a friend R1250 at 4 per cent. simple interest and tell him to keep it until principal and interest amount to R1666. 10*a*. 8*p*., how long will he have it ?

ANSWERS.

Ex. 1. (p. 4.)

1. Fifteen; twenty-nine; fifty-one; seventy-six; eighty-five; ninety-two; forty-nine; ninety-seven; thirty-three; twenty-five; sixty-seven; forty-five; thirty.
2. 3 hun. eighteen; 7 hun. seven; 5 hun. ninety-four; 4 hun. twenty; 5 hun. seventy-six; 5 hun. ten; 8 hun. thirty-two; 2 hun. forty-five; 2 hun. eight; 4 hun. seven; 8 hun. sixty; 7 hun. one.

Ex. 2. (pp. 5-6.)

1. 7 thd. 903; 8 thd. 25; 2 thd. 107; 3 thd. 3; 9 thd. 67; 5 thd. 988; 6 thd. 250.
2. 29 thd. 61; 70 thd. 635; 29 thd. 143; 39 thd. 672; 69 thd. 2; 80 thd. 103.
3. 123 thd. 456; 706 thd. 709; 880 thd. 87; 567 thd. 890; 800 thd. 721.
4. 5 mil. 189 thd. 879; 9 mil. 95 thd. 56; 4 mil. 94 thd. 704; 5 mil. 206 thd. 650; 4 mil. 870 thd. 320.
5. 79 mil. 38 thd. 25; 80 mil. 753 thd. 45; 19 mil. 800 thd. 769; 36 mil. 850 thd. 67.
6. 805 mil. 566 thd. 931; 860 mil. 370 thd. 635; 900 mil. 300 thd. 300.
7. 8 thd. 729 mil. 639 thd. 167; 2 thd. 906 mil. 150 thd. 216; 3 thd. 727 mil. 529 thd. 61.
8. 30 thd. 216 mil. 917 thd. 720; 21 thd. 900 mil. 186 thd. 692; 70 thd. 80 mil. 90 thd. 300.
9. 960 thd. 530 mil. 975 thd. 643; 7 bil. 6 thd. 398 mil. 7 thd. 346.
10. 90, 2; 400, 30, 5; 3000, 700, 5; 30000, 700. 20, 5; 90000, 2000, 300, 70, 5; 100000, 3000, 500, 90, 5; 100000, 70, 5; 3000000, 50000, 6; 6000000000, 700000000, 30000000, 200000, 5000, 800, 90.
11. 9 mil. 999 thd. 999; 10 mil.

Ex. 3. (p. 6.)

1. 53; 72; 37; 89; 17; 95; 44; 26; 60.
2. 300; 405; 960; 503; 624; 403; 807.

Ex. 4. (p. 7.)

1. 8000; 7400; 6002; 2530; 4744; 5001.
2. 57974; 39180; 48207; 67073; 40700; 23009.
3. 935473; 724008; 602045; 400404; 909090.
4. 4738685; 7020099. **5.** 9203161; 5019380; 7006009.
6. 72316115; 99909090; 70077007; 30040008.
7. 372072720; 209000110; 296008060.
8. 1260705201; 7005052070; 9000009009.
9. 30370360306; 52012109098. **10.** 890123556096; 700070007007.
11. 8000080080080; 7000000000707; 5000005005005.
12. 7556797249725. **13.** 510000079007007; 703056070820000.
14. 9999999999; 100000000000.

Ex. 5. (p. 8.)

1. 24 thd. 3 hun. 92; 75 thd. 3 hun. 81; 90 thd. 6 hun. 13; 90 thd. 6 hun. 11; 53 thd. 5 hun. 45; 72 thd. 1 hun. 31.
2. 29 lacs 6 thd. 96; 32 lacs 1 thd. 7 hun. 6; 3 lacs 34 thd. 56; 1 crore 30 lacs 70 thd. 6; 9 lacs 1 hun. 94.
3. 3 crores 2 lacs 16 thd. 9 hun. 17; 30 crores 40 lacs 24 thd. 3 hun. 40; 2 crores 96 lacs 8 hun. 36; 9 crores 99 lacs 99 thd. 9 hun. 99.
4. 7 crores 71 lacs 96 thd 1 hun. 76; 5 crores 80 lacs 26 thd. 2 hun. 60; 88 crores 30 lacs 70 thd.
5. 519604; 3108054; 95780705; 70500000; 573009005.
6. 1705000003; 30990501020. **7.** 100; 3 thd.; 500.
8. 1 hun-thd. billions; sankha.

Ex. 6. (p. 9.)

1. 6; 14; 24; 99; 98. **2.** 159; 276; 339; 540; 590.
3. 690; 628; 609; 499; 721; 429.
4. VIII, XV, XXVII, XXXV, XLVI, LI, LXII, LXXVII, XCIV, CVIII, CXXVII.
5. XIX, XXIX, XXXIX, XLIX, LIX, LXIX, LXXIX, LXXXIX, XCIX, CIX, CXXXIX.
6. CCXVII, CDXCIV, DLXVII, MMCXXXIX, MMDCCLXVIII, $\overline{\text{VII}}$ICMLIII, $\overline{\text{LXVII}}$CCXLI, $\overline{\text{XL}}$LXIX.

Ex. 7. (pp. 11-12.)

4. 36. **5.** 35. **6.** 28. **7.** 44. **8.** 46. **9.** 32.
10. 23. **11.** 21. **12.** 50. **13.** 30. **14.** 42. **15.** 38.

16. 33. **17.** 31. **18.** 42. **19.** 73. **20.** 52. **21.** 57.
22. 39. **23.** 15. **24.** 40. **25.** 15.

Ex. 8. (pp. 14-16.)

1. (1) 160. (2) 73. (3) 201. (4) 205 (5) 201.
(6) 143. (7) 156. (8) 225. (9) 63. (10) 220.
(11) 160. (12) 180. (13) 2549. (14) 2059. (15) 2261.
(16) 2216. (17) 1178. (18) 1457. (19) 586. (20) 2467.
(21) 1893. (22) 30506. (23) 25494. (24) 23369. (25) 6340.
(26) 24277. (27) 41446. (28) 13724. (29) 37973. (30) 23281.
(31) 343110. (32) 232630. (33) 114967. (34) 245533.
(35) 249903. (36) 278064. (37) 188828. (38) 1523265.
(39) 1938800. (40) 3119315. (41) 942242. (42) 957730.
(43) 1438041. (44) 345676544. (45) 2666736664.
(46) 165664385. (47) 317265555. (48) 93014275.
2. (1) 346029. (2) 4700037. (3) 16335199.
(4) 10431010. (5) 80088050. (6) 79325069834.
3. (1) 5473624 (2) 22758936. (3) 6189584002. (4) 3478145.
(5) 1008987500. (6) 10001371174240. **4.** 176662. **5.** 485616.
6. 878092. **7.** 148222636. **8.** 55055844468817. **9.** 7081913902.
10. 622. **11.** 286. **12.** 2354162. **13.** 3153.
14. 365. **15.** 61293. **16.** 24761. **17.** 1878.
18. 47 ; 19. **19.** 423360. **20.** 655344. **21.** 427.
22. 174753979. **23.** 62483. **24.** 301. **25.** 10595.

Ex. 9. (pp. 17-18.)

7. 11. **8.** 17. **9.** 20. **10.** 31. **11.** 40.
12. 13. **13.** 10. **14.** 14. **15.** 14. **16.** 30.

Ex. 10. (pp. 19-21.)

1. (1) 37. (2) 30. (3) 210. (4) 432. (5) 128. (6) 422.
(7) 207. (8) 391. (9) 208. (10) 170. (11) 53152. (12) 36038.
(13) 34115. (14) 40369. (15) 54734. (16) 5879. (17) 2226765.
(18) 964706. (19) 2803339. (20) 776998. (21) 5092198. (22) 6723435.
(23) 46926299. (24) 12713999. (25) 6621988. (26) 6779941.
(27) 5432099. (28) 2623188. (29) 2177478. (30) 2239329.
(31) 9007663. (32) 8512239. (33) 7326224. (34) 4970309.
(35) 4581447. (36) 1233866. (37) 22309347.
(38) 536646440. (39) 261197629. (40) 155765897.

2. (1) 33013919; 235740951. (2) 11998822; 36696941.
(3) 864197532; 53815307. (4) 318824605; 135898997.
(5) 312167728; 10129850.

3. (1) 57357772; 721038. (2) 370915475; 71234782.
(3) 10959983; 35297197. (4) 103394997; 24874267.

4. 5433102. **5.** 94398; 4300000; 1051574.

6. 9999771; 9999496; 9992368; 9922095; 9769996. **7.** 3493913.

8. 60236. **9.** 30. **10.** 12571. **11.** 2400. **12.** 20415.

13. 4957. **14.** 52. **15.** 63. **16.** 2514. **17.** 5947402.

18. 48. **19.** 16331. **20** 1039603. **21.** 635481. **22.** 157.

23. 116. **24.** 1495, 191. **25.** 22. **26.** 1837, 1887.

27. 27864000. **28.** 887365.

Ex. 11. (p. 22.)

1. 4. **2.** 24. **3.** 665. **4.** 3520. **5.** 31058.

6. 475. **7.** 81, 676, 5968, 43211.

Ex. 12. (p. 25.)

2. 65. **3** 85. **4.** 144. **5.** 72. **6.** 102. **7.** 195.

8. 240. **9.** 153. **10.** 72. **11.** 225. **12.** 12; 34; 154.

13. 126. **14.** 208. **15.** 607. **16.** 284.

Ex. 13. (pp. 26-27.)

1. (1) 64312; 96468; 128624; 160780; 192936; 225092; 257248; 289404.
(2) 6835801; 7812344; 8788887; 10741973; 11718516; 12695059; 13671602; 14648145.
(3) 17307276; 28845460; 40383644; 51921828; 63460012; 74998196; 86536380; 98074564; 109612748.
(4) 18584386; 37168772; 55753158; 74337544; 92921930; 111506316; 130090702; 148675088; 167259474; 176551667; 185843860.

2. (1) 6050114; 7458960; 9136305. (2) 53419506; 8087365; 12123124.
(3) 3059604; 6693988; 108219225.
(4) 867216; 716482; 1908724990.

3. (1) 6729780; 67297800; 672978000; 6729780000.
(2) 14699300; 146993000; 1469930000; 14699300000.
(3) 20845710; 27794280; 34742850; 41691420; 48639990; 55588560.
(4) 208457100; 277942800; 347428500; 416914200; 625371300.
(5) 91904120; 1002590400; 10861396000; 133678720000.

4. 77625600; 596160000. **5.** 9051048; 253074. **6.** 13844690.
7. 122773. **8.** 2160. **9.** First, 2190; youngest, 14148. **10.** 5295.

Ex. 14. (pp. 29-30.)

1. (1) 41487255; 61769913; 82052571; 13274562; 17758153.
(2) 657232365; 812038860; 45750319830; 56334868710.
(3) 5917763181; 7891509720; 5775721000; 6267701090.

2. (1) 6517947673083. (2) 8718916617800. (3) 5776374258223.
(4) 6367923228416. (5) 2910195338300. (6) 3375824060900.
(7) 2647684986520. (8) 31028191436. (9) 1567924859234 56.
(10) 1114580299 7120. (11) 2710898334086. (12) 7356254297418 0.
(13) 10679841779 1156. (14) 292849150619367.
(15) 259062146860 7007. (16) 26963692373 82. (17) 930142601333.
(18) 343278572130 00. (19) 1379992070627 43. (20) 1689802489883 3.

3. (1) 32245362; 22407561. (2) 815947008; 5940111105.
(3) 7903377953520; 4696957448662.
(4) 12009157671325269; 33672050580121500.
(5) 70498687429049005 0000; 3613984653761230749000.

4. 1675520. **5.** 351468. **6.** 305152. **7.** 77872310.
8. 19034. **9.** 1776000. **10.** 4384375. **11.** 15075540.
12. 15081800. **13.** 579975. **14.** 53597460. **15.** 497040.

Ex. 15. (p. 31).

1. (1) 23400. (2) 15120. (3) 109440. (4) 66990. (5) 742560.
(6) 288000. (7) 4342500. (8) 3450000. (9) 1377215424.
(10) 2497500. (11) 9371250000. (12) 20353998109. **2.** 720 **3.** 2350.
4. 20160. **5.** 1929375. **6.** 2158080. **7.** 365625. **8.** 156800.

Ex. 16. (pp. 31-32).

1. (2) 15625, 105625, 275625, 525625.
2. (1) 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728, 2197, 2744, 3375, 4096, 4913, 5832, 6859, 8000, 27000, 125000, 729000.
(2) 389017, 857375, 1601613, 2685619, 16974593.
3. (1) 35. (2) 10. (3) 326. (4) 6913
4. (1) 1099. (2) 110. (3) 154. (4) 128. (5) 0. (6) 14940.

Examples A. (p. 32).

1. 8324975. **2.** 118407. **3.** 2441411. **4.** 63268180.
5. R184896. **6.** 14150 **7.** R59925. **8.** R86575.
9. Borrows R58203. **10.** 405. **11.** R109801. **12.** 14856 mi. **13.** 14144mi.

Ex. 17. (pp. 34-35.)

4. 4. **5**. 15. **6**. 19. **7**. 16. **8**. 12 **9**. 13. **10**. 11.
11. 12. **12**. 15. **13**. 7 ; 23. **14**. 19 **15**. Nothing. **16**. R7.

Ex. 18. (pp. 35-36.)

1. (1) 369, 123, 82 ; 2108, 766-6.
(2) 32451-1 ; 19470-4 ; 13907-5 ; 10817-1 ; 8850-4 ; 6490-4.
(3) 28828-1 ; 19218-9 ; 17740-5 ; 15375 ; 14414-1 ; 13566-3.
(4) 227634-5 ; 202341-8 ; 151756-5 ; 130076-13 ; 113817-5 ; 101170-17 ; 95846-3
3. 1802. **4**. 31 pice. **5**. 385. **6**. 354 ; 347.

Ex. 19. (pp. 37-38.)

1. (1) 54187. (2) 61625. (3) 190132. (4) 105812. (5) 1274997. (6) 577157. (7) 81454. (8) 1140994. (9) 71340986. (10) 8736504; 3744216 ; 2299080 ; 1899240 ; 1820105 ; 1541736 ; 1337220.
2. (1) 234915. (2) 58468-56. (3) 704745. (4) 33798-51.
(5) 13972-297. (6) 8215-613. (7) 76983-218. (8) 62656-753.
(9) 922459-504. (10) 8637-32. (11) 22151-637.
(12) 21972-1407. (13) 10002-2105. (14) 368-626.
(15) 103-3522. (16) 1203-3484. (17) 115-27034. (18) 2613-8061.
(19) 158-58991. (20) 510-20331. (21) 17774-317687.
(22) 27027-42598. (23) 1370-50317. (24) 6084. (25) 874360-247173.
(26) 764095. (27) 21956-38272709. (28) 22611-794005.
(29) 210849-24244848. (30) 307930337-2578859.
3. 1598 ; 1081 ; 782. **4**. 1123. **5**. 90. **6**. 12. **7**. 126. **8** 234915.
9. 5912. **10**. 65. **11**. 40. **12**. 2456. **13** 4972. **14**. 121. **15** 320.

Ex. 20. (p. 39.)

1. (1) 4567-8, 2283-18, 1522-18, 913-28, 652-38, 507-48.
(2) 890-12, 445-12, 222-212, 178-12, 148-212.
(3) 4515-623, 4139-323, 3820-1123, 3104-723.
(4) 5173-1312, 4629-312, 418-17412, 20-255412.
(5) 1203-21306, 1085-8206, 863-25506.
(6) 89-4380, 235-180, 147-710, 42-5960.
(7) 714-53900, 127-139900, 65-506900.
2. (1) 6608-41, 5966-9, 5113-69, 4474-57.
(2) 52770-91, 35180-91, 24996-163, 14072-91.
(3) 7518687-16, 1708792-247, 1566393-79, 1038765-751.
(4) 1122400-415, 982100-415, 872978-175, 714254-1135.

Examples B. (p. 40.)

1. 8. **2.** 10. **3.** 20. **4.** 16. **5.** 800. **6.** 406. **7.** 50. **8.** 440. **9.** 599. **10.** 322. **11.** 38. **12.** 667. **13.** 392. **14.** 3581. **15.** 1220.

Ex. 21. (pp. 41-42.)

1. (1) 12. (2) 36. (3) 4. (4) 6. (5) 8. (6) 425938. **2.** 1498. **3.** 2436. **4.** 52001. **5.** 9780. **6.** 8015. **7.** (1) 1303. (2) 94303. (3) 64. (4) 165336. (5) 0. (6) 36. (7) 225. (8) 81. (9) 454.

Ex. 22. (pp. 44-45.)

1. 4057, 1567. **2.** 777, 48. **3.** 618, 382.
4. House, R15929 ; garden, R9700. **5.** Ram 40, Gopal 34, Hari 18.
6. R602, R148, R48. **7.** 164259. **8.** 2260676. **9.** 324.
10. 437. **11.** 541. **12.** 1279. **13.** 164. **14.** 271. **15.** 820.
16. 2124. **17.** 836. **18.** 156. **19.** 2601 ; 2550. **20.** 100 pice.
21. 10063 ; 99936. **22.** 99999024 ; 10001670.

Ex. 23. (pp. 47-48.)

1. (1) 419660, 2098300, 6294900, 10491500.
(2) 48319125, 241595625, 1207978125.
(3) 8367634836, 84437042436, 845131118436.
(4) 696198, 7426112, 74792626, 748815084.

2. (1) 1225, 2025, 3025, 4225, 5625, 7225, 9025, 13225. 15625, 21025, 70225, 180625.
(2) 1764, 3364, 6889, 9409, 11236, 17424, 24336, 13689, 29584, 33489, 38025, 38809. **3.** 396144 ; 734760 ; 1989000.

4. 999572, 999927054. **5.** 12780192 ; 19500543 ; 316352970.

Ex. 24. (p. 49.)

1. 9256·1, 925·31, 92·281. **2.** 156492·45, 15649·145, 1564·4645.
3. 37712·16, 12570·66, 6285·66. **4.** 19745·20, 6581·270, 2820·645.
5. 252·290 ; 69·7340.

Ex. 25. (p. 50.)

1. 62. **2.** R7. **3.** R640. **4.** 4440. **5.** *A* R200, *B* R400, *C* R600.
6. 1250, 2250, 2750. **7.** R60. **8.** R675.
9. 11 hrs. ; 275 mi., 220 mi. **10.** 6 hrs. ; 30 miles from Calcutta.

Miscellaneous Examples 1. (pp. 51-56.)

1. (*a*) 259007. (*b*) 540421. **2.** 44709. **3.** 311501. **4.** 5191847. **5.** 2028. **6.** (1) 69748835. (2) 96974834. **7.** 26889. **8.** R26500.

9. 138. **10.** 56 yrs.; 17 yrs. **11.** R425. **12.** 34. **13.** R7.
14. R21; R33. **15.** 17. **16.** 320. **17.** *A*, 29 yrs.; *B*, 34 yrs.
18. 624 mi. **19.** R132. **20.** *A*, R82; *B*, R119; *C*, R174.
21. 323. **22.** R40. **23.** (*a*) 2159. (*b*) 1579. **24.** 195804. **25.** 3473603.
26. 11441476. **27.** 19. **28.** *A*, R3568; *B*, R1166. **29.** 592 mds.
30. 40704. **31.** 2341. **32.** 5365. **33.** 12658-3666. **34.** 464. **35.** 7.
36. R17950. **37.** 560. **38.** R562500. **39.** 561 mi. **40.** 5 days; 75 mi. from Calcutta. **41.** R16525. **42.** R33790. **43.** R7.
44. R328. **45.** 22320 *kalsis*. **46.** 62088. **47.** 550. **48.** 309.
49. 3240 mi. **50.** 15 mi. **51.** 63. **52.** R563; R437. **53.** 1073.
54. 319. **55.** 652727. **56.** 42. **57.** R2. **58.** 7128900. **59.** 150 mi.
60. 427. **61.** R10760. **62.** 49. **63.** 201. **64.** 300. **65.** R4100.
66. R330. **67.** R590. **68.** R140. **69.** R450. **70.** R1020
71. *A*, 74; *B*, 46; *C*, 25. **72.** 6 yrs. **73.** (*a*) 36. (*b*) 12236.
74. *A*, 776; *B*, 1164; *C*, 1358; *D*, 1552. **75.** 13 pice.

77.

2	9	4
7	5	3
6	1	8

78.

1	8	10	15
12	13	3	6
7	2	16	9
14	11	5	4

79. 390. **80.** 7728. **81.** 425.

Ex. 26. (pp. 59-60.)

1. (1) 512*a*., 752*a*., 1008*a*., 1312*a*., 1600*a*., 4160*a*.
(2) 200*a*., 518*a*., 1357*a*., 4517*a*.

2. (1) 1536*ps*., 4608*p*.; 2368*ps*., 7104*p*.; 6336*ps*., 19008*p*.; 15424*ps*., 46272*p*.; 28800*ps*., 86400*p*.; 46720*ps*., 140160*p*.
(2) 2940*ps*., 8820*p*.; 6568*ps*., 19704*p*.; 16992*ps*., 50976*p*.; 57909*ps*., 173727*p*. 64120*ps*., 192360*p*.; 605*ps*., 1815*p*.; 886*ps*., 2658*p*.; 1483*ps*., 4449*p*.; 3335*ps*., 10005*p*.; 1330*ps*., 3990*p*.; 6017*ps*., 18051*p*.; 9079*ps*., 27237*p*.

3. 8000*g*., 32000*c*.; 11840*g*., 47360*c*.; 24000*g*., 96000*c*.; 33920*g*., 135680*c*.; 5060*g*., 20240*c*.; 45300*g*., 181200*c*.; 66520*g*., 266080*c*.; 130860*g*., 523440*c*.; 288860*g*., 1155440*c*.; 2330*g*., 9320*c*.; 6345*g*., 25380*c*.; 32950*g*., 131800*c*.

4. 8000*ps*., 24000*p*.; 7360*ps*., 22080*p*.; 5952*ps*., 17856*p*.; 850*ps*., 2550*p*.; 20160*ps*., 60480*p*.; 11328*ps*., 33984*p*.; 627968*ps*., 1883904*p*.; 144920*ps*., 434760*p*.

5. 9350, 18700, 37400; 15240, 30480, 60960, 3346, 6692, 13384; 9724, 19448, 38896; 4912, 9824, 19648; 52102, 104204, 208408; 92596, 185192, 370384; 182492, 364984, 729968.

6. 1514, 6056; 3034, 12136; 6128, 24512; 44806, 179224; 22542, 90168: 45332, 181328; 4509, 18036; 30315, 121260.

7. 8500*s.*, 17840*s.*; 81220*s.*; 18498*s.*; 22393*s.*; 37855*s.*; 29052*s.*

8. (1) 35040*d.*; 40080*d.*; 48720*d.*; 454800*d.*; 1790400*d.*; 2357760*d.*
(2) 37164*d.*; 43380*d.*; 52212*d.*; 99996*d.*; 194340*d.*
(3) 8539*d.*; 22715*d.*; 27799*d.*; 170888*d.*

9. 677796*q.*; 77423979*q.*; 1691206*q.*; 8275247*q.*

10. (1) 646080, 1292160; 6086400, 12172800; 63816, 127632; 295968, 591936; 540912, 1081824.
(2) 22264, 44528; 28602, 57204; 357142, 714284; 672969, 1345938; 749205, 1498410; 3900801, 7801602; 581171, 1162342.
(3) 108, 216; 1008, 2016; 15072, 30144; 158460, 316920; 174000, 348000, 102400, 204800; 112392, 224784; 574560, 1149120; 946080, 1892160.

11. (1) 6000, 4500, 3000; 17600, 13200, 8800; 24320, 18240, 12160; 59200, 44400, 29600; 88960, 66720, 44480; 152960, 114720, 76480.
(2) 936, 702, 468; 9644, 7233, 4822; 9656, 7242, 4828; 124832, 93624, 62416; 369748, 277311, 184874.

12. 2324. **13.** 5724. **14.** 289. **15.** 3888.

Ex. 27. (p. 61.)

1. R9218. 12*a.* 1*ps.*, R611. 14*a.* 1 *ps.*; R265. 10*a.* 3*ps.*; R510. 1*a.* 1*ps.*; R267. 5*a.* 3 *ps.*; R1099. 13*a.* 3*ps.*

2. (1) R 5. 6*a.* 5*p.*; R483. 10*a.* 8*p.*; R372. 14*a.* 4*p.*; R159. 8*a.*, R283. 2*a.* 2*p.*; R365. 11*a.* 1*p.*
(2) R361. 11*a.* 2*p.*, R 15. 15*a.* 2*p.*; R375. 10*a.* 11*p.*; R335. 13*a.* 8*p.*; R209. 15*a.* 8*p.*; R1761. 4*a.* 2*p.*

3. (1) R31. 4*a.*, R579. 9*a.* 2*bu.*; R615. 8*a.* 9*g.* 2*c.*; R110. 7*a.* 13*g.*
(2) R736. 10*a.* 3*bu.*; R114 1*a.* 2*c.*; R240. 12*a.* 3*ps.*; R20. 10*a.* 13*g.*
(3) R29. 9*a.*; R7441. 5*a.*; R96. 5*a.*; R239. 3*a.*

4. (1) £386. 18*s.* 8*d.*; £298. 6*s.* 4*d.*; £127. 12*s.*; £226. 10*s.* 2*d.*; £292. 11*s.* 1*d.*; £289. 7*s.* 2*d.*
(2) £751. 7*s.* 4*d.*; £71. 3*s.* 10¾*d.*; £52. 10*s.* 6½*d.*; £316. 14*s.* 8*d.*; £614. 11*s.* 9*d.*; £10. 0*s.* 1½*d.*
(3) £74. 6*s.* 3¼*d.*; £10. 0*s.* 11*d.*; £126. 5*s.* 2½*d.*; £30. 13*s.* 9¾*d.*; £99. 10*s.* 1¾*d.*; £18. 1*s.* 4¼*d.*

5. £100802. 2s.; £239411. 0s. 6d.; £4336. 5s.; £59452. 12s. 6d.; £57511. 7s.; £1090. 12s. 6d.; £570. 0s. 4d.; £920. 3s. 6d.; £3289. 18s.

6. R82. 10a. 7. £1442. 14s. 9d. 8. £3084. 15s.

Ex. 28. (p. 62.)

1. 507, 1014; 1026, 2052; 1261, 2522; 4840, 9680; 2467, 4934.
2. 605; 887; 643, 169. 3. 20, 40; 80, 160; 240, 480; 960, 1920; 4800, 9600; 9600, 19200; 28800, 57600.
4. 41; 481, 9601. 5. R415; R717. 8a.; R45; R853. 4a.
6. £178-15-5; £8-17-7; £4-12-8. 7. R39915. 8. R1837. 8a.

Ex. 29. (pp. 63-65.)

1. (1) R2. 3a. 9p. (2) R3. 1a. 8p. (3) R2. 5a. 6p.
(4) R2. 8a. 10p. (5) R3. 1a. 7p. (6) R3. 0a. 6p.
(7) R61. 13a. 1p. (8) R80. 13a. 9p. (9) R321. 14a. 11p.
(10) R58. 1a. 9p. (11) R1116. 14a. (12) R429. 6a. 8p.
(13) R745. 8a. 8p. (14) R1429. 8a. 2. 35777. 0a. 7p.
3. (1) £63. 4s. 4d. (2) £192. 3s. 5d. (3) £179 8s. 3d.
(4) £338. 14s. 6d. (5) £2778. 7s. 3¼d. (6) £2686. 12s. 6¼d.
(7) £2924. 1s. 9d. (8) £2070. 10s. 3¾d. 4. £227824. 2s. 1½d.
5. R11977. 11a. 8p. 6. £5. 19s. 7¾d. 7. R51.

Ex. 30. (pp. 66-67.)

1. (1) R7. 4a. 1p. (2) R9. 2a. 1p. (3) R7. 3a. 2p. (4) R21. 3a. 1p.
(5) R7. 3a. 1p. (6) R61. 2a. 3p. (7) R81. 13a. 10p.
(8) R21. 15a. 9p. (9) R3. 4a. 10p. (10) R2. 13a. 10p.
(11) R7. 13a. 9p. (12) R3. 14a. 10p. (13) R9. 13a. 7p.
(14) 5a. 4p. (15) R10. 9a. 11p. (16) R9. 14a. 11p.
2. (1) £13. 2s. 2d. (2) £93. 18s. 8d. (3) £9. 18s. 2d. (4) 12s. 4d.
(5) £87. 19s. 8d. (6) £99. 8s. 1¼d. (7) 19s. 11½d.
(8) £81. 8s. 3¾d. (9) £1. 17s. 10¾d. (10) £7. 6s. 10¾d.
(11) £725. 17s. 6d. (12) £92. 16s. 10¾d. (13) £32. 18s. 10¾d.
(14) £7927. 18s. 11¼d. (15) £5. 14s. 2½d. (16) £113. 19s. 11¾d.
3. (1) R3. 1a. 4p. (2) R17. 11a. 9p. (3) R90. 12a. 9p.
(4) £5. 18s. 8d. (5) £100. 17s. 9¾d. (6) £8. 17s. 4d.
4. R5. 14a. 10p.; R52. 11a. 10p. 5. R192. 2a. 3p. 6. £486. 9s. 4d.
7. R27. 7a. 8. £152. 8s. 10d. 9. £1. 2s. 6d. 10. 14s. 11d.
11. R2800. 15a. 3p. 12. R13191. 15a. 13. £1396. 6s. 9d. 14. £7.

Ex. 31. (pp. 68-69.)

1. (1) R4. 7*a*. 2*ps*. ; R11. 2*a*. 3*ps*. ; R24. 9*a*. 1*ps*. ; R29. 0*a*. 3*ps*.
(2) R31. 2*a*. ; R46. 11*a*. ; R93. 6*a*. ; R116. 11*a*. 2*ps*.
(3) R33. 2*a*. 3*p*. ; R37. 14*a*. ; R75. 12*a*. ; R85. 3*a*. 6*p*.
(4) R130. 14*a*. 8*p*. ; R196. 6*a*. ; R245. 7*a*. 6*p*. ; R327. 4*a*. 8*p*.
(5) £70. 14*s*. ; £106. 1*s*. ; £123. 14*s* 6*d*. ; £159. 1*s*. 6*d*.
(6) £235. 5*s*. $7\frac{1}{2}$*d*. ; £250. 19*s*. 4*d*. ; £282. 6*s*. 9*d*. ; £313. 14*s*. 2*d*.
(7) R149. 13*a*. 4*p* ; R323. 13*a*. 4*p*. ; R468. 13*a*. 4*p*. ; R526 13*a*. 4*p*.
(8) R5755. 1*a*. ; R6666. 10*a*. 10*p*. ; R7633. 12*a*. 6*p*.
(9) £406. 8*s*. $10\frac{1}{2}$*d*. ; £820. 11*s*. $1\frac{1}{2}$*d*. ; £851. 4*s*. $7\frac{1}{2}$*d*.
(10) £59953. 1*s*. 6*d*. ; £52697. 5*s*. ; £54223. 10*s*.

2. (1) R797. 8*a* ; R1096. 9*a*. ; R1395. 10*a*. ; R1794. 6*a*.
(2) R3841. 4*a*. ; R4390 ; R4755. 13*a*. 4*p*. ; R5121. 10*a*. 8*p*.
(3) £943. 9*s*. $9\frac{3}{4}$*d*. ; £1029. 5*s*. 3*d*. ; £1486. 14*s*. 3*d*. ; £1715. 8*s*. 9*d*.
(4) £2505. 7*s*. 6*d*. ; £5938. 13*s*. 4*d*. ; £9279. 3*s*. 4*d*. ; £11135.

3. (1) R463 1*a*. 3*p* ; R913. 10*a*. 3*p*. ; R1289. 1*a*. 9*p*. ; R1414. 4*a*. 3*p*.
(2) £228. 17*s*. $5\frac{1}{4}$*d*. ; £396. 6*s*. $10\frac{1}{4}$*d*. ; £463. 6*s*. $7\frac{1}{4}$*d*. ; £876. 8*s*. $4\frac{3}{4}$*d*.
(3) R55084. 2*a*. 8*p*. ; R105642. 1*a*. 4*p*. ; R54033. 15*a*. 4*p*.
(4) £46301. 16*s*. $4\frac{1}{2}$*d*. ; £34994. 17*s*. ; £61686. 5*s*. $7\frac{1}{2}$*d*.

4. (1) R28. 11*a*. (2) R55. 2*a*. (3) R41. 8*a*.
(4) R56. 4*a*. (5) R286. 3*a*. 5*p*. (6) R1708. 8*a*.
(7) £176. 14*s*. (8) £285. 7*s*. 6*d*. (9) £672. 9*s*. $4\frac{1}{2}$*d*.
(10) £654. 2*s*. (11) £1503. 6*s*. 8*d*. (12) £5549. 10*s*.

5. (1) R555. 12*a*. 6*p*. (2) R11738. 10*a*. 4*p*. **6.** R68907. 6*a*.

7. £4684. **8.** R78. 12*a*. **9.** R19874. 8*a*. 6*p*.

Ex. 32. (pp. 71-72.)

1. (1—3) R2. 13*a*. 3*ps*. (4) R2. 13*a*. 8*p*. (5) R2. 13*a*. 9*p*. (6) R2. 14*a*. 11*p*.
(7) R3. 7*a*. 9*p*. (8) R3. 5*a*. 1*p*. (9—12) R3. 4*a*. 9*p*.
(13, 14) R39. 12*a*. 9*p*. (15) R48. 13*a*. 8*p*. (16) R48. 11*a*. 4*p*.
(17) £4. 19*s*. 8*d*. (18) £4. 19*s*. 10*d*. (19) £1. 13*s*. $3\frac{3}{4}$*d*.
(20) £5. 4*s*. 8*d*. (21) £7. 10*s*. 4*d*. (22) £12. 18*s*. 10*d*.
(23) £11. 19*s*. 6*d*. (24) £3. 17*s*. $7\frac{1}{4}$*d*. (25) £12. 1*s*. $7\frac{3}{4}$*d*.
(26) £12. 2*s*. $2\frac{1}{4}$*d*. (27) £19. 19*s*. $11\frac{3}{4}$*d*. (28) £1. 3*s*. $10\frac{1}{4}$*d*.
(29) £1. 14*s*. 2*d*. (30) £1. 0*s*. $9\frac{1}{2}$*d*.

2. (1, 2) R3. 8*a*. 3*p*.
(3) R3. 8*a*. 9*p*. (4) R9. 12*a*. 6*p*. (5, 6) R39. 14*a*. 11*p*.
(7, 8) £37. 15*s*. $4\frac{3}{4}$*d*. (9) £35. 1*s*. $5\frac{1}{4}$*d*. (10) £194. 12*s*. 3*d*.

3. (1, 2) R3. 4*a*. 3*ps*. (3) R3. 4*a*. 8*p*. (4) R5. 7*a*. 7*p*.
(5) £12. 18*s*. 11*d*. (6) £21. 17*s*. $2\frac{1}{4}$*d*. (7) £963. 1*s*. 8*d*.
(8) £450. 4*s*. $10\frac{1}{4}$*d*.

4. (1) R3. 2*a*. 1*p*. (2) R2. 3*a*. 4*p*.

(3) R5. 4*a*. 5*p*. (4) £5. 6*s*. 8*d*. (5) £7. 6*s*. 4*d*.
(6) £5. 9*s*. 11*d*. **5**. R5. 2*a*. 1*p*. **6**. R3. 2*a*. 4*p*. **7**. £1. 5*s*. 5*d*.
8. £2. 7*s*. 1½*d*. **9**. R15. 5*a*. 3*p*. **10**. R450. 13*a*. 6*p*.

Ex. 33. (pp. 72-73.)

1. (1) 166 ; 45. (2) 84 ; 85. (3) 15 ; 192. (4) 6 ; 57. (5) 29 ; 29.
2. (1) 20. (2) 25. (3) 75. (4) 90. (5) 4954.
3. 71. **4**. 5000. **5**. 47. **6**. 163. **7**. 1005. **8**. 109.

Ex. 34. (pp. 75-77.)

1. 93385 to., 74708 ka. ; 16745, 13396 ; 57265, 45812 ; 63495, 50796.
2. 77md. 8ch. 1to. 1308md. 6sr. 14ch. 1ka. ; 1478md. 35sr. 9ch. ; 77922md. 1po. ; 712md. 37 sr. 9 ch.
3. 99807 to. ; 1544 kan. 3 md. 2 vis. 3 sr. 6 pa. 2 to.
4. 20173024 dh. ; 627 kan. 14 md. 24 sr. 15 tan. 2 ma. 5 ra.
5. 27996dh. ; 40to. 7ma. 4ra. 1dh. **6**. 5184dh. ; 175to. 5 ma. 1ra.
7. 71680000 ; 9605120 ; 5870592 ; 11316992 ; 4441024 ; 19487838 ; 8733152.
8. 7tons. 19cwt. 15lb. 9oz. 9dr. ; 98 tons. 3cwt. 1qr. 14lb. 15oz. 14dr. ; 16 tons 3 cwt. 24lb. 14 oz. ; 25107tons. 15cwt. 2qr. 5lb. ; 25 tons 9 cwt. 3 qr. 21 lb. 8 oz. 2 dr.
9. 198720 ; 159816 ; 195264 ; 275417. **10**. 105lb. ; 51lb. 5oz. 6dwt. 10lb. 20gr. ; 16lb. 8oz. 17dwt. 4gr. ; 1lb. 3oz. 9dwt. 5gr.
11. 86400 ; 113100 ; 97840. **12**. 21lb. 9℥. 6ʒ. 1℈. 18gr. ; 85lb. 4℥. 2℈. ; 164lb. 3 ℥. 2ʒ. 11gr. **13**. 79200 ; 287640. **14**. 1qr. 8lb. ; 3qr. 24lb. ; 4cwt. 3qr. 8lb. ; 9cwt. 2qr. 16lb. **15**. 14md. ; 70 md.
16. 46 Bombay md.; 560 Madras md.; 1ton 6cwt. 13lb.; 2tons 14cwt.3qr.
17. (1) 1154md. 8sr. 13ch. (2) 93md. 4sr. 7ch.
(3) 3lb. 5 oz. 13dwt. 14gr. (4) 236 lb. 7 oz. 2dwt. 13gr.
(5) 119tons 12cwt. 8lb. 8 oz. (6) 222tons 13cwt. 2qr. 24lb. 15 oz. 14dr.
18. (1) 12md. 36sr. 6ch. ; 215md. 18sr. 14ch.
(2) 1 oz. 18dwt. 21gr. ; 13lb. 8 oz. 17dwt.
(3) 2 lb. 1 oz. 2dr. ; 26lb. 2 oz. ; 2 cwt. 3qr. 2lb. ; 165 tons 3qr.
19. 65106md. 30sr. **20**. 3277md. 29sr. 8ch. **21**. 5md. 4sr. 9ch.
22. 2153. **23**. 88. **24**. 251. **25**. 3840. **26**. 736md. 16sr. 43tan.
27. 123lb. 4 oz. 12dwt. 12gr. **28**. 64. **29**. 193 tons 10 cwt.
30. 3qr. 17lb. 13 oz. **31**. 1392. **32**. 4043tons. 8cwt. 2qr. 7lb.
33. 1cwt. 2qr. 7lb. **34**. 143. **35**. 100. **36**. £29. 2*s*. 5*d*. **37**. Cork ; 1240gr.

Ex. 35. (pp. 78-79.)

1. (1) 83200 ; 81000 ; 1980 ; 2475.
(2) 950gj. 1ht. 2gi. 1an. ; 454gj. ; 5914gj. 4gi.
2. (1) 44286 ; 248514 ; 478728. (2) 1582398 ; 3665970 ; 3635257.
3. 1fur. 9po. 4yd. 1ft. 6in. ; 1fur. 17po. 1yd. 2ft. 6in. ;
1mi. 1fur. 15po. 5yd. 11in. ; 7mi. 2fur. 14po. 9in. ;
1mi. 6fur. 13po. 2yd. 1ft. 6in. ; 12mi. 3fur. 25po. 5yd. 1in. ;
38mi. 6fur. 25po. 1yd. 2ft. ; 113mi. 6fur. 34po. 4yd. 2ft. 2in. ;
145mi. 3fur. 18po. 2yd. 1ft. 6in. ; 1146mi. 1fur. 7po. 5yd.
4. (1) 34mi. 160yd. (2) 25kr. 640ht. (3) 52yd. 9in.
(4) 47ft. 3in. (5) 364n. (6) 444n. (7) 135in. (8) 56000lk.
5. (1) 68yd. 1ft. 9in. (2) 4fur. 19po. (3) 235mi. 30po. 5yd. 2in.
6. (1) 16yd. 1ft. 10in. (2) 34po. 4yd. 2ft. 3in. (3) 5mi. 30p. 4yd. 1ft.
7. 73mi. 7f. 2yd. 2ft. ; 211mi. 24p. 1yd. 1ft. ; 295mi. 4f. 1po. 5yd. 6in.
8. 70yd. 2ft. 10in. ; 5fur. 14po. 1yd. 2ft. 7in ; 1m. 6f. 32po. 4yd. 1ft. 10in.
9. 83yd. 2qr. 2n. 2in. 10. 328mi. 1520yd. 11. 272yd. 8in. 12. 66560.

Ex. 36. (pp. 81-82.)

1. 178240, 255660 ; 325095 ; 241840 ; 134397. 2. 23bi. 6kat. 5ch. ;
147bi. 4kat. 3ch. 6ga. ; 644bi. 14kat. 5ch. ; 88bi. 2kt. 14ch. 8ga.
3. 122000 ; 220320 ; 1143100. 4. 36000 ; 29520 ; 58905.
5. 22394880000 ; 30595276800 ; 642064032200.
6. (1) 5bi. 14bisw. 1knaw. (2) 1713gh. 1bi. 3ka. 17ma. 7sa.
(3) 5ca. 13gd. 2199sq. ft. 5 sq. in. (4) 9641ch. 19ru. 1bi. 10pa. 1kt.
7. 108800 ; 177600 ; 486080. 8. (1) 159408 ; 59328 ; 25458 ; 16146.
(2) 8389656 ; 382631040 ; 2730166560 ; 94089600 ; 878169600 ;
100362240000. (3) 111496176 ; 267280848 ; 654896016.
9. 3 ro. 31po. 1 sq. yd. 2 ft. 36 in. ; 2 ac. 27 po. 16 sq. yd. 6 ft. 36 in. ;
11 po. 23 sq. yd. 7 sq. ft. 81 sq. in. ; 3 ro. 32 po. 23 sq. yd. ;
14 ac. 2 ro. 35 po. 6 yd. 8 ft. 36 in. 10. 8781696 ; 154306944.
11. 1600 ac. ; 7865 bi. ; 400 ac. ; 8000 ac. ; 146410 bi.
12. 64000 ; 320000 ; 2592000 ; 9680000.
13. (1) 97 bi. 12 ch. (2) 124 sq. yd. 1 sq. ft. 36 sq. in.
(3) 255 ac. 1 ro. 32 po. 12 sq. yd. 4 sq. ft. 72 sq. in.
14. (1) 2 bi. 14 ka. 14 ch. (2) 14 ac. 2 ro. 31 po.
(3) 6 ac. 2 ro. 38 po. 29 sq. yd. 2 sq. ft. 36 sq. in.
15. (1) 313 bi. 10 ka. 5 ch. ; 752 bi. 8 ka. 12 ch.
(2) 134 ac. 31 po. 26 sq. yd. 2 sq. ft. 36 sq. in.

16. (1) 2 bi. 10 ka. 14 ch. ; 1 bi. 3 ka. 2 ch. ; 6 ka. 14 ch.
(2) 11 sq. yd. 8 ft. 118 in. ; 37 ac. 3 ro. 39 po. 15 yd. 1 ft. 18 in.

17. 13552 bi. ; 13440 ac.

Ex. 37. (p. 83.)

1. 4595 ; 3327 ; 2745. **2.** 508044 ; 1295228 ; 2425484.

3. 8865 chow. 52 c. cub. ; 10543 chow. 49 c. cub.

4. 102 c. yd. 12 c. ft. 1598 c. in. ; 2019 c. yd. 9 c. ft. 595 c. in.

Ex. 38. (pp. 83-84.)

1. 4920 ; 10160 ; 2432000 ; 10570240 ; 47900.

2. 417952 md. 4 do. 1 pa. 1 ch. ; 176 kh. 3 bs. 3 kt. 1 do. 1 pa.
147275 md. 1 ch. ; 7 kh. 2 bs. 9 ar. 3 kt. 1 rk.

3. 68800 ol. ; 48256 ol. ; 23 ph. 1 ma. 4 pu. 4 ol. ;
18 ph. 4 ma. 2 pu. ; 38 ph. 4 ma. 5 pu.

4. 552960 ta. , 128000 tp. , 584928 ta. ,
256 ka. 6 ph. 11 pa. 2 sr. 1 tp. 34 ta. 144 mu. 15 ph. 10 py. 1 tp.

5. 87 ; 5752 , 12032 , 36396.

6. 7310 last 1 bu. 2 pk. ; 212 last 1 qr. 5 bu. 2 pk. ;
150335 last 4 qr. 2 bu. 3 pk. . 5786 last 1 qr. 6 bu.

Ex. 39 (pp. 84-85.)

1. 828 ; 21824 ; 32160 , 72224 ; 144564.

2. 65552 , 38016 58896. **3.** 1152000 ; 1891200 , 1226400.

4. 208 pi. 1 hd. 60 gal. 2 qt. 1 pt. 2gl. ; 3676 tuns. 1 pi. 1 hd. 44gal. 1pt. ,
485 pi. 41 gal. 3 qt. , 408 tuns. 1 pi. 1 hhd. 25 gal. 1 pt.

5. 2477 tuns 1bt. 37gal. , 566 tuns 47gal. 2qt. ; 41 tuns 1 hhd 31 gal. ;
12 tuns 1 hhd. 36 gal. 2 qt. 1 pt. **6.** 127 lb. 8 oz. **7.** 24062 lb. 8 oz.

Ex. 40. (pp. 87-88.)

1. (1) 1476000. (2) 12476700. (3) 64800000. (4) 623160000.
(5) 435780000. (6) 1412640000. (7) 604407000.

2. (1) 1358100 ; 31614300 22176000. (2) 4733100 ; 8455556.

3. (1) 128 pal. 21 bip. 45 anu. (2) 2072 dan. 49 pal. 8 bip.
(3) 28 pra. 1 dan. 42 pal. 20 bip. (4) 2617 din. 6 dan. 41 pal.

4. (1) 1299 hr. 41 min. 41 sec. (2) 742 da. 3 hr. 31 min.
(3) 286 da. 17 hr. 40 min. (4) 459 wk. 23 hr. 46 min.

5. (1) 6155 dan. (2) 18500 hr.

6. (1) 56 sap. 4 din. 6 pra. (2) 62 dan 22 pal. 38 bip. (3) 103 da. 5 dan.
(4) 2 da. 20 hr. 14 min. 10 sec. (5) 388 da. 7 hr. 32 min. 20 sec.
(6) 127 wk. 2 da. 12 hr. 12 min.

7. (1) 22 dan. 59 pal. 59 bip. 49 anu. (2) 6 din. 7 pra.
(3) 22 din. 7 pra. 6 dan. 30 pal. (4) 2 hr. 53 min. 56 sec.
(5) 44 da. 22 hr. 51 min. 54 sec. (6) 6 da. 23 hr. 4 min.

8. (1) 260 din. 4 pra. 6 dan. 434 din. 2 pra. 5 dan.;
694 din. 7 pra. 3 dan. 30 pal.
(2) 87 da. 15 hr. 45 min.; 157 da. 18 hr. 45 min.; 280 da. 12 hr.

9. (1) 3 sap. 4 din 3 pra. 1 sap. 2 pra.; 3 din. 5 pra.
(2) 63 wk. 1 da. 21 hr. 40 min., 38 wk. 2 da. 10 hr. 13 min. 20 sec.;
28 wk. 5 da. 7 hr. 40 min.

Ex. 41. (p. 88.)

1. (1) 56415″ (2) 290730.″ (3) 262104″.
2 (1) 13 rt. ang. 66°51′. (2) 19 rt. ang. 16°7′1″. (3) 77° 1′.
(4) 75°12′. (5) 209 rt. ang. 26°46′59″.

Ex. 42. (pp. 90-91.)

1. £1 8s. **2.** 61 md. 17 sr. **3.** 15s. **4.** 14 yd. **5.** R5. 5a.
6. R330. **7.** 5 sacks. **8.** 166 mi. **9** 250. **10.** 143 days.
11. R781-5-9 **12.** £1126-3-5¼. **13.** R13411-8-9. **14.** £915-13-4.
15. R421-5-7. **16.** £2262-3-2¾. **17.** 1222md-10-7.
18. 3 ton. 13 cwt. 1 qr. 21 lb. **19.** 166 srs. **20.** 6.
21. 255 srs. **22.** 31. **23.** 2153. **24.** 1120. **25.** 640.
26. 350. **27.** 390. **28.** £5164-8-3 **29.** 2400. **30.** £360-2-6.

Ex. 43. (pp. 92-93.)

1. 1 mi. 1530 yd. **2.** 3 mi. 920 yd. 1 ft. 1 in. **3.** 5 yd. 1 ft. 3 in.
4. 5 yd. 7 in. **5.** 3432. **6.** 2640. **7.** 14400.
8. 3960; 2640. **9.** 6 ft. **10.** 660. **11.** 12 ft. **12.** 110 sec.

Ex. 44. (p. 94.)

1. R509. 12a. 3p. **2.** R231-12-9 gain. **3.** 4a. 9p. **4.** Lost R28. 14a.
5. 6a. 4p. **6.** Gains £10. 10s. 8d. **7.** Gains R57. 8a.
8. 3d. **9.** 1s. 10d. **10.** 1763. **11.** 450. **12.** 46 md. 34 sr.

Ex. 45. (pp. 95-96.)

1. R76. 0a. 8p. **2.** 4s. 7d.; 5s. 5d. **3.** 4s. 3d. **4.** R2. 14a.
5. 2a. 1p. **6.** 75 sr. **7.** 23 sr. **8.** 224 gal.
9. 36 gal. **10.** 17s. 6d. **11.** R69-4-3. **12.** R40-0-8 nearly.

Ex. 46. (pp. 96-97.)

1. R5760. **2.** £184. **3.** 1408sr. **4.** 78.
5. Gains £2. 12*s.* 4*d.* **6.** *A* should give *B* R37. 8*a.* **7.** R1. 4*a.*
8. Pays £9848. 12*s.* **9.** 3*s.* 4½*d.* **10.** Loses R340.

Ex. 47. (pp. 98-99.)

1. *A*, R503-7-8 ; *B*, R261-1-3. **2.** *A*, £2038-19-1. ; *B*, £2710-17-6.
3. Each man, R107. 1*a.* ; woman, R100
4. *A*, 8985 yd. 1 ft. 4 in. ; *B*, 8759 yd. 1 ft. 8 in.
5. *A*, R299. 14*a.* 8*p.* ; *B*, R274. 10*a.* 6*p.* ; *C*, R252. 4*a.* 2*p.*
6. *A*, 15 tons 14 cwt. ; *B*, 12 tons 16 cwt. 3 qr. ; *C*, 11 tons 5 cwt. 1 qr.
7. *A*, £2883. 16*s.* 11*d.* ; *B*, £2421. 6*s.* 3*d.* ; *C*, £2937. 18*s.*
8. Man, R2. 8*a.* ; woman, R2. 4*a.* boy, R2.
9. Man, 11 tons , woman, 10 tons 10 cwt. ; boy, 10 tons.
10. *A*, 200 md. ; *B*, 300 md. **11.** 45 md. 4 sr. 8 ch ; 20 md. 2 sr. 4 ch.
12. *A*, R128. 6*a.* 4*p.* , *B*, R96 4*a.* 9*p.* *C*, R32. 1*a.* 7*p.*
13. *A*, £363 ; *B*, £580. 16*s.* ; *C*, £508. 4*s.*
14. *A*, £434. 12*s.* 6*d.* ; *B* or *C*, £144. 17*s.* 6*d.*
15. 100. **16.** 100. **17.** 125.
18. Rupees, 100 ; half-rs., 300 , quarter-rs., 400 two-anna pieces, 500.
19. 44. **20.** 200 md.

Miscellaneous Examples II. (pp. 99-103.)

1. 200. **2.** 129000. **3.** R1050. **4.** 1259 mi. 2 fur. 15 po.
5. 40. **6.** 5*s.* 4½*d.* **7.** 28000. **8.** R100. 15*a.*
9. £122. 10*s.* **10.** R24912. **11.** 9 mi. 810 yd.
12. R56763. 10*a.* 6*p.* **13.** R2102400. **14.** R3. 5*a.* 3*p.*
15. £1387. 10*s.* **16.** *A*, R5. 13*a.* 8*p.*; *B*, R12.; *C*, R8. 10*a.* 10*p.*
17. *A*, 48 cwt. 6 lb. ; *B*, 75 cwt. 2 qr 2 lb. , *C*, 67 cwt. 3 qr. 12 lb.
18. 3652. hr. **19.** 7*a.* 6*p.* **20.** 5*d.* **21.** 15360 **22.** R232. 8*a.*
23. 300. **24** 160 md. ; 80 md. **25.** 7 hr. 52 min. **26.** 2. **27.** 28.
28. Gains, £1. 10*s.* 6*d.* **29.** R13. 2*a.* **30.** 5 hr. 15 min. **31.** 20 qt.
32. 21 yr. 4 mo. 12 da. **33.** 12*a.* **34.** R1. 10*a.* 3*p.* **35.** 8*a.*
36. R1403. 10*a.* **37.** Loses £10. 5*s.* **38.** R110-9 ; R55-4-6 ; R18-6-10.
39 8 lb. 8 oz. **40.** 960. **41.** 3*s.* **42.** 77 yd.
43. 9240. **44.** 11242. **45.** 10lb. 8oz.. **46.** R3048. 7*a.* 6*p.*
47. 20 sec. **48.** 5541lb. 8 oz. ; 7600. **49.** 23 mi. 20 yd.

50. 7 yd. 4 in. **51.** 36840. **52.** 15 ft. **53.** 17th Sept.
54. Tuesday. **55.** £33. 13*s.* 11¼*d.* **56.** 28th Mar. 1882.
57. 5th Nov. 1888. **58.** 18 hr. **59.** 5. **60.** 23 yr. 3 mo.
61. 20 yr. 10 mo. **62.** 11484. **63.** £684. 4*s.* 6*d.* **64.** 36.
65. 92877000 miles. **66.** 2⅜ miles. **67.** 28 yrs. 1 mo. 13 days.
70. 676 half-rupees and 324 quarter-rupees.
71. 112 mds. at R8-8. ; 144 mds. at R6-12.
72. 96 mds. at R8-8 ; 160 mds. at R6-12.

Ex. 48. (p. 107.)

2. (1) $3^2, 5, 11$; $2, 3, 5^3, 7$; $2^2, 3^2, 17$; $2, 3, 5^3$; $3^2, 11^2$; $3^5, 5$; $2^4, 3^2, 13$; $2, 3, 7, 11, 13$; $3^2, 7, 11, 13$.
(2) $3, 5, 7, 11, 13$; $2, 7, 11, 13, 17$; $3, 7, 11, 13, 19$; $2^2, 7, 11, 13, 17$; $2, 3^2, 13, 317$. $2^4, 11, 13, 43$; $3^3, 7^2, 11, 13$.
(3) $3^2, 7^2, 11$; $2^4, 3^4, 5$, $3^2, 7, 11^2$; $2^3, 3^4, 13$; $2^9, 3^5$; $2^3, 3, 5^2, 71$ $3^2, 5^2, 7^2$; $3^2, 11^2, 13$.
(4) $2^2, 3, 7, 11, 13, 353$, $5, 7^2, 11, 13, 151$; $2, 5, 11, 13, 4481$; $2^3, 3, 5^2, 7, 11, 13, 41$; $2, 5, 7, 11, 13, 19, 449$.
3. (1) 29, 37, 101, 113 are primes ; 3^4 ; 3×31 ; 3×37 ; 7×17 ; 7×23 $3\times5\times11$; $3^2\times19$; 11×17.
(2) 503 is prime ; $3^2\times23$. 13×23 ; 257×2 ; 11×61 , $2\times3^4\times5$; $2^5\times5\times7$, 13^3 ; $2^3\times11\times13$; $3\times11\times71$.

Ex. 50. (p. 108.)

1. 15. **2.** 27. **3.** 16. **4** 18. **5.** 41. **6.** 55. **7.** 6. **8.** 240.
9. 6. **10.** 17. **11.** 36. **12.** 39. **13.** 25. **14.** 22. **15.** 8. **16.** 9.
17. 8 **18.** 2. **19.** 28. **20.** 4. **21.** 2. **22.** 11.

Ex. 51. (p. 109.)

1. 13. **2.** 17. **3.** 19. **4.** 21. **5.** 24. **6.** 27. **7.** 31.
8. 32. **9.** 34. **10.** 36. **11.** 256. **12.** 5. **13.** 909. **14.** 7013.
15. 2. **16.** 10111. **17.** 101. **18.** 23. **19.** 3. **20.** 5. **21.** 13.

Ex. 53. (pp. 110-111.)

1. 21. **2.** Moidore. **3.** 22 yd. **4.** R2. 13*a*. **5.** 21 gal.
6. 21 qr. **7.** 1 tola. **9.** 56. **10.** 25. **11.** 9.
12. 5134, 23103 ; 7701, 20536 ; 10268, 17969 ; 12835, 15402.
13. R65000. **14.** Half-guinea. **15.** 317. **17.** 999797 ; 100023.
18 99997 ; 100274.

Ex. 54. (p. 112.)

2. (1) 546. (2) 180. (3) 48. (4) 195. (5) 1620. (6) 8736. (7) 3556. (8) 21021. (9) 24024. (10) 23400 (11) 6720. (12) 275625 (13) 940500. (14) 2010960. (15) 45276.

Ex. 55. (p. 112.)

1. 7119. **2.** 36163. **3** 10120. **4.** 4374. **5.** 3531. **6.** 38440. **7.** 99789. **8.** 5898230. **9** 5641864. **10.** 25194.

Ex. 56. (p. 113.)

1. 42. . 126. **3.** 112. **4.** 180. **5.** 120. **6.** 315. **7.** 1260. **8** 157872 **9.** 1482. **10.** 3360. **11.** 55440. **12.** 4641. **13.** 63000. **14.** 5775. **15.** 65520. **16.** 960. **17.** 3780. **18.** 6300. **19.** 45360. **20** 98280. **21.** 2520. **22.** 5405400. **23.** 151200. **24.** 100800. **25.** 453600.

Ex. 57. (pp. 114-115.)

1. 360. **2.** 72 hr. **3.** 420. **4.** 90 pt. **5.** R100. **6.** 24 hr. **7.** 1440 sec **8** £189. **9.** 245 ft. 4 in **10.** 4800 pages, 77 chap. **11.** 280 ft.; 120, 112, 105 **12.** 13650 days. **13.** 12 min.; 3, 4, 6. **14.** 370 ft. **15.** 1821. **16.** 467. **17.** 15. **18.** 396. **19.** 13497. **20.** 1643 yd. **21.** 2 ft.; 1 ft. 6 in. **22.** 1636325 **23.** 9765. **24.** 109395. **25.** 100798, 995398.

Ex. 58. (p. 117.)

1.

2. $\frac{120}{10}$. $\frac{240}{16}$; $\frac{360}{20}$; $\frac{475}{25}$.

3. ; $\frac{57}{3}$; $\frac{95}{5}$, $\frac{114}{6}$; $\frac{171}{9}$.

4. $\frac{30}{60}$; $\frac{40}{60}$; $\frac{45}{60}$; $\frac{48}{60}$, $\frac{42}{60}$, $\frac{32}{60}$.

5.

6. $\frac{7}{9}$; $\frac{5}{4}$; $\frac{4}{9}$ · $\frac{3}{25}$.

Ex. 59. (pp. 118-119.)

2 (1) $\frac{4}{25}$; 15 , $\frac{7}{9}$; $\frac{375}{903}$; $\frac{35}{52}$; $\frac{5}{6}$.

(2) $\frac{9}{10}$; 5 ; 183 ; 354 ; $\frac{301}{603}$; $\frac{1754}{3877}$. $\frac{4}{6}$.

(3) $\frac{14}{15}$; $\frac{26}{225}$; $\frac{523}{6452}$; $\frac{15}{16}$; 48 ; 13.

(4) $\frac{3}{4}$; 127 : $\frac{2113}{4321}$; $\frac{71}{111}$; $\frac{51}{107}$; $\frac{40}{225}$.

(5) $\frac{5}{7}$; $\frac{17}{379}$; $\frac{647}{741}$, 1000 ; $\frac{39}{40}$; $\frac{901}{1000}$

(6) $\frac{101}{111}$; $\frac{37}{270}$; $\frac{15}{29}$; $\frac{7}{13}$;

3. (1) $\frac{9}{25}$; ; $\frac{16}{1875}$; (2) 1 ; $\frac{9}{160}$; $\frac{1}{21}$; $\frac{7}{30}$. (3) $\frac{3}{16}$; $\frac{8}{189}$; $\frac{2}{21}$; $\frac{1}{20}$.

Ex. 60. (p. 120.)

2. (1) $70\frac{7}{17}$; $60\frac{21}{25}$; $18\frac{27}{26}$; $16\frac{1}{2}$; $12\frac{1}{6}$; 5 ; $22\frac{171}{385}$; $15\frac{591}{281}$; $82\frac{45}{73}$.

(2) $1325\frac{50}{71}$, $8835\frac{1}{992}$; $1122\frac{1047}{1517}$; $116\frac{477}{7818}$; $1111\frac{1000}{1111}$; $4047\frac{232}{305}$.

4. (1) $\frac{2569}{15}$; $\frac{2006}{21}$; $\frac{314}{25}$; $\frac{1624}{33}$; $\frac{7837}{92}$; $\frac{1921}{112}$.
(2) $\frac{78312}{211}$; $\frac{141891}{307}$, $\frac{13562}{71}$; $\frac{651248}{807}$; $\frac{20011}{97}$.
(3) $\frac{989948}{99}$; $\frac{9989092}{999}$; $\frac{99880926}{9999}$; $\frac{9999804763}{99999}$.

Ex. 61. (p. 121.)

1. $\frac{1}{3}$; $\frac{1}{2}$; $\frac{1}{6}$; $\frac{4}{9}$; $\frac{1}{4}$; $\frac{8}{27}$. **2.** $\frac{1}{11}$; 5; $\frac{55}{126}$; $\frac{5}{14}$. **3.** $\frac{5}{3}$; $\frac{3}{8}$; $\frac{12987}{20}$.
4. $\frac{5}{54}$. **5.** $\frac{105}{4}$. **6.** 25. **7.** $\frac{30}{7}$. **8.** $\frac{595}{288}$. **9.** $\frac{845}{248}$.
10. $\frac{437}{9}$. **11.** $\frac{15}{13}$. **12.** $\frac{17263}{14}$. **13.** $\frac{66885}{32}$. **14.** $\frac{301455}{1024}$. **15.** $\frac{26307}{180}$.

Ex. 62. (p. 122.)

1. $\frac{6}{12}, \frac{4}{12}, \frac{3}{12}$. **2.** $\frac{15}{30}, \frac{20}{30}, \frac{24}{30}$. **3.** $\frac{18}{60}, \frac{21}{60}, \frac{22}{60}$. **4.** $\frac{150}{180}, \frac{165}{180}, \frac{72}{180}$.
5. $\frac{9}{66}, \frac{4}{66}, \frac{24}{66}, \frac{21}{66}$. **6.** $\frac{30}{36}, \frac{16}{36}, \frac{14}{36}, \frac{33}{36}$. **7.** $\frac{20}{30}, \frac{6}{30}, \frac{27}{30}, \frac{12}{30}$.
8. $\frac{60}{72}, \frac{42}{72}, \frac{44}{72}, \frac{27}{72}$. **9.** $\frac{120}{216}, \frac{84}{216}, \frac{132}{216}, \frac{93}{216}$. **10.** $\frac{12}{72}, \frac{63}{72}, \frac{64}{72}, \frac{51}{72}$.
11. $\frac{171}{180}, \frac{54}{180}, \frac{113}{180}, \frac{100}{180}, \frac{120}{180}$. **12.** $\frac{1980}{2448}, \frac{1584}{2448}, \frac{2312}{2448}, \frac{1700}{2448}, \frac{912}{2448}$.
13. $\frac{297}{792}, \frac{616}{792}, \frac{648}{792}, \frac{748}{792}, \frac{690}{792}$. **14.** $\frac{64}{32}, \frac{120}{32}, \frac{7}{32}$. **15.** $\frac{93}{180}, \frac{94}{180}, \frac{18}{180}, \frac{30}{180}$.
16. $\frac{196}{660}, \frac{600}{660}, \frac{95}{660}, \frac{132}{660}, \frac{540}{660}$.

Ex. 63. (p. 123.)

1. $\frac{4}{5}$; $\frac{6}{7}$; $\frac{27}{30}$; $\frac{37}{42}$; $\frac{22}{25}$; $\frac{27}{30}$ **2** $\frac{8}{25}$; $\frac{60}{33}$; $\frac{1}{2}$, the first.
3 (1) $\frac{1}{7}, \frac{3}{10}$; $\frac{9}{10}, \frac{7}{11}$; $\frac{1}{20}, \frac{11}{15}$. (2) $\frac{17}{24}, \frac{28}{44}$; $\frac{73}{80}, \frac{13}{24}$; $\frac{7}{5}$, $\frac{18}{90}$.
(3) $\frac{67}{88}, \frac{44}{71}$; $\frac{53}{54}, \frac{8}{9}$; $\frac{61}{66}, \frac{14}{33}$. (4) $\frac{6}{7}$ of $\frac{9}{10}$, $\frac{2}{5}$ of $\frac{3}{4}$; $\frac{5}{6}$ of $\frac{11}{12}$, $\frac{5}{8}$ of $\frac{2}{5}$.
(5) $\frac{5}{6}$ of $\frac{6}{7}$, $\frac{4}{7}$ of $\frac{4}{5}$; $\frac{5}{9}$ of $\frac{3}{4}$, $\frac{12}{25}$ of $\frac{10}{9}$.

4 (1) $\frac{2}{3}$, $\frac{7}{12}$, $\frac{8}{15}$, $\frac{9}{20}$. (2) $\frac{53}{60}$, $\frac{33}{40}$, $\frac{18}{25}$, $\frac{53}{75}$. (3) $\frac{7}{24}$, $\frac{13}{48}$, $\frac{17}{64}$, $\frac{4}{15}$.
(4) $\frac{9}{20}$, $\frac{7}{20}$, $\frac{13}{40}$, $\frac{7}{16}$. (5) $\frac{35}{88}$, $\frac{27}{77}$, $\frac{7}{22}$, $\frac{18}{66}$. (6) $\frac{23}{54}$, $\frac{25}{63}$, $\frac{5}{24}$, $\frac{7}{36}$.
(7) $\frac{23}{26}$ of $\frac{39}{46}$, $\frac{7}{9}$ of $\frac{13}{28}$, $5\frac{1}{5}$ of $\frac{2}{39}$, $\frac{3}{8}$ of $\frac{3}{25}$.
(8) $\frac{22}{25}$ of $\frac{5}{44}$, $\frac{3}{40}$ of $1\frac{7}{33}$ of $\frac{32}{33}$, $\frac{5}{18}$ of $\frac{3}{7}$ of $\frac{30}{60}$, $\frac{4}{55}$ of $\frac{44}{76}$.

Ex. 64. (p. 125.)

2. (1) $2\frac{5}{12}$, $1\frac{13}{204}$; $2\frac{185}{504}$; $2\frac{1}{20}$. (2) $1\frac{137}{338}$; $2\frac{77}{684}$; $1\frac{79}{96}$; $2\frac{29}{54}$.
(3) $3\frac{31}{45}$. (4) $19\frac{5}{16}$ (5) $9\frac{29}{45}$. (6) $3\frac{13}{90}$. (7) $3\frac{17}{36}$.
(8) $8\frac{65}{308}$. (9) $5\frac{11}{12}$. (10) $3\frac{133}{220}$. (11) $11\frac{23}{30}$ (12) $10\frac{507}{560}$.
(13) $80\frac{97}{132}$. (14) $49\frac{23}{153}$. (15) $3\frac{1}{15}$. (16) $32\frac{43}{78}$. (17) $52\frac{63}{285}$.
(18) $19\frac{5}{6}$ (19) $61\frac{25}{72}$. (20) $36\frac{29}{70}$. (21) $45\frac{7}{24}$. (22) $429\frac{11}{15}$.
(23) $70\frac{1}{3}$. (24) $8\frac{527}{1200}$. (25) $52\frac{37}{40}$.

Ex. 65. (p. 126.)

2. (1) $\frac{11}{54}$; $\frac{14}{65}$; $\frac{85}{108}$; $\frac{34}{105}$; $\frac{341}{480}$. (2) $1\frac{107}{210}$; $\frac{13}{15}$; $\frac{29}{40}$; $\frac{7}{8}$; $12\frac{3}{100}$.
(3) $33\frac{13}{72}$; $\frac{7}{8}$; $105\frac{9}{20}$; $3\frac{356}{441}$.
3. (1) $2\frac{1}{2}$. (2) $7\frac{59}{60}$. (3) $\frac{19}{28}$. (4) $8\frac{5}{9}$. (5) $5\frac{53}{80}$. (6) $9\frac{103}{150}$. (7) $4\frac{55}{90}$.
(8) $4\frac{1}{5}$ (9) $1\frac{5}{36}$. (10) $17\frac{53}{140}$. (11) $39\frac{37}{600}$. (12) $15\frac{419}{432}$. (13) 0.

Ex. 66. (p. 127.)

2. (1) $3\frac{1}{8}$; $6\frac{1}{4}$, $12\frac{1}{2}$; 50. (2) 285; 380; 665.
(3) $1\frac{6}{11}$; $5\frac{2}{5}$; $22\frac{2}{5}$; $45\frac{1}{5}$. (4) $14\frac{1}{2}$; $43\frac{1}{2}$; $72\frac{1}{2}$.
(5) $88\frac{4}{5}$; $125\frac{4}{5}$. (6) $56\frac{5}{11}$; $155\frac{1}{4}$; $776\frac{1}{4}$.
(7) $11\frac{1}{2}$; 23; $57\frac{1}{2}$; $80\frac{1}{2}$. (8) $38\frac{1}{8}$; $68\frac{5}{8}$; $91\frac{1}{2}$; $152\frac{1}{2}$.

3. (1) $69\frac{3}{10}$; $79\frac{1}{5}$; $89\frac{1}{10}$; 99; $148\frac{1}{2}$. (2) $98997\frac{3}{7}$; 439990.
(3) $768\frac{22}{25}$; $1098\frac{2}{5}$; $1647\frac{3}{5}$; 5492; 21968.
(4) $12487\frac{1}{5}$; $24974\frac{2}{5}$; $37461\frac{3}{5}$; $49948\frac{4}{5}$; $74923\frac{1}{5}$; $99897\frac{3}{5}$.

Ex. 67. (pp. 128-129.)

1. (1) $\frac{1}{4}$; $\frac{1}{9}$; $\frac{1}{16}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{5}{27}$; (2) $\frac{2}{15}$; $\frac{1}{2}$; $\frac{4}{5}$; $\frac{2}{9}$; $\frac{4}{5}$.
(3) $\frac{33}{40}$; $\frac{3}{10}$; $\frac{32}{117}$; $42\frac{1}{2}$; $27\frac{29}{30}$. (4) $1\frac{1}{8}$; $2\frac{923}{1536}$; $2\frac{1}{18}$. (5) $\frac{731}{2100}$.

2. (1) $\frac{5}{8}$. (2) $40\frac{5}{48}$. (3) $7\frac{2}{7}$. (4) $\frac{2}{7}$. (5) $10\frac{1}{2}$.
(6) $2\frac{29}{35}$. (7) $\frac{1}{20}$. (8) $\frac{108}{325}$. (9) $6426\frac{27}{28}$. (10) $33366\frac{2}{3}$.

3. (1) $12\frac{608}{825}$. (2) 14. (3) $\frac{39}{448}$. (4) $\frac{1000}{1331}$.

Ex. 68. (pp. 129-130.)

2. (1) $\frac{5}{73}$; $\frac{5}{146}$; $\frac{5}{219}$; $\frac{5}{292}$. (2) $\frac{7}{41}$; $\frac{5}{41}$; $\frac{1}{82}$. (3) $\frac{7}{47}$; $\frac{7}{94}$; $\frac{3}{47}$; $\frac{1}{47}$.
(4) $\frac{5}{259}$; $\frac{25}{2331}$; $\frac{5}{777}$. (5) $\frac{2}{9}$, $\frac{2}{27}$, $\frac{2}{45}$. (6) $\frac{1}{7}$; $\frac{1}{14}$; $\frac{1}{28}$.

4. (1) $\frac{7}{12}$. $\frac{7}{10}$; $1\frac{1}{2}$; $\frac{35}{66}$. (2) $1\frac{1}{2}$; 1; $1\frac{1}{7}$. (3) $1\frac{1}{11}$; $\frac{9}{11}$; $\frac{21}{44}$.
(4) $\frac{2}{3}$; $1\frac{1}{9}$; $\frac{196}{243}$. (5) $3\frac{5}{9}$; 2; $3\frac{1}{9}$. (6) $5\frac{1}{2}$; $2\frac{318}{325}$; $1\frac{402}{445}$.
(7) $2\frac{6}{7}$; $1\frac{11}{14}$; $\frac{25}{28}$. (8) $4\frac{1}{12}$; $2\frac{13}{18}$; $4\frac{23}{24}$. (9) $\frac{3}{7}$. (10) $\frac{5}{21}$.
(11) $\frac{89}{108}$. (12) $\frac{1}{4}$. (13) $7\frac{47}{64}$. (14) $24\frac{4}{5}$. (15) $\frac{3}{28}$. (16) $\frac{337}{403}$.

5. (1) $19\frac{1}{15}$. (2) $78\frac{29}{32}$. (3) $\frac{168}{451}$. (4) $\frac{7}{32}$. (5) $2\frac{23}{27}$.

Ex. 69. (pp. 130-131.)

1. $1\frac{13}{64}$; $\frac{3}{4}$; $\frac{61}{320}$; $25\frac{1}{2}$; 4; $\frac{345}{418}$; 4; $3\frac{3}{10}$.
2. $1\frac{6}{7}$; $\frac{1}{3}$; $33\frac{2}{3}$; $\frac{9}{200}$. **3.** $\frac{875}{8112}$; $2\frac{2}{15}$; $16\frac{2}{3}$. **4.** $\frac{4312}{10125}$.
5. 4; $1\frac{29}{100}$; $\frac{6325}{16666}$. **6.** $\frac{7}{40}$; $\frac{121}{765}$.

Ex. 70. (p. 131.)

1. $\frac{7}{15}$. **2.** $\frac{56}{513}$. **3.** $\frac{4}{11}$. **4.** $\frac{34}{155}$.
5. $\frac{264}{409}$. **6.** $1\frac{1}{4}$. **7.** $\frac{70}{79}$. **8.** $2\frac{97}{138}$.

Ex. 71. (pp. 131-133.)

1. $2\frac{1}{4}$. **2.** $8\frac{13}{18}$. **3.** $20\frac{167}{605}$. **4.** $\frac{2}{27}$. **5.** $\frac{11}{14}$. **6** $2\frac{113}{168}$.
7. 1. **8.** $13\frac{59}{80}$. **9.** $15\frac{59}{80}$. **10.** $54\frac{13}{48}$. **11.** $1\frac{7}{30}$. **12** $2\frac{1}{4}$.
13. $1\frac{195}{1000}$. **14.** $3\frac{13}{30}$. **15.** $\frac{23}{40}$. **16** $\frac{69}{410}$. **17.** $6\frac{21}{22}$. **18.** $\frac{73}{165}$.
19. $10\frac{67}{180}$. **20.** $4\frac{1}{8}$. **21.** $2\frac{1}{2}$. **22.** $1\frac{210}{1061}$. **23.** $4\frac{23}{56}$. **24.** 1.

25 0. **26.** $2\frac{13}{16}$. **27.** $\frac{37}{650}$. **28.** $\frac{33}{112}$. **29.** $5\frac{1}{40}$. **30.** $39\frac{187}{265}$.
31. $\frac{1}{3}$. **32** $1\frac{248}{819}$. **33.** $4\frac{3}{7}$. **34.** $\frac{9}{16}$. **35.** 1. **36.** $\frac{7}{13}$.
37. $1\frac{543}{850}$. **38.** $1\frac{1}{24}$. **39.** 1. **40.** $3\frac{3}{7}$. **41.** $2\frac{299}{540}$. **42.** $3\frac{10}{11}$.
43. $\frac{343}{2353}$. **44.** $\frac{31}{174}$. **45.** 1. **46.** 1. **47.** $\frac{278}{495}$. **48.** $19\frac{118}{195}$.

Ex. 72. (p. 134.)

1. (1) $\frac{1}{36}$; 21. (2) $\frac{1}{78}$; 30. (3) $\frac{8}{231}$; $2\frac{2}{11}$. (4) $\frac{19}{40}$; $5\frac{7}{10}$.
(5) $\frac{5}{297}$; $33\frac{1}{3}$. (6) $\frac{3}{4}$; $472\frac{1}{2}$. (7) $\frac{13}{675}$, $17\frac{1}{3}$. (8) $\frac{8}{63}$; 528.
2. R4 4*a.* ; 55, 64. **3.** $27\frac{1}{2}$ hr. **4.** $1766\frac{1}{4}$ hr. **5.** $307\frac{1}{2}$ ft. **6.** 354 yds.

Ex. 73. (pp. 135-136.)

1. $\frac{24}{25}$. **2.** $\frac{27}{95}$. **3.** $14\frac{41}{60}$. **4.** $1\frac{10}{11}$; $\frac{11}{21}$; $2\frac{100}{231}$; $1\frac{89}{231}$. **5.** $27\frac{11}{72}$; $\frac{1955}{18825}$.
6. $\frac{97}{180}$, $\frac{43}{180}$. **7.** $16\frac{2371}{3000}$. **8.** $\frac{11}{24}$. **9.** $3\frac{5}{8}$. **10.** R225. **11.** $85\frac{7}{24}$.
12. $\frac{5}{44}$. **13.** 720. **14** $\frac{275}{546}$. **15.** $9\frac{11}{18}$. **16.** $15\frac{202}{441}$. **17.** 84.
18. $6\frac{5}{6}$. **19.** $422\frac{14}{33}$. **20.** 18 ; 22. **21.** *A*, $80\frac{3}{4}$; *B*, $95\frac{19}{20}$; *C*, $63\frac{1}{2}$.
22. $\frac{1}{4}$. **23.** 3600. **24.** 200 ; 160 ; 120 ; 120. **25.** 25 ft.
26. *A*, $\frac{2}{27}$; *B*, $\frac{13}{15}$; *C*, $\frac{8}{135}$. **27.** $\frac{1}{16}$. **28.** 348.

Ex. 74. (pp. 136-137.)

1. (1) R54-0-$7\frac{58}{693}$. (2) R199-1-$1\frac{11}{18}$. (3) £64-3-$8\frac{3}{7}$. (4) £53-11-$4\frac{167}{180}$.
2 (1) R2-14-$7\frac{5}{9}$. (2) R9-14-$10\frac{38}{55}$. (3) 19*s.* $1\frac{102}{117}$*d.* (4) £2-19-$9\frac{50}{63}$.
3. (1) R35-13-10 ; R62-12-$2\frac{1}{2}$. R98-10-$0\frac{1}{2}$; R147-15-$0\frac{3}{4}$.
(2) £139-6-$6\frac{12}{17}$; £247-13-$10\frac{10}{17}$; £526-7 ; £789-10-6.
(3) 136 md. $2\frac{1}{2}$ ch. ; 181 md. 13 sr. $8\frac{3}{5}$ ch. ; 544 md. 10 ch.
(4) 112 tons 14 cwt. 2 qr. 8 lb. ; 225 tons 9 cwt. 16 lb. ;
338 tons 3 cwt. 2 qr. 24 lb.
4. (1) R1. 4*a.* $7\frac{5}{7}$*p.* ; R1. 1*a.* $8\frac{46}{49}$*p.* ; 11*a.* $9\frac{27}{40}$*p.*
(2) £1. 10*s.* $1\frac{5}{16}$*d.* 10*s.* $10\frac{20}{67}$*d.* ; 6*s.* $10\frac{142}{361}$*d.*
(3) $18\frac{309}{1180}$; $5\frac{583}{1260}$. (4) $26\frac{4787}{13793}$; $4\frac{9346}{17001}$.

Ex. 75. (p. 138.)

1. (1) 12*a.* ; 3*a.* : R1-12 ; R47-8. (2) 12*a* ; R26-4 ; R61-4 ; R48-6-$4\frac{4}{5}$.
(3) 15*s.* ; 17*s.* 6*d.* ; £1. 13*s.* 9*d.* ; 9*s.* ; 15*s.* 9*d.* ;
£1. 2*s.* $10\frac{2}{7}$*d.* £29. 5*s.* ; £21. 6*s.* 8*d.* ; £10. 16*s.* 8*d.*
(4) £1. 8*s.* $6\frac{6}{7}$*d.* ; £72. 16*s.* ; £28 ; £8. 8*s.*
(5) 30 sr. ; 35 sr. ; 2 md. 10 sr. ; 25 md. 8 sr. $5\frac{1}{3}$ ch.
(6) 2 oz. 8 dwt. ; 10 dwt. 3 gr. ; 2 lb. 1 oz. ; 5 cwt. 1 qr. $13\frac{1}{15}$. lb.
(7) 37 yd. 3 in. ; 4 mi. 1184 yd. ; 7 ac. 3 ro. ; 2 ac. 20 po.
(8) $1\frac{3}{8}$ gal. ; 1 hhd. 31 gal. 2 qt. 1 pi. 42 gal. ; 65 gal.

2. (1) R2-14-9$\frac{3}{5}$. (2) £3-9-2. (3) 2 md. 7 sr. 6$\frac{2}{5}$ ch. (4) 1 mi. 1310 yd.
(5) 2 ro. 5 po. 4 yd. 5 ft. 36 in. (6) 1 qr. 3$\frac{1}{24}$ bu. (7) 16 min. 46 sec.

Ex. 76. (p. 139.)

1. (1) R7. 0*a*. 6$\frac{1}{3}$*p*. R2. 4*a*. 2$\frac{1}{28}$*p*. R6. 14*a*. 9$\frac{15}{61}$*p*. R6. 8*a*. 5$\frac{89}{320}$.
(2) R11. 9*a*. 4$\frac{1}{27}$*p*. R24. 7*a*. 8$\frac{29}{30}$*p*.; R62. 12*a*. 7$\frac{7}{24}$*p*.
(3) £6-4-2$\frac{8}{35}$, £6-2-11$\frac{29}{105}$; £40-8-11$\frac{11}{60}$; £174-15-3$\frac{29}{40}$.

2 (1) R29-5-8$\frac{11}{20}$; R35-10-9$\frac{187}{192}$: R29-12-0$\frac{213}{296}$; R39-10-8$\frac{71}{74}$.
(2) R3. 3*a*. 0$\frac{3}{25}$*p*; 13*a*. 3$\frac{40}{513}$*p*., 6*p* 9$\frac{701}{873}$*p*.
(3) £176. 14*s*. 2*d*.; £5. 17*s*. 9$\frac{2}{3}$*d*; £3. 17*s*. 3$\frac{23}{32}$*d*.; £1. 0*s*. 7$\frac{47}{120}$*d*.

3 (1) R22-2-5$\frac{1}{5}$; 14*a* 9*p*. (2) £10-13-4; £72-13-1$\frac{1}{5}$; £141-19-8$\frac{4}{5}$.
(3) 60 md 2-8; 28 md. 25-6. (4) 18 cwt.; 2 cwt. 7 lb.
(5) 6 fur. 13 po; 104 mi. 3 fur. 24 po. 2 yd. 1 ft. 1$\frac{1}{3}$ in.
(6) 110 da. 6 hr. 56 min. 15 sec: 75 wk. 4 da. 4 hr. 48 min.

4. (1) R11. 4*a*. 9$\frac{1}{5}$*p*. (2) R61. 0*a*. 8$\frac{5}{15}$*p*. (3) £170. 13*s*. 2$\frac{17}{32}$*d*.
(4) 14*s*. 1$\frac{1}{2}$*d*. (5) 10 md. 32 sr. 5$\frac{1}{27}$ ch. (6) 13 tons 2 cwt. 3 qr. 17 lb.

Ex. 77. (p. 140.)

1. R$\frac{3}{32}$, R$\frac{11}{64}$; R$\frac{43}{192}$, R$\frac{67}{128}$; R$\frac{167}{192}$; R$\frac{85}{144}$; R$\frac{387}{512}$.
2. £$\frac{13}{160}$; £$\frac{4}{15}$; £$\frac{23}{80}$; £$\frac{123}{160}$; £$\frac{11}{16}$; £$\frac{5}{8}$; £$\frac{143}{240}$.
3. R7$\frac{17}{24}$; R13$\frac{11}{12}$; R6$\frac{7}{16}$; **4.** £13$\frac{101}{240}$; £4$\frac{47}{80}$; £12$\frac{109}{480}$.
5. 5$\frac{341}{1024}$ md.; 5$\frac{3}{16}$ tons. **6.** 5$\frac{1}{3}$ mi.; 6$\frac{5}{7}$ pi.

Ex. 78. (p. 141.)

1. $\frac{23}{32}$; $\frac{23}{60}$. **2.** $\frac{2}{3}$, $\frac{1}{3}$. **3.** $\frac{16}{25}$; $\frac{32}{25}$. **4.** $\frac{75}{94}$; $\frac{3}{4}$. **5.** $\frac{873}{1178}$; $\frac{291}{1304}$.
6. $\frac{2}{3}$; $\frac{6}{16}$. **7.** $\frac{1}{5}$. **8.** $\frac{2182}{2629}$. **9.** $\frac{1574}{2071}$. **10.** $\frac{25}{88}$.
11. $\frac{73}{80}$; $\frac{73}{160}$. **12.** $\frac{721}{1710}$. **13.** $\frac{14014}{57019}$. **14** $\frac{1000}{1029}$. **15.** $\frac{45}{44}$.
16. $\frac{1}{144}$. **17.** $\frac{11}{30}$. **18** $\frac{5}{36}$. **19.** $\frac{128}{105}$. **20.** $\frac{4391}{4480}$.
21. $\frac{1}{2}$. **22.** $\frac{1}{4}$. **23.** $\frac{2}{3}$. **24.** $\frac{2}{5}$; 1$\frac{3}{5}$. **25.** $\frac{1384}{1431}$.

Ex. 79. (pp. 143-144.)

1. 706 yd. 4 in. **2.** 87 md. 7 sr. 2 ch. **3.** 329 tons 3 cwt.
4. 468 mi. 1320 yd. **5.** £11. 15*s*. 11*d*. **6.** 5*d*. **7.** 16 md. 17 sr. 8 ch.
8. 26$\frac{2}{3}$. **9.** £181. 9*s*. 2*d*. **10.** £140. 16*s*. 8*d*. **11.** 333$\frac{1}{2}\frac{62}{24}$yd.
12. R47. 4*a*. 9*p*. **13.** £1. 14*s*. 2$\frac{1}{16}$*d*. **14.** £53. 4*s*.
15. R2. 14*a*. 2$\frac{2}{3}$*p*. **16.** 10 sr. **17.** R512100. **18** R33.13*a*. 4*p*.
19. 54 mi. **20.** R11011. **21.** 2 ft. 1$\frac{1}{2}$ in. **22.** 8$\frac{4}{15}$ oz.
23. 6 ch. **24.** 4 mo. **25.** R4000. **26.** 36 days.
27. 18 ch. **28.** 5 men. **29.** 24 days. **30.** R280.

Ex. 80. (pp. 145-146.)

1. $6\frac{2}{3}$hr. **2.** $37\frac{1}{2}$ min. **3.** $3\frac{6}{13}$ days. **4.** $\frac{11}{30}$. **5.** 12 hr.
6. 36 min. **7.** 16 min. **8.** $12\frac{1}{2}$ min. **9.** 25 hr.
10. 11-40 A. M. **11.** 4 A. M. **12.** $6\frac{2}{3}$ hr. **13.** 150 hr.

Ex. 81. (pp. 149-150).

1. $1\frac{1}{5}$ da. **2.** $3\frac{10}{27}$da. **3.** 24 hr. **4.** 15 da. **5.** $1\frac{11}{13}$ da.
6. 8 hr. **7.** $4\frac{4}{23}$ da. **8.** 24 da. **9.** 42 hr. **10.** 18 da.
11. $\frac{5}{6}$. **12.** 4 hr. **13.** 160 da. **14.** $5\frac{2}{5}$ da. **15.** 6 da.
16. 25 hr. **17.** $2\frac{22}{73}$ da ; $5\frac{11}{41}$ da. **18.** $\frac{1}{4}$. **19.** 121 da. **20.** 105 hr.

Ex. 82. (p. 150.)

1. 41. **2.** $\frac{91}{1000}$. **3.** 10 md. **4.** $1\frac{1}{640}$ ft. **5.** 2 qr. 5lb. 4 oz.

Ex. 83. (pp. 151-152.)

1. 210. **2.** 10. **3.** 18. **4.** 7*s*. **5.** 400 yd.
6. R9. 6*a*. **7.** 45. **8.** 90 hr. **9.** 8 hr. **10.** £2000.

Ex. 84. (pp. 154-155.)

1. 1st greatest, 3rd least. **2.** $\frac{6}{7}$ of R6. 4*a*. ; $\frac{5}{7}$ of R5. 2*a*. ; $\frac{4}{7}$ of R4. 3*a*.
3. $\frac{7}{23}$. **4.** $\frac{1}{2}$. **5.** $\frac{37}{40}$. **6.** $1\frac{11}{28}$. **7.** 21*s*. **8.** 9 yd. $2\frac{2}{5}$ in.
9. £12 4*s*. 6*d*. **10.** $2\frac{2}{5}$ mi. **11** $\frac{5}{24}$. £5617. 1*s*. **12.** R1060.
13. £5400. 10*s*. 6*d*. **14.** R40. **15.** R55 11*a*. **16.** £11250. 9*s*. $4\frac{1}{2}$*d*.
17. *A*, R6. 4*a*. ; *B*, R3. 12*a*. . *C*, R1. 4*a*. **18.** R30. **19.** $\frac{1111}{1200}$.
20 40. **21.** 4*s*. **22.** R32. 13*a*. 8*p*. **23.** £5859. 3*s*.
24. £11666. 13*s*. 4*d*. **25.** 3 hr. 16 min. **26.** $\frac{1}{5}$.
27. R5. 14*a*. , R6. 10*a*. ; R7. 6*a*. ; R8. 2*a*. **28.** R90.
29. R2. 10*a*. $6\frac{2}{83}$*p*. **30.** R14741. 3*a*.

Miscellaneous Examples III. (pp. 155-159).

1. 1. **2.** $11\frac{13}{380}$. **3.** 12. **4.** $5\frac{19}{25}$. **5.** $60\frac{35}{38}$. **6.** $\frac{145}{168}$.
7. *A*, $\frac{49}{75}$; *B*, $\frac{26}{75}$. **8.** R30. **9.** $\frac{4}{5}$. **10.** $\frac{81}{1280}$.
11. £1600 ; £1866-13-4 ; £2133-6-8. **12.** *A*, R180 ; *B*, R120 ; *C*, R45.
13. $5\frac{85}{86}$. **14.** R28. 12*a*. $9\frac{3}{5}$*p*.
15. *A*, R16666. 10*a*. 8*p*. ; *B*, R12500 ; *C*, R10000 ; *D*, R10833. 5*a*. 4*p*.
16. $\frac{3}{5}$. **17.** *A*, R32 ; *B*, R16 ; *C*, R24 ; *D*, R28. **18.** £52. 10*s*.
19. 780 yds. **20.** *A*, R2400 ; *B*, R900, *C*, R240. **21.** R32. 3*a*.
22. 17 ; $\frac{20}{1083}$. **23.** $16\frac{1}{4}$ hr. **24.** R8. 5*a*. 4*p*. **25.** $12\frac{1}{2}$ mi.

26. $\frac{1}{12}$; man £2-0-6; wo., 11*s.* 3*d.* **27.** *A*, £30-16-3; *B*, £28-15-2.
28. £1. 2*s.* 5*d.* **29.** 24960. **30.** 9.
31. *A*, R100-8-6; *B*, R33-8-2; *C*, R16-12-1. **32.** $52\frac{1}{2}$ da.; 21, 28, 30.
33. *P*, £138. 19*s.* $9\frac{1}{8}$*d.*; *Q*, £88. 8*s.* $11\frac{1}{2}$*d.*; *R*, £70. 15*s.* 2*d.*
34. 450 gal. **35.** $8\frac{2}{11}$ min. **36** *A*, R15; *B*, R12; *C*, R3.
37. $\frac{10}{9}$. **38.** $4\frac{1}{2}$da.
39. 8*a.* $3\frac{3}{5}$*p.*; 10*a.* $9\frac{3}{5}$*p.*; 13*a* $3\frac{3}{5}$*p.*; 15*a.* $9\frac{3}{5}$*p*; R1. 2*a.* $3\frac{3}{5}$*p.* **40.** $33\frac{1}{3}$da.
41. $6\frac{2}{7}$ da. **42.** The former; £4. 13*s.* 4*d.* **43** 3 hr. **44.** R7. 15*a.* $4\frac{2}{7}$*p.*
45. 35 days. **46.** 5*s.* 3*d.* **47.** 2 da. more. **48.** 6 min.
49. Man, R23-12-4; wo., R17-13-3; boy, R6-12-8; girl, R5-1-6.
50. 4770. **51.** 3125. **52** Sov., $123\frac{171}{623}$ gr.; shil., $87\frac{3}{11}$ gr.; pen. $291\frac{2}{3}$gr.
53. £32. 19*s.* $7\frac{1}{3}$*d.* **54** £5. 2*s.* **55.** $154\frac{2}{7}$ ft.; 63, 64.
56. Man, R20; woman, R15; boy, R10; girl, R9.
57. *A*, £36; *B*, £60; *C*, £35 **58.** 24. **59** R45.
60 £22. 6*s.* 6*d.* **61.** Home-made cloth. **62.** 96. **63.** $4333\frac{1}{3}$ gal.
64. 32 days. **65.** £3060. **66.** 87. **67.** 2750.
68. 9; £30000. **69.** 2250. **70.** 7000.

Ex. 85. (pp. 161-162.)

1. (1) $\frac{81}{10}$; $\frac{7219}{100}$; $\frac{94701}{100}$; $\frac{42001}{1000}$; $\frac{5761123181}{10000000}$; $\frac{6712346711}{10000000000}$.
(2) $\frac{3}{10}$; $\frac{31}{100}$; $\frac{121}{1000}$; $\frac{1461}{10000}$; $\frac{1}{100}$; $\frac{3}{1000}$; $\frac{21}{1000000}$; $\frac{41}{100000000}$; $\frac{71}{10000000000}$.
2. (1) $\frac{1}{2}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{4}{5}$; $\frac{3}{25}$; $\frac{7}{20}$; $\frac{7}{50}$; $\frac{1}{25}$; $\frac{1}{200}$; $\frac{1}{40000}$; $\frac{81}{4}$.
(2) $\frac{16761}{250}$; $\frac{461}{8}$; $\frac{29}{40000}$; $\frac{65961}{10000}$; $\frac{150013}{25000}$.
(3) $\frac{529029}{40000}$; $\frac{41}{1000000000000}$; $\frac{3528000601}{8000000}$.
3. (1) $72\frac{157}{250}$; $756\frac{1}{120}$; $7\frac{5283}{100000}$; $27\frac{3}{5000}$; $2\frac{1579}{10000}$.
(2) $67\frac{1313}{250000}$; $141\frac{7}{2000}$; $478\frac{14141}{200000}$; $30\frac{1001}{5000000}$; $7521\frac{4}{125}$.
4. (1) ·1, ·3, ·5, 1·1, 25·1, 374·1, ·03, ·31, ·005, ·0511.
(2) ·0007, ·00051, 274·1641, ·0004781, ·000059, 7741·261.
(3) ·0712, 21·96, 9·4618, ·021416, ·0000001, ·051215.
(4) 1·7, ·25, ·051, ·0621, 4·51324, ·072416, ·00000000461.

Ex. 86. (p. 162.)

1. ·9, 9, 90, 900, 9000, 90000. **2.** ·27, 2·7, 27, 270, 270000.

Ex. 87. (p. 162.)

1. 5·1, ·51, ·051, ·0051, ·00051, ·000051.
2. ·0057, ·000057, ·0000057, ·00000057, ·000000000057.
3. ·025006, ·0025006, ·0000025006, ·000000025006.

Ex. 88. (pp. 163-164.)

1. (1) 31·98. (2) 32·90443261. (3) 4·296396. (4) 450·4731896.
(5) 20·65121971. (6) 4183·86 (7) 660·079170071. (8) 6906·94678.
(9) 1776·300416. (10) 7538·040853051.

2. (1) 1052·6973. (2) 187·112367. (3) R6121·977476. (4) £886·192043.
(5) 852·8245md. (6) 7·260712cwt. (7) 1549·088ft. (8) 86·001442741days.

Ex. 89. (pp. 164-165.)

1. (1) ·1837 ; 17·1559 ; 3·2993. (2) ·01 ; ·001 ; ·001 ; ·001.
(3) ·0611 ; ·99853 ; 1·998581. (4) 2·40959948 ; 571·99359 ; 8·01.

2. (1) ·0025 ; 12·481 ; 1·0932. (2) 2·409 ; 8·82 ; 1·99991.
(3) 599·94 ; 15·999984. 3 (1) R2·5246 ; R·01619 ; R·0376.
(2) £11·923 ; £·117864 ; £·00992235. (3) ·001 md. ; ·096cwt ; ·693yd.

4. (1) 13·189. (2) 17·948556. (3) 68·57184. (4) 16·092385.
(5) 6·8914. (6) 6·294924. 5 24·7893. 6. 57·8425.

Ex. 90. (pp. 165-166.)

1. (1) 149·625 ; 172·71 ; 2009·25 ; 20·0925 ; ·200925.
(2) 38·5065, ·0385065, 3·85065, ·385065, 6·93117, ·693117, ·000693117.
(3) 85·0299 ; 8·50299 ; ·850299 ; ·0850299 ; ·000850299.
(4) 289·69375 ; 289693·75 ; ·00000002896975.
(5) ·00000032 ; ·000000096 ; ·0000000016 ; ·000000008.
(6) ·0000449882114 ; ·000449882114 ; ·449882114.
(7) 25554113·75 ; 650·46835 ; 112·4381005 ; 11173633·80744.
(8) 35148·519433005 ; 351485·19433005 ; 3·5148519433005.

2. (1) 53·7219 ; 2441·8 ; ·00254364 ; 570·744126.
(2) 93618 ; 43·5306329 ; ·0000083559.
(3) 1·672 ; ·015975 ; 1164·8582. (4) ·000074024 ; ·0010419699.

3. (1) 21·45868733. (2) 13·5274268. (3) 1·53029799.
(4) 17·19975. (5) 5·2305. (6) 38·44584.

Ex. 91. (pp. 168-169.)

1. (1) 1·2 ; 307·05 ; ·002 ; ·001. (2) 7·2 ; 13·13 ; ·305 ; 1743.
(3) 679·85 ; ·017 ; ·0003.
(4) 125 ; ·03125 ·078125 ; ·05 ; ·1 ; ·2 ; ·02 ; ·000002.
(5) 1·2375 ; 12·375 ; 123·75 ; 1237·5 ; 12375.
(6) ·0027 ; ·27 ; ·00027 ; ·0000027 ; ·0000000027 ; 27.
(7) 1·8 ; ·144 ; 18 ; 1·44 ; 14400 ; ·00018.
(8) ·005 ; ·015625 ; 12·8 ; ·00000128.

(9) ·0014641 ; ·014641 ; ·14641 ; ·00014641 ; ·00000014641.
(10) ·038 ; 4·07 ; ·0046. (11) ·0021 ; ·314 ; 417 ; ·417.
(12) 31·94 ; 3·194 ; ·1 ; 6·8 ; 170.

2. (1) 8·5997 ; 4·1923 ; 1·0882. (2) 1·8972 ; ·0073 ; 90196·3888.
(3) 24·6870 ; 21·1804 ; 516·1290. (4) ·0637 ; 912226·9841 ; 18257·6744.

3. (1) 7·210125 ; 48·0675 ; 36·050625 , 2·403375 ; 1·80253125 ; 90·1265625 , 80·1125.
(2) 3246·48 ; 108·216 ; 194·7888 ; 38·0446875 ; 338·175 ; 19022·34375 ; 6492·96.

4. (1) ·67442 ; 67·44285 ; 674·42857 ; 33·72142 ; 3372·14285 ; ·00337.
(2) 16321·94092 ; 6·95601.

5. (1) ·5 ; ·25 ; ·125 ; ·375 ; ·75 ; ·1875 ; ·28 ; ·85 ; ·1122 ; ·296.
(2) 5·734375 ; 7·35546875 ; 45·3828125 ; 237·0944 ; 37·906225 58·1504 ; 729·76.

6. (1) $\frac{3}{5}, \frac{1}{2}, \frac{1}{3}$. (2) $\frac{3}{4}, \frac{4}{7}, \frac{8}{9}$ (3) $\frac{17}{21}, \frac{3}{8}, \frac{3}{10}$. (4) $\frac{2}{7}, \frac{5}{18}, \frac{3}{10}$.
(5) [illegible], $\frac{7}{8}, \frac{4}{9}$. (6) $\frac{13}{15}, \frac{11}{131}$ (8)

Ex. 92. (p. 170.)

1. ·002275 ; 13·640625 ; ·0012056 ; ·1590038.
2. ·0002052 ; ·0000154 ; ·00000101578125.
3. 1830·183. **4.** 8·91. **5.** 768. **6.** ·27512. **7.** 476. **8** 777·2.

Ex. 93. (p. 170.)

1. (1) ·05. (2) ·008. (3) ·009. (4) ·056. (5) ·025.
2. (1) 840. (2) 72. (3) 168·3. (4) 277·2.

Ex. 94. (p. 171.)

1. $\frac{21}{125}$; $\frac{7}{16}$; $\frac{27}{80}$; $\frac{93}{16}$; $\frac{44}{110}$; $\frac{28}{17}$ **2.** 20, 25, 32, 40, 50, 64.

Ex. 95. (p. 172.)

1. $\cdot\dot{3}$; $\cdot 8\dot{3}$; $\cdot\dot{2}8571\dot{4}$; $\cdot\dot{4}$; $\cdot\dot{9}\dot{0}$; $\cdot 6\dot{4}2857\dot{1}$; $\cdot 7\dot{3}$; $2\cdot 5\dot{3}5714\dot{2}\dot{8}$; $5\cdot\dot{2}3076\dot{9}$; $4\cdot\dot{4}11764705882352\dot{9}$; $\cdot\dot{7}\dot{5}$; $2\cdot 884615\dot{3}$.
2. $\cdot 031\dot{3}14285\dot{7}$; $2\cdot\dot{2}0\dot{7}$; $\cdot 24\dot{3}$; $25\cdot\dot{4}8\dot{3}$; $\cdot 7\cdot 1469\dot{5}9\dot{4}$; $7\cdot\dot{5}9\dot{4}$; $4\cdot 62\dot{5}$; $10\cdot 349\dot{6}$.
3. $1\cdot 0\dot{3}0\dot{3}$; $19\cdot\dot{1}3394\dot{7}$; $33\cdot\dot{0}5624\dot{1}$; $44\cdot 0002\dot{5}71428\dot{8}$, $11\cdot\dot{2}0\dot{7}$; $\cdot 00018\dot{5}$.

Ex. 96. (p. 175.)

1. (1) $\frac{2}{11}$; $\frac{2}{111}$; $\frac{7}{333}$; $\frac{5}{333}$; $\frac{43}{108}$; $\frac{1}{270}$; $\frac{37}{9900}$; $\frac{49}{90}$; $\frac{1}{6}$; $\frac{8}{4995}$.
(2) $\frac{1}{300}$; $\frac{807}{1111}$; $\frac{95}{111}$; $\frac{731}{900}$; $\frac{31}{198}$; $\frac{1}{7}$; $\frac{5}{7}$; $\frac{109}{180}$.
(3) $\frac{11}{108}$; $\frac{5}{14}$; $\frac{6}{7}$; $\frac{3}{14}$; $\frac{54}{7}$. (4) $\frac{512}{117}$; $\frac{48}{13}$; $\frac{111}{220}$; $\frac{50403}{700}$; $\frac{9707}{20800}$.
2. 4 ; 4·3 ; 7·1 ; 20 ; 16 ; ·43 ; 50 ; 17·5.

Ex. 97. (p. 176.)

1. $136{\cdot}993\dot{6}4246\dot{0}$. **2.** $41{\cdot}5437\dot{4}94131395 12\dot{1}$. **3.** $15{\cdot}35\dot{3}0715684181\dot{0}$.
4. $18{\cdot}116\dot{5}2709324986\dot{5}$. **5.** $28{\cdot}6\dot{6}3181\dot{3}$.

Ex. 98. (p. 177.)

1. $103{\cdot}393346$. **2.** $3714{\cdot}488021$. **3.** $118{\cdot}594147$.
4. $189{\cdot}367007$. **5.** $796{\cdot}963077$.

Ex. 99. (pp. 177-178.)

1. (1) $21{\cdot}\dot{9}\dot{5}$. (2) $3{\cdot}19\dot{3}1900307147\dot{8}$. (3) $\cdot 2303\dot{5}3\dot{6}$.
(4) $\cdot\dot{0}2\dot{3}$. (5) $\cdot 0\dot{1}$. (6) $9{\cdot}5\dot{2}1576\dot{1}$.
2. (1) $4{\cdot}175135$. (2) $\cdot 336356$. (3) $\cdot 266340$. (4) $2{\cdot}044044$.
(5) $2{\cdot}167188$. (6) $1{\cdot}383582$. **3** $\cdot 672\dot{5}73\dot{9}$; $\cdot\dot{5}743\dot{1}$; $\cdot 0179\dot{6}17\dot{8}$.

Ex. 100. (p. 179.)

1. $22{\cdot}\dot{8}\dot{7}$; $36{\cdot}\dot{6}\dot{0}$. **2.** $84{\cdot}\dot{6}$; $112{\cdot}\dot{8}$. **3.** $20{\cdot}261\dot{8}$; $27{\cdot}01\dot{5}\dot{7}$.
4. $\cdot 261\dot{8}$; $\cdot 294\dot{5}$. **5.** $25{\cdot}4\dot{6}$; $27{\cdot}7\dot{8}\dot{1}$. **6.** $\cdot\dot{1}8\dot{2}$; $\cdot\dot{1}9\dot{5}$.
7. $84{\cdot}5761\dot{0}\dot{9}$. **8.** $\cdot 0892\dot{9}\dot{6}$. **9.** $\cdot 0020116$. **10.** $\cdot 0401454$.
11. $399{\cdot}057\dot{1}$. **12.** $\cdot 00\dot{1}1274\dot{9}$. **13.** $\cdot 1\dot{7}6\dot{9}$. **14.** $114{\cdot}\dot{2}\dot{1}$.
15. $1{\cdot}\dot{1}2239\dot{5}$. **16.** $\cdot 000027$. **17.** $\cdot 070\dot{9}8043\dot{4}$. **18.** $\cdot 044\dot{8}$.

Ex. 101. (pp. 179-180.)

1. $\cdot 08\dot{3}$; $0\dot{5}$. **2.** $\cdot 08\dot{3}\dot{1}$; $\cdot 0266\dot{0}\dot{2}$. **3.** $\cdot\dot{1}\dot{5}$ **4.** $7{\cdot}36\dot{7}\dot{4}$.
5. $\cdot 0\dot{0}0704\dot{5}$. **6.** $\cdot 1$. **7.** $\cdot 52\dot{3}80\dot{9}$.
8. $\cdot\dot{0}0199\dot{9}$; $\cdot 19990\dot{0}$. **9.** $856{\cdot}\dot{9}85\dot{6}$; $\cdot\dot{8}56\dot{9}$. **10.** $15{\cdot}57291\dot{6}$; $216{\cdot}\dot{6}$.
11. $\cdot\dot{2}\dot{7}$; $\cdot 41\dot{3}$. **12.** $\cdot 0\dot{3}$; $31{\cdot}2\dot{9}8\dot{1}$. **13.** $\cdot\dot{4}$.
14. $\cdot 0\dot{5}$; $5{\cdot}06\dot{3}$. **15.** $10{\cdot}\dot{5}$, $15{\cdot}\dot{3}$. **16.** $\cdot 01\dot{2}$; $26{\cdot}\dot{8}\dot{1}$.

Ex. 102. (pp. 180-181.)

1. $2{\cdot}5$. **2.** $\cdot 53125$. **3.** $680{\cdot}8$. **4.** $1{\cdot}\dot{7}\dot{2}$. **5.** $70{\cdot}875$.
6. 12. **7.** $1{\cdot}94375$. **8.** $1{\cdot}7\dot{4}9639\dot{2}$. **9.** $1{\cdot}44$. **10.** 1.
11. $1{\cdot}63\dot{8}$. **12.** $8{\cdot}5$. **13.** 0. **14.** $2{\cdot}08\dot{3}$. **15.** 1.
16. $\cdot 125$. **17.** $19{\cdot}72$. **18.** $\cdot 6$. **19.** 5. **20.** $\cdot 125$. **21.** 1000.
22. $\cdot 25$. **23.** $\cdot\dot{7}1428\dot{5}$. **24.** $\cdot 5$.

Ex. 103. (pp. 182-183.)

1. (1) $1128p$. ; $4608{\cdot}0528p$. ; $3294{\cdot}24p$. ; $\cdot 0336p$.
(2) $6864q$. ; $2{\cdot}472q$. ; $3042q$. $270q$.
(3) $64p$. ; $63{\cdot}850\dot{6}p$. ; $7120{\cdot}\dot{7}2\dot{0}p$. ; $38{\cdot}419\dot{2}p$.

(4) $8778{\cdot}\dot{6}q$. ; 4000q. ; $10802{\cdot}\dot{4}\dot{2}q$. ; $5333{\cdot}\dot{3}q$. ; $30893{\cdot}\dot{9}3\dot{3}q$.
(5) 13680 24 to. ; 319176 in. ; 716 8 dr. ; 33393·6 sec.

2. (1) R3. 5a. 7·2p. ; R4. 11a. 7·2p. ; R2. 1a. ; R3. 1a. $1{\cdot}75\dot{0}\dot{3}p$.
(2) £3. 12s. 3d ; £2. 4s. $3\frac{1}{4}d$. ·2q. ; $6\frac{1}{3}d$. ·4q ; 13s. 4d.
(3) 2 md. 23 sr. ; 7 kr. 3000 hat. ; 4 hr. 58 min. 48 sec. ; 1 ft. 6·432 in.
(4) 4 ac. 1 ro. 29 yd. 51 84 in. ; 1 mi. 7 fur. 31 po. 1 yd. 3·6 in.
(5) 5 cwt. 1 qr. 13 lbs. 11 oz. 8·32 dr. ; £5. 4s. 5d. ·28q.

3. (1) R2. 0a. 2 4p. ; R5 0a. 6p. ; R10. 13a. 7 2p. ; R12. 6a. 4 8p. R13. 12a. 8·448p. ; R140. 10a. 4·32p., R160. 8a. 3 84p.
(2) £10. 0s. 10d.·512q. ; £26. 15s. 7d.·032q. ; £43. 10s. $3\frac{3}{4}d$.·552q. ; £223. 6s. 9d. ; £315 6s. ; £177. 12s. ; £178. 17s. 7d. ·8q.
(3) 1 md. 9 sr. ; 9 cwt. 1 qr. 14lb. ; 15 mi. 1 fur. 16 po. ; 4 hr. 12 min.
(4) R65. 14a. $10{\cdot}0\dot{3}\dot{6}p$. ; £132. 2s. ; 104 mi. ; 78 da. 14 hr. 24 min.
(5) 9a. 1·5p ; R1. 7a. 0 25p. ; 5·3248p.
(6) R3. 13a. 4·125p. ; R43. 10a. 0·7p. ; R27. 5a. 3 22p.
(7) £11. 10s. 1 875d. ; £78 7s. 10d. ; £339. 1s. $4\frac{1}{2}d$. ·72q.
(8) 11s. $5\frac{1}{3}d$. ·24q. ; 3s. $9\frac{3}{4}d$. ·95q. ; £89. 17s. $4\frac{1}{4}d$. ·88q.
(9) $2\ 95\dot{9}\dot{7}5p$. ; $8\frac{1}{2}d$.·8q. ; R23. 4a. $3{\cdot}\dot{5}9\dot{2}p$.
(10) 3 md. 3 sr. 9 75 ch. ; 23 tons 5 cwt. 1 qr. 3lb. 1 oz. 4·48 dr.
(11) R4-9-4·32. (12) £35-6-4·2. (13) R19-15-3·24. (14) R4-9-2.
(15) £3 5s. $1\frac{1}{2}d$. ·75q. (16) £4. 9s. 11d. 5q. (17) £4. 5s. $2\frac{1}{2}d$. 08q.

4. (1) R23·765625 ; £19·2625 ; £44·509375 ; R582·515625.
(2) 439·79375 mds. ; 33 040625 tons ; $11\ 21041\dot{6}$lb ; $14{\cdot}6407670\dot{4}\dot{5}$mi.
(3) $R{\cdot}1\dot{6}$; $R{\cdot}\dot{3}$; $R\ \dot{6}$; R·46875 ; $R3\ 291\dot{6}$
(4) $£{\cdot}1\dot{6}$; $£\ \dot{3}$; $£\ \dot{6}$; £ 375 ; £·875 ; $£3{\cdot}4\dot{6}$

5. R130. 10a. **6.** £254. **7.** £41. 7s. 6d.

Ex. 104. (pp. 183-185.)

1. (1) ·58203125. (2) ·446875. (3) ·2197265625. (4) $\cdot 541\dot{6}$.
(5) $\cdot 7935\dot{6}\dot{0}$. (6) $\cdot 62458\dot{3}$. (7) 1·25. (8) 5.
(9) ·28125. (10) ·725. (11) ·375. (12) ·23175.
(13) ·3. (14) $\cdot 17\dot{8}5714\dot{2}$. (15) ·375. (16) $\cdot 3\dot{3}1168\dot{8}$.
(17) $\cdot 07589\dot{2}8571\dot{4}$. (18) $\cdot\dot{3}$. (19) $\cdot 36458\dot{3}$. (20) ·625
(21) ·5. (22) ·75. (23) $\cdot 2\dot{9}97\dot{7}$. (24) $\cdot\dot{7}\dot{2}$. (25) $1\ 0\dot{9}4\dot{5}$.

2. $\cdot 204\dot{5}$. **3.** ·0310546875. **4.** $276{\cdot}\dot{9}9512\dot{1}$. **5.** ·42.
6. ·2. **7.** $\cdot\dot{0}\dot{6}$. **8.** $\cdot 0072 91\dot{6}$. **9.** ·05.
10. 1 077. **11.** $\cdot 2\dot{5}$. **12.** $1{\cdot}697819\dot{3}$. **13.** ·284375 **14.** ·57.
15. $\cdot 672\dot{6}194\dot{7}$. **16.** $\cdot\dot{3}$. **17.** $1{\cdot}351\dot{6}$. **18.** ·25. **19.** ·5625.
20. (1) $\cdot\dot{7}\dot{5}$. (2) $\cdot 42\dot{6}$. (3) $\cdot 8\dot{1}4285\dot{7}$. (4) ·50375.

Ex. 105. (pp. 187-188.)

1. (1) 29549·304 ; 70·769. (2) ·002 ; 52·900. (3) ·32951 ; 2·08739.
2. (1) 1·18884. (2) 1436·391013. (3) ·00000624. (4) 1·1647727.
3. (1) 2·000000. (2) 1·2500000. (3) 1·41068613. (4) ·178572.

Ex. 106. (p. 189.)

1. 6fl. 6c. $6\cdot\dot{6}$m. ; 4fl 6c. 5·625m. ; £11. 4fl. 5c. 6·25m.
2. £1-4-7·248 ; £10-13-0·552 ; £12-17-10·2 ; £120-11-4·86.
3. 2108500 ; 25090 ; 809·5825.
4. £3909. 8fl. 8c. (1) £310. 7fl. 4c. $5\cdot5\dot{6}$m. (2) 19421·535...

Miscellaneous Examples IV. (pp. 190-193.)

1. 60·49. **2.** ·01296. **3** 5. **4.** $1\cdot\dot{9}2307\dot{6}$.
5 ·00093141 ; $\cdot0369\dot{6}071428\dot{5}$. **6.** ·49406275822. **7.** 41·1255 ; 33·8745.
8. $\cdot496\dot{2}\dot{5}$. **9.** $1\frac{2}{3}$ft. **10.** R6. 8*a*. **11.** ·15625.
12. R10. 8*a*. **13.** R500. **14.** R43·2. **15.** 40. **16.** $\cdot\dot{1}6\dot{2}$.
17. *A*, R2500 ; *B*, R1000 ; *C*, R400. **18.** R2200. **19.** R1050.
20. 3 md. **21.** 60. **22.** 9454. **23.** 90, 70, 55, 17, 12, 6.
24. R72. **25.** R1720. 6*a*. 4*p*. **26.** 120. **27.** R3200.
28. $\frac{4}{5}$ day. **29.** 63. **30.** *A*, £7200 ; *B*, £2700 ; *C*, £720.
31. $\cdot\dot{0}1587\dot{3}$. **32.** 4*a*. **33.** In 9·7875 hr. more. **34.** 1000.
35. 5·29375. **36.** (1) 111571776. (2) ·346573590. **37.** 100·8 sec.
38. 1*s*. 3*d*. **39.** *A*, $26\frac{2}{17}$ da. ; *B*, 12 da. **40.** R10489. 7*a*.
41. Tea, R1. 5*a*. 4*p*. ; coffee, 13*a*. 4*p*. ; sugar, 2*a*. 4*p*. **42.** 7911·685...
43. £5. 4*s*. 6*d*. ; ·25. **44.** $7^0 : 8^0$; £179. 4*s*.
45. 10*d*. from 1st., 4*s*. $3\frac{1}{4}$*d*. from 2nd. **46.** $4\frac{2324}{4039}$*p*.
47. $\frac{40}{41}$ hr. ; $3\frac{11}{41}$mi. **48.** 1171875 **49.** R25. **50.** £1000 ; £7000.

Ex. 107. (pp. 196-197.)

1. R20. 12*a*. 6*p*. **2.** R34. 13*a*. 4*p*. **3.** R76. 0*a*. 3*p*.
4. R45. 10*a*. 8*p*. **5.** R1099. 4*a*. **6.** R260. 1*a*. 1*p*.
7. R6591. 8*a*. 4*p*. **8.** R11224. 15*a*. 6*p*. **9.** R6939. 2*a*. $3\frac{1}{2}$*p*.
10. R1018. 15*a*. $7\frac{1}{2}$*p*. **11.** R20951. 4*a*. **12.** R14787. 3*a*. $11\frac{1}{2}$*p*.
13. R6038. 11*a*. $2\frac{1}{2}$*p*. **14.** R12522. 5*a*. 6*p*. **15.** R90817. 11*a*. 9*p*.
16. R29396. 8*a*. 6*p*. **17.** R7436. 14*a*. 9*p*. **18.** R147251. 10*a*. 9*p*.
19. R2321. 7*a*. $0\frac{1}{2}$*p*. **20.** R8420. 2*a*. $2\frac{1}{2}$*p*. **21.** R5371. 4*a*. $5\frac{1}{3}$*p*.
22. £167. 10*s*. 4*d*. **23** £690. 13*s*. 4*d*. **24.** £541. 12*s*. 6*d*.

25. £583. 17*s*. 6*d*. **26.** £655. 0*s*. 8*d*. **27.** £2893.
28. £1931. 19*s*. 7*d* **29.** £649. 19*s*. $11\frac{1}{8}$*d*. **30.** £16658010. 8*s*. 4*d*.
31. £1509. 10*s*. $5\frac{1}{4}$*d*. **32.** £7879. 19*s*. $11\frac{1}{4}\frac{4}{5}$*d*. **33.** £31572. 19*s*. 4·24*d*.
34. £54662. 11*s*. 7*d*. **35.** £4650. 6*s*. $6\frac{3}{4}$*d*. **36.** £2973. 0*s* $6\frac{3}{4}$*d*. ·9*q*.
37. £1238. 12*s*. $8\frac{7}{10}$*d*. **38.** £2559. 15*s* $9\frac{3}{8}$*d*. **39.** £13828. 0*s*. 7*d*.

Ex. 108. (pp. 198-200.)

1. R246-14-$11\frac{3}{16}$. **2** R380-11-$3\frac{3}{4}$. **3.** £165-2-$4\frac{3}{8}$.
4. £318-18-$10\frac{7}{8}$. **5.** £1688-8-$3\frac{1}{4}$. **6** R18166-5-$1\frac{1}{16}$.
7. R1360-12-$10\frac{8}{9}$. **8.** £1896-0-$2\frac{1}{2}\frac{1}{4}$. **9.** R369473-9-$10\frac{1}{2}$.
10. £14955-5-$11\frac{3}{10}$. **11.** £2736-6-$2\frac{1}{4}$. **12** R3720-15-0.
13. £12528-17-$4\frac{4}{7}$. **14.** £1908-4-8. **15.** £122-2-$5\frac{7}{12}$.
16. £275-16-$7\frac{7}{16}$. **17** £800-18-$3\frac{93}{128}$. **18** R12865-0-$1\frac{3}{5}$.
19. £930-12-$5\frac{4}{27}$. **20.** R128-10-$10\frac{1}{2}$. **21.** £4411-9-$4\frac{7}{20}$.
22. R83-15-9. **23.** R4386-9-0. **24.** 8141 md. 36 śr. $8\frac{1}{2}$ ch.
25. R360-15-6·525. **26.** 503 cwt. 3 qr. 18lb. 2 oz. 8 dr. **27.** R111-14-6·6.
28. R5761-14-$9\frac{1}{3}$. **29.** £632-7-11. **30.** R96-12-$9\frac{1}{2}$.
31. £2325-5-$8\frac{5}{8}$. **32.** R1055-9-$4\frac{7}{8}$. **33.** £4766-0-$3\frac{3}{4}$.
34. R13251-7-$7\frac{4}{7}$. **35.** £13522-9-$4\frac{1}{2}$. **36.** R62701-13-$6\frac{3}{4}$.
37. R226-6-$10\frac{175}{192}$. **38** R6771-1-$3\frac{3}{8}$. **39.** £261-11-8.

Ex. 109. (pp. 201-202.)

1. R76. 1*a*. 10*p*. **2.** R299. 1*a*. **3.** R196. 6*a*. 6*p*. **4.** R754. 5*a*.
5. £24. 13*s*. 11*d*. **6.** £26. 7*s*. 4*d*. **7.** £51. 1*s*. 3*d*. **8.** £37. 6*s*. 7*d*.

Ex. 110. (p. 202.)

1. 86, 32571, 243969. **2.** 300, 90000, 56725, 62524.

Ex. 111. (p. 203.)

2. 36, 65, 460, 1710, 672, 315, 1530.
3. 6, 42, 462, 1365, 23205. **4.** 900.

Ex. 112. (p. 205.)

1. (1) 105, 231, 252, 616, 729, 999. (2) 908, 5432, 1575, 3528, 3343.
(3) 15367, 31605, 17325, 72765. (4) 30507, 28703, 739000, 302550.
2. (1) 7·39, 11·23, 38·52, 77·77. (2) 143·58, 53·1441, 727·65.
(3) ·0151, ·00503, ·013509, ·000251.
3. 326. **4.** 265. **5.** 1225. **6.** 1098.

Ex. 113. (pp. 206-207.)

1. (1) 1·4142, 1·7320, 2·2360, 2·6457, 3·3166, 4·2426, 4·8989, 5·6568, 7·5498, 8·4852, 8·9442.

(2) 14·9331, 49·2747, 1930·7374, 959·9833, 1651·7027, 8926·5977.

2. (1) ·316, ·447, ·632, ·948, 1·264, 1·483, 2·213, 2·134, 2·728, ·881, ·426, 2·538.

(2) ·154, 29·409, 6·415, 16·981, 31·869, 31·758.

(3) ·031, ·009, ·022, 6·375, 161·383.

(4) 2·078, 1·536, 1·326, ·172, ·041, 6·813, 17·329.

3 17·936840580, 8·021221852, ·041569219, 3·464101615. 3·605551275, 1·252996408, 1·490301982, 6·654622453.

Ex. 114. (p. 207.)

1. (1) $\frac{9}{10}$, $\frac{11}{12}$, $\frac{13}{15}$, $\frac{16}{19}$, $\frac{7}{8}$, $\frac{37}{78}$, $\frac{139}{126}$, $\frac{151}{253}$. (2) $12\frac{7}{41}$, $4\frac{7}{19}$, $6\frac{13}{23}$, $6\frac{1}{4}$, $23\frac{4}{5}$, $166\frac{1}{3}$.

2. (1) 2·403, 2·209, 4·515, 4·516, 20·519, 290·896.

(2) 22·092, 15·914, 6·471, 6·115, 53·054, 21·511.

3. $3\cdot\dot{6}$, $6\cdot\dot{3}$, $116\cdot\dot{6}$, $28\cdot\dot{3}$.

Ex. 115. (pp. 210-212.)

1. (1) 400 sq. ft. (2) 782 sq. ft. (3) 12299 sq. ft. (4) 345 sq. ft. (5) 625 sq. ft. (6) 1173 sq. ft. 136 sq. in. (7) 145 sq. ft. $94\frac{1}{2}$ sq. in. (8) 176 sq. ft. $44\frac{7}{16}$ sq. in.

2 (1) 684 sq. ft. (2) 2028 sq. ft. (3) 256 sq. ft. 96 in. (4) 30 sq. yd.

3. (1) 10 ft. 4 in. (2) 17 ft. 3 in. (3) 8 ft. 3 in. (4) 8 ft. $7\frac{1}{2}$ in.

4. 15 ft. ; 18 ft. 4 in., 12 ft. 9 in. **5.** 43 ac. 2 ro. 141 sq. yd.

6. $3\frac{3}{4}$ ac. **7.** 250. **8.** 128. **9.** 1875.

10. 22. **11.** 775. **12.** 68640. **13.** 7560.

14. 352. **15.** 3360 ft. **16.** 4500. **17.** 60.

18. (1) R63-10-$6\frac{2}{9}$. (2) £7-16-$10\frac{2}{9}$. (3) R310-14-$5\frac{1}{3}$. (4) £25-10.

19. (1) R24-7-$1\frac{1}{3}$. (2) R62-5-6. (3) £7-1-$7\frac{1}{2}$. (4) £3-0-$5\frac{1}{3}$.

20. 25 ft. **21.** Length, 33 ft. ; breadth, 11 ft.

22. Length, 15 ft. **23.** 15 ft. 8 in.

24. 22 ft. 6 in. **25.** 11520. **26.** 330 yd. **27.** 1792 ; R840.

28. R50. **29.** 3600 sq. yd. **30.** 32000 sq. ft. **31.** R4950.

32. 10 min. **33.** £127. 17*s.* 6*d.* **34.** 3872.

Ex. 116. (pp. 213-215.)

1. (1) 1248 sq. ft. (2) $919\frac{1}{8}$ sq. ft. (3) $1462\frac{1}{6}$ sq. ft. (4) $965\frac{1}{4}$ sq. ft.

2. (1) $149\frac{1}{3}$ yd. (2) 194 yd.-1-$1\frac{5}{6}$. (3) 154 yd.-2-9. (4) 464 ft.

3. (1) R84-10-$6\frac{34}{81}$. (2) R443-3-$5\frac{7}{9}$. (3) £194-19-$8\frac{2}{3}$. (4) £188-11-$6\frac{1}{4}$.
4. (1) R53-5-4. (2) R64. (3) £6. (4) £16. 16*s.*
5. £34. 15*s.* $5\frac{1}{3}$*d.* **6.** R1613. 5*a.* 4*p.* **7.** 640 sq. ft.; 80 md.; R1000.
8. R31. 15*a.* **9.** 221184. **10.** 20 ft. **11.** Length 20 ft.; breadth 10 ft.
12. 10 ft 6 in. **13.** 20 ft. 4 in. **14** Breadth 15 ft.; height 18 ft.

Ex. 117. (pp. 217-218).

1. 2016 sq. yd. **2.** R2079. 11*a* **3.** $37532\frac{4}{25}$. **4.** 38 ac. 4780 sq. yd.
5. 5364. **6.** 1200. sq. ft. **7.** R359. 11*a.* $5\frac{2}{3}$*p.* **8.** R385. 8*a.* 8*p.*
9. 4656 sq. yd., £1952. **10.** R34. 10*a.* **11.** £10. 5*s.*
12. R69. 13*a.* 4*p.* or R63. 1*a.* 4*p.* **13.** £1531. 13*s.* 4*d.* **14.** R6. 10*a.*

Ex. 118. (pp. 220-222.)

1. (1) 1520 c. ft. (2) 325 c. ft. (3) 60 c. ft.
(4) 167 c. ft. 1584 c. in. (5) 3295 c. yd. 11 c. ft.
2. 421 c. ft. 1512 c. in. **3.** 12 ft. **4.** 1 ft. 4 in. **5.** R266. 10*a.* 8*p.*
6. 111 tons 12 cwt. 16lb. **7.** 84480. **8.** 60. **9.** 48.
10. 7 md. 20 sr., R116. 4*a.* **11.** 96768. **12.** 6 hr. 45 min.
13. 18 ft. **14.** 112 sq. ft. **15.** £96. 5*s.* **16.** R267. 12*a.*
17. 15 ft. **18.** $96\frac{3}{4}$ in. **19.** $3\frac{1}{8}$ ft. **20.** 1296000. **21.** 15 ft.
22. 256. **23.** $2\frac{1}{2}$ ft. **24.** $15\frac{5}{16}$ ft. **25.** 11 yd. **26.** 1350 yd.
27. $36\frac{7}{9}$ md.; R579. 4*a.* **28.** 288. **29.** 34635 lb. 5 oz.
30. R23140. **31.** 10 ft. **32.** 44 md. 15 ch. **33.** 8 ft.
34. 30. **35.** 1536. **36.** R26730.

Ex. 119. (p. 223.)

1. (1) 15 ft. 7′; 24 ft. 5′ 8″; 23 ft. 11′ 8″.
(2) 15 sq. ft. 10′ 9″; 45 sq. ft. 11′ 1″ 4‴; 26 sq. ft. 3′ 11″ 4‴.
(3) 45 c. ft. 3′ 2″ 5‴; 26 c. ft. 6′ 6″ 11‴; 10′ 7″ 1‴.
2. (1) 29 ft. $8\frac{3}{4}$ in.; 231 ft. $7\frac{3}{5}$ in.; 31 ft. $4\frac{15}{16}$ in.
(2) 61 sq. ft. $79\frac{3}{5}$ in.; 46 sq. ft. $107\frac{3}{4}$ in.; 42 sq. ft. $67\frac{53}{71}$ in.
(3) 58 c. ft. 684 c. in.; 4 c. ft. 668 c. in; 56 c. ft. $1270\frac{11}{12}$ c. in.

Ex. 120. (p. 224.)

1. (1) 48 sq. ft. 10′ 8″. (2) 111 sq. ft. 11′ 8″.
(3) 182 sq. ft. 1′ 6″. (4) 90 sq. ft. 10′ 2″ 8‴.
(5) 208 sq. ft. 4′ 11″ 3‴. (6) 150 sq. ft. 4′ 3″ 10‴ 9⁗.
2. (1) 453 c. ft. 7′ 9″. (2) 3181 c. ft. 6′ 2″ 8‴.
(3) 1117 c. ft. 8′ 5″. (4) 612 c. ft. 3′ 6″ 6‴.

Ex. 121. (p. 225.)

1. 30 bi. 2. 283 bi. 5 kat. 3. 272 bi. 8 ch.
4. 141 bi. 17 kat. 4 ch. 5. 31369 bi. 16 kat. 6. 1 bi. 6 kat.
7. 19 bi. 4 kat. 8. 35 bi. 4 kat.

Ex. 122. (p. 225.)

1. 3200 c. cubits. 2. 10625 c. cubits. 3. 97500 c. cubits.
4. 47360 c. cubits. 5. 25920 c. cubits. 6. 9856 c. cubits.

Ex. 123. (pp. 226-227.)

1. 11 yd. 2. $11\frac{1}{4}$ mi. 3. 20 yd. 4. 6 min. 5. 160 yd.
6. *C* ; $3\frac{3}{4}$. 7. 18 points. 8. *B* ; 1 point. 9. 5 points.
10. *C* by $125\frac{5}{7}$ yd. 11. $5\frac{1}{4}$ min. 12. 300 yd. 13. 5.
14. 40 yd. 15. *A*, $8\frac{1}{2}$ min. ; *C*, $11\frac{5}{63}$ min
16 *A*, 4 min. 46 sec ; *B*, 5 min. $26\frac{6}{7}$ sec. ; *C*, 5 min 52 sec.

Ex. 124. (p. 228.)

1 $26\frac{2}{3}$°*C*. 2. $129\frac{1}{5}$°*F*. 3. 113°*F*. 4. 75°*C*.
5. 40°*R*. 6. 75°*C*. 7. $73\frac{3}{5}$°*R*.

Ex. 125. (p. 229.)

1. (1) (i) $16\frac{4}{11}'$; $49\frac{1}{11}'$. (ii) 12 o'clock. (iii) $32\frac{8}{11}'$.
(2) (i) $21\frac{9}{11}'$ $54\frac{6}{11}'$. (ii) $5\frac{5}{11}'$. (iii) $38\frac{2}{11}'$.
(3) (i) $27\frac{3}{11}'$; 3 o'clock. (ii) $10\frac{10}{11}'$. (iii) $43\frac{7}{11}'$.
(4) (i) 3 o'clock ; $32\frac{8}{11}'$. (ii) $16\frac{4}{11}'$. (iii) $49\frac{1}{11}'$.
(5) (i) $5\frac{5}{11}'$; $38\frac{2}{11}'$. (ii) $21\frac{9}{11}'$. (iii) $54\frac{6}{11}'$.
(6) (i) $10\frac{10}{11}'$; $43\frac{7}{11}'$. (ii) $27\frac{3}{11}'$. (iii) 6 o'clock.
(7) (i) $16\frac{4}{11}'$; $49\frac{1}{11}'$. (ii) $32\frac{8}{11}'$. (iii) 6 o'clock.
(8) (i) $21\frac{9}{11}'$; $54\frac{6}{11}'$. (ii) $38\frac{2}{11}'$. (iii) $5\frac{5}{11}'$
(9) (i) $27\frac{3}{11}'$; 9 o'clock. (ii) $43\frac{7}{11}'$. (iii) $10\frac{10}{11}'$.
(10) (i) 9 o'clock ; $32\frac{8}{11}'$. (ii) $49\frac{1}{11}'$. (iii) $16\frac{4}{11}'$.
(11) (i) $5\frac{5}{11}'$; $38\frac{2}{11}'$. (ii) $54\frac{6}{11}'$. (iii) $21\frac{9}{11}'$.
(12) (i) $10\frac{10}{11}'$; $43\frac{7}{11}'$. (ii) 12 o'clock. (iii) $27\frac{3}{11}'$.

2. 5 times. 3. 5 times. 4. 5 times.

Ex. 126. (pp. 231-232.)

1. 590 days. 2. 852 days. 3. 52′ $58\frac{1}{8}$″ past 4 P. M.
4. 38′ $33\frac{3}{4}$″ after 4 P. M. 5. 120 da. ; 12-24′, 11-24′. 6. 15″.
7. 8 P. M., 14 July. 8. $9\frac{7}{121}$ min. to 10 P. M. 9. After 30 da.

10. Saturday, 1 A. M. **11.** 2nd June ; 7-12′.
12. Back, $17\frac{13}{371}'$; forward, $16\frac{67}{68}'$. **13.** $5\frac{1}{5}$ min. **14** $38\frac{3}{4}'$ past 3.
15. In 24 days. [Thursday]. **16.** 18 days after ; 11-54′, 10-24′.
17. 12 days before ; 528 days. **18.** $16\frac{4}{11}$ min. ; at right angles.
19 (i) $3\text{-}29\frac{5}{11}'$ P. M. ; (ii) $3\text{-}12\frac{96}{253}'$ P. M. ; (iii) $3\text{-}46\frac{134}{253}'$ P. M.
20. $2\text{-}48\frac{12}{341}'$ P. M. **21.** $12\text{-}35\frac{13}{22}'$ P. M.

Ex. 127. (pp. 235-239.)

1. $6\frac{1}{4}$ hr **2.** 39 mi. **3.** 3-30 P. M. next day ; 1120 mi. from Calcutta.
4. 21 hr. ; 168 mi. **5.** $11\frac{1}{8}$ min. ; $1\frac{41}{48}$ mi. **6.** $15\frac{1}{2}$ hr.
7. 11 hr. 41 min. **8.** 15 hr. 20 min. **9.** $11\frac{1}{4}$ mi. **10.** 10-14 A.M.
11. $12\frac{1}{2}$ mi. **12.** 500 mi. **13.** 20 mi. ; 15 mi **14.** 810 mi.
15. 20 mi. **16.** 11 hr. **17.** $4\frac{1}{2}$ mi. **18.** P to Q ; 10 min.
19. $4\frac{1}{2}$ mi. ; $1\frac{1}{2}$ mi. **20.** $13\frac{1}{3}$ hr. **21.** 10 hr. **22.** 4 hr.
23. 5 hr. **24.** 8 hr. **25.** 6 mi. ; 4 mi. **26.** 720. **27.** 245.
28. 8 mi. ; 6 mi. **29.** 4 hr. ; 24 mi. **30.** 15 mi. ; 10 mi.
31. $42\frac{3}{91}$ mi. **32.** 1140 ft. per sec. **33.** 18 min. 18 sec
34. $4\frac{7}{32}$ mi per hr. **35.** $8\frac{23}{41}$ mi. **36.** 23 min. **37.** $802\frac{2}{27}$ hr.
38. 7 sec. **39.** 81 sec. **40.** 24 mi. ; 16 mi.
41. 35 mi. ; 25 mi. **42.** (i) $4\frac{1}{2}$ sec. (ii) $22\frac{1}{2}$ sec. **43.** 2 mi.
44. 40 mi. **45.** 125 mi. **46.** 60 mi.

Miscellaneous Examples V. (pp. 240-244.)

1. £22. 8*s.* ; £16. 14*s.* $2\frac{2}{3}d.$ **2.** £270. 16*s.* 8*d.* **3.** £5. 17*s.* $0\frac{1}{3}d.$
4. 420 or 392. **5.** 16 ft. 4 in. **6.** 45 ft. **7.** 8*s.* $7\frac{1}{8}d.$ **8** 24 ft.
9. After 4 hr. 53′ 20″ ; 6, 5, 4. **11.** £20. 6*s.* $1\frac{1}{2}d.$ **12.** £4. 11*s.* 8*d.*
13. *C*, by $\frac{25}{204}$ yd. **14.** 2*s.* 4*d.* **15.** 492 yd. **16.** £10. 5*s.*
17. 11339 c. in. **18.** $87\frac{703}{1231}$ yd. **19.** 5 mi. **20.** 203280 c. yd.
21. $6\frac{2}{3}$ mi. from *B*. **22.** *A*, £25 ; *B*, £33. 6*s.* 8*d.* ; *C*, £41. 13*s.* 4*d.*
23. 16*g.* 1*c.* **24.** 80 mi. from London. **25.** 4800, 3600, 3200, 3000.
26. $3\frac{3}{4}$ hr. **27.** 2·4494 ; ·8164. **28.** 4 tons 9 cwt. 2 qr. 16lb.
29. 11 mi. ; 9 mi. **30.** 1176. **31.** 80 ; 24. **32.** 32 sr. **33.** 120 yd.
34. $25\frac{7}{9}$ min. **35.** At 10 o'clock. **36.** *A*, £200 ; *B*, £160 ; *C*, £150.
37. 89 hr. $59\frac{25}{26}$ min. **38.** 36 mi **39.** $9\frac{17}{25}$ sec. **40.** R3.
41. R4100. **42.** 45 yrs. **43.** 110 yd. ; 88 yd. **44** $17\frac{7}{9}$ mi.
45. 36 ; 24. **46.** 10 ft. ; 8 ft. **47.** 16 ft. **48.** 12 ft. ; 15 ft.
49. 215. **50.** 220. **51** Centres 48 ; outers 31. **52.** 14.
53. 129. **54.** 6 men. **55.** 30 hr. **56.** $5\frac{17}{64}$ gal.

57. Silk 80 yd., velvet 40 yd. **58.** 30 copies ; R82. 8*a*.
59. 105 mangoes , R8. 4*a*. **60** 28′ 12″ after 5 P. M. on the 5th day.

Ex. 128. (pp. 249-252.)

1. 99 yd. **2.** 120 da. **3.** 9*s*. $8\frac{1}{2}$*d*. ; £68. 8*s*. $9\frac{1}{2}$*d*. **4.** £7. 18*s*.
5. R4200. **6.** 113 ft. 3 in. **7.** 1890. **8.** R32. **9.** 24 mo.
10. 10 da. **11.** 15. **12.** R7. 8*a*. **13.** 300 mi. **14.** R20.
15. 35 da. **16.** 18. **17.** $\frac{2}{5}$ mi. **18.** 120. **19.** R24000.
20. 10-22$\frac{98}{381}$ A. M. ; 11-4$\frac{56}{381}$ A. M. **21.** $4\frac{1}{2}$ mi. **22.** 7 da.
23. $1\frac{2}{3}$ mi. **24.** 80. **25.** 3 hr. 17 min. **26.** £1040. **27.** $3\frac{1}{5}$ da.
28 15*s*. **29.** Brandy, £1. 4*s*. ; rum, 18*s*. ; gin, 10*s*. (a gal.).
30. 1568 yd. **31.** 594 yd. **32.** Hare, 200 ; hound, 120.
33. Brandy, 18*s*. ; rum, 14*s*. ; gin, 10*s*. 6*d*. (a gal.). **34.** 3′ 10″.
35. 1 lb. **36.** 300. **37.** R2200. **38.** 240 mi.

Ex. 129. (pp. 253-257.).

1. 8 men. **2** $5\frac{1}{4}$ hr. **3** 20 days. **4.** R162. **5.** R77.
6. R3640. 14*a*. $2\frac{2}{5}$*p*. **7.** 312 md 20 sr. **8.** 8 hr. **9.** 10 hr.
10. 30 da. **11.** 9 men. **12.** 36 women. **13.** 30 boys.
14. 110 mi. **15.** R1000. **16.** 9 oz. **17.** 27 da. **18.** 40.
19. 5040 yds. **20.** R800. **21.** £50. 10*s*. **22.** 112 reams.
23. 7654500. **24.** 168. **25.** R252. **26.** 16 oz. **27.** 120.
28. $1\frac{2}{5}$lb. **29** 15. **30.** 800 **31.** R3125. **32.** 200.
33 75. **34.** 15. **35.** 840. **36.** 800 more men.
37. 1080 reams. **38.** 1980. **39** 405. **40.** 366 boys. **41.** 2000 men.
42. $30\frac{10}{11}$ ft. **43.** 2800. **44.** $72\frac{1}{2}$lbs. **45.** R2133. 5*a*. 4*p*. **46.** 24.
47. 6. **48.** $62\frac{1}{2}$ da. **49.** 154 yd. ; $\frac{10}{13}$ min. **50.** 456 women.

Ex. 130. (pp. 258-259.)

1. R960. **2.** R4056. **3.** R16000. **4.** R55344.
5 £7200. **6.** R900. **7.** R480. **8.** £8077. 18*s* $0\frac{3}{7}$*d*.
9. R1566. **10.** £7200. **11.** £408. **12.** R400.
13. $6\frac{1}{2}$*d*. ; £1126. **14.** £4289. 13*s*. 4*d*.

Ex. 131. (pp. 260-262.)

1. (1) 10. (2) $31\frac{1}{2}$. (3) 4 md. 20 sr. (4) R225 (5) $4\frac{1}{6}$ cwt. (6) $6\frac{3}{4}$ hr.
2. (1) 48. (2) $7\frac{13}{16}$. (3) $16\frac{4}{31}$. (4) $26\frac{2}{3}$.
(5) $82\frac{1}{2}$. (6) $9\frac{1}{2}$. (7) $1\frac{1}{2}$. (8) $31\frac{2}{5}$. (9) 20.

3. R183. **4.** 1617182. **5.** $\frac{1}{2}$. **6.** $40\frac{60}{71}$.
7. $16\frac{2}{3}$. **8.** 20. **9** $10\frac{2230}{2467}$. **10** R880.
11. 19480. **12.** R16666. 10*a*. 8*p*. **13.** R51200.
14 Copper $64\frac{92}{97}$ p. c. ; zinc $35\frac{5}{97}$ **15.** 120000 **16** $88\frac{8}{9}$
17. Increase, 1·8 p. c. **18.** 988. **19.** 260.

Ex. 132. (pp. 262-265.)

1. 4*a*. 6*p*. **2.** 1*s*. $1\frac{1}{2}$*d*. **3.** 20. **4.** R62 8*a*. **5.** $15\frac{6}{13}$ gain.
6 $8\frac{1}{4}$. **7.** 25. **8.** $16\frac{2}{3}$. **9.** £2. 10*s*. **10.** R2. 12*a*.
11. $9\frac{1}{11}$. **12.** R43. 12*a*. **13.** Gain, 12. **14.** $31\frac{1}{4}$. **15.** $11\frac{1}{9}$.
16. Gain, $6\frac{2}{3}$. **17.** 5. **18.** 9*s*. $2\frac{1}{2}$*d*. **19.** 15. **20.** R53. 8*a*.
21. $18\frac{2}{21}$. **22** R7. **23.** R4160. **24** R=1. **25.** R400.
26 48. **27.** 44. **28.** Gain, $2\frac{2}{5}$. **29.** R5250 **30.** 25.
31. Gain, $5\frac{5}{9}$. **32.** Loss, $6\frac{1}{4}$. **33.** $18\frac{1}{8}$. **34.** R6 14*a*.
35. 15 ; $17\frac{11}{17}$. **36** $37\frac{1}{2}$. **37.** £3. 2*s*. $4\frac{1}{2}$*d*. **38** 20.
39. R8-15-4. **40** R8 12*a*. **41** R17. 9*a* **42** 70.

Ex. 133. (pp. 266-268.)

1. (1) R39 ; R52 ; R65. (2) £96. 15*s*. ; £134. 15*s*. £211. 15*s*.
(3) 5 md. ; 1 md. 10 sr. ; 20 sr. ; 10 sr. ; $6\frac{2}{3}$ sr.
(4) 66 lb. ; 14 lb. ; 10 lb.
2. 1 sr. **3.** 400 lb. ; 308 lb.
4 Nitre, $60\frac{4}{5}$ cwt , sulphur, $7\frac{3}{5}$ cwt. , charcoal, $11\frac{3}{5}$ cwt.
5 Pot., 1 md 20 sr. . phos. of lime, 3 md 8 sr ; chl. of pot , 10·24 ch. ; earthy phosphates, 4 md. 18 sr. ; silica, 2 sr. ; metalic oxides, 1 sr.
6. Cop., $15\frac{3}{4}$ md. ; zinc , $7\frac{3}{4}$ md. **7** *A*, R100, *B*, R120, *C*, R140, *D*, R160.
8. R20. 0*a*. 10*p* ; R8. 9*a*. 6*p*
9. 2nd son, 236 bi. 3 kat. 14 ch. ; 3rd son, 180 bi. 12 kat. 6 ch. , whole farm, 694 bi. 13 kat. 12 ch.
10. *A*, 4 tons 16 cwt. ; *B*, 48 tons ; *C*, 50 tons 8 cwt.
11. *A*, R120 ; *B*, R40 ; *C*, R13. 5*a*. 4*p*.
12. *A*, $466\frac{2}{3}$; *B*, $933\frac{1}{3}$; *C*, 1400. **13.** 38400 sq. yd. **14.** $58\frac{3}{4}$ sr.
15. Oxygen, 6 md. 37 sr. 13 ch. ; hydrogen, 34 sr. 11 ch.
16. Man, 791 md. 22sr. ; woman, 527 md. 28 sr. ; child, 263 md. 34 sr.
17. R86. 10*a* 8*p* **18.** 800, 6400, 21600, 51200 (tons). **19.** $52\frac{1}{2}$ cub.
20 *A*, 28800 md. ; *B*, 14400 md. , *C*, 19200 md. , *D*, 16000 md.
21. 80 guineas ; 66 sovereigns ; 168 crowns.

22. 45 rupees, 72 half-rupees; 96 quarter-rupees.

23. 480 moidores; 620 guineas; 800 sovereigns.

Ex. 134. (pp. 269-270.)

1. R1092; R858. **2**. R1890. 14*a*. **3**. *A*, £200; *B*, £100; *C*, £300.

4. *A*, £391. 10*s*.; *B*, £522; *C*, £783; *D*, £913. 10*s*.

5. *A*, $157\frac{1}{2}$ md.; *B*, $337\frac{1}{2}$ md.; *C*, $207\frac{1}{2}$ md.

6. *A*, £48; *B*, £52; *C*, £60. **7**. *A*, R$2527\frac{7}{9}$; *B*, R$1971\frac{3}{5}$; *C*, R$2072\frac{7}{9}$.

8. *A*, R16800, *B* and *C*, R100800. **9**. *A*, R1140; *B*, R1134; *C*, R1716.

10. *A*, R288; *B*, R270, *C*, R216, *D*, R126.

11. *A*, R200, *B*, R210; *C*, R250. **12**. *A*, R2400; *B*, R1350.

13. *A*, R28. 12*a*.; *B*, R37. 6*a*. **14**. *A*, R3434-10-8; *B*, R1165-5-4.

15. R35400. **16**. *A*, £23170; *B*, £8771. 10*s*.; *C*, £8275.

17. 95 men; 192 women; 352 boys. **18**. *A*, R$4\frac{5}{6}$; *B*, R$6\frac{1}{6}$; *C*, R$5\frac{1}{2}$.

Ex. 135. (pp. 271-272.)

1. 12*a*. **2**. £3697. 4*s*. **3**. R4763. 7*a*. **4**. £528.

5 £5569. 8*s* $6\frac{3}{4}$*d*. **6**. 14*a*.; *A*'s loss R800. *B*'s R900, *C*'s R1000.

7. R12812. 8*a*.; R1640. 10*a*. **8**. 13*a*. 4*p*. **9**. £3642.

10. 11*a*. $2\frac{2}{5}$*p*.; R5284. 8*a*. **11**. £31916. 13*s*. 4*d*. **12**. 15*s*. $6\frac{3}{4}$*d*.

13. R16800, R10500. **14**. £9000, £6000. **15**. R896.

16. £18144. **17**. $5\frac{3}{5}$*p*., *A* loses R8037, *B*, R6916, *C*, R950.

Ex. 136. (pp. 273-274.)

1. (1) $16\frac{1}{2}$. (2) $45\frac{2}{3}$. (3) $3\frac{121}{300}$. (4) $30\cdot8\dot{1}4285\dot{7}$.

2. 38471. **3**. 61 yd. **4**. 70 mi. **5**. 10 st. **6**. $11\frac{1}{2}$ st.

7. 12 st. **8**. R3 13*a*. 4*p*. **9**. 25·18. **10**. 11 63. **11**. 7812.

12. $6\frac{1}{4}$%. **13** 546710. **14**. 12 yrs. **15**. 12 yrs. 6 mo.

16. 47°. **17**. R16. 2*a*. 8*p*.; R18. 12*a*. 8*p*.

Ex. 137. (pp. 274-275.)

2. $5\frac{11}{30}$ mo. **3**. $175\frac{157}{775}$ days. **4**. 3rd May.

5. 5 mo. **6**. 10 mo. **7**. $51\frac{1}{7}$ days. **8**. 9 mo.

Ex. 138. (pp. 276-277.)

1. 1 : 3. **2**. 10 : 7. **3**. 3 : 2. **4**. 39 sr.; 27 sr.

5. 20 md.; 45 md. **6**. 48 gal.; 64 gal. **7**. 1 md. 14 sr.; 5 md. 28 sr.

8. 9 : 2. **9**. 28 tolas; 10 tolas. **10**. 44 md.; 32 md.

11. 1 : 1 : 4. **12**. 1 : 1 : 1 : 1 or 3 : 7 : 7 : 3. **13**. 2 : 7.

14. 7 : 1. **15**. $22\frac{1}{2}$ sr.; $17\frac{1}{2}$ sr. **16**. 15 gal.; 65 gal.

Miscellaneous Examples VI. (pp. 280-285.)

1. $3\frac{1}{5}$ hr. **2.** R72. **3.** 30. **4.** 6·75 lb. **5.** £2093-16-$3\frac{9}{80}$.
6. 3200 md. **7.** 3 mo. **8.** R5-12-2. **9.** 1·15625.
10. R2-14-8. **11.** $4\frac{13}{30}$. **12.** 30. **13.** Loss 1 p. c. **14.** 20 days.
15. R745-0-6. **16.** 1 p. c. **17.** R50-7-$2\frac{1}{4}$. **18.** £1-1-$11\frac{13}{48}$.
19. Man, R100; woman, R75; boy, R50; girl, R45.
20. 8-35 P. M. **21.** 13268·337$\dot{3}$8329$\dot{2}$. **22.** $13\frac{1}{3}$ oz.
23. 2·236067; R59. 10*a*. 1*p*. nearly. **24.** $123\frac{1}{5}$ gr.
25. 3rd Aug. 2-$40\frac{40}{101}$ A. M.; 2-$13\frac{67}{101}$ A. M.; 3-$13\frac{67}{101}$ A. M. **26.** £600.
27. R39187-8, increase. **28.** $2^2.3^2.5.11.13^2$; $2^5.3^3 7^2.11$; 1st × 55; 2nd × 66.
29. R4. 11*a*. $8\frac{52}{81}$*p*. **30.** $86\frac{5}{8}$. **31** R2. 9*a*. 2*p*. **32.** $1\frac{4}{5}$ hr. after return.
33. $\frac{352}{1125}$ mi. **34.** ·156 sq. in. **35.** 3000.
36. 4 days. **37.** 16 ft. **38.** *A*, R1005; *B*, R50. 4*a*.; *C*, R62. 13*a*.
39. 254 sq. ft. $57\frac{1}{24}$ sq. in.; R143. 1*a*. $6\frac{23}{32}$*p*.
40. Length, 20 ft. $9\frac{3}{5}$ in.; width, 13 ft. $10\frac{2}{5}$ in.; height, 17 ft. 4 in.
41. R20. **42.** R9000. **43.** 14 cub. ft. 197 cub. in.
44. $4\frac{10}{21}$. **45.** 10 wk.; R69-4-4. **46.** R270. 7*a*. 8*p*.
47. *B* comes first; $40\frac{87}{97}$ ft. **48.** $2\frac{1}{3}$ min., 1232 yd.
49 10655. **50.** Wine : water :: 3 : 1. **51.** 259200.
52. Eldest, R100 less; 2nd, R36 less; 3rd, R44 more; 4th R92 more.
53. $12\frac{114}{121}$ sec. **54.** 25 ; 8 : 5.
55. Diminished $7\frac{43}{51}$ p. c. **56.** *A*, 9 da.; *B*, 18 da.; *C*, 9 da.
57. $37\frac{3}{25}$ min. **58.** 60. **59.** 80 mi. from Howrah.
60. 8 mi.; 12 mi. **61.** *A*, R1040; *B*, R780; *C*, R1180.
62. 6 p. c. **63.** *A*, £160; *B*, £175. **64.** 120 gal.
65. *A*, £21175; *B*, £15246, *C*, £21780; *D*, £18480.
66. Height, $12\frac{1}{2}$ ft. **67.** £303. 15*s*. **68.** 1 : 2.
69. $12\frac{1}{2}$ p.c.; $33\frac{1}{3}$ p. c. **70.** 8-$4\frac{1291}{1601}$ A. M. **71.** £66. 13*s*. 4*d*.
72. 1 : 1 : 1. **73.** R2. **74.** 30. **75.** $36\frac{4}{11}$.
76. 5*s*. $1\frac{2}{3}$*d*. **77.** 80 min. **78.** $33\frac{1}{3}$. **79.** R15.
80. 24 men. **81.** 4 md.; 2 md. 30 sr.
82. Man, $7\frac{1}{5}$ hr.; boy, 18 hr.; man and boy $5\frac{1}{7}$ hr. **83.** 15 ft.
84. 2·4 ft. **85.** £460. 4*s*. **87.** *A*, 30 yr.; *B*, 35 yr.
88. $25\frac{18}{17}$ mi. **89.** 15 mi.: $1\frac{2}{3}$ hr. **90.** At $\frac{20}{21}$ of the 2nd ebb tide.

Ex. 139. (p. 287.)

1. R14. **2.** R21. 8*a*. **3.** R26. 7*a*. 6*p*.
4. R42. 14*a*. 3*p*. **5.** R177. 3*a*. **6.** R831. 4*a*.

Ex. 140. (p. 288.)

1. (1) R6. (2) R20. (3) R14. 4*a*. (4) R28.
(5) R100. 14*a*. (6) R128. 6*a*. (7) £14. (8) £34. 10*s*.
(9) £45. 9*s*. (10) £54. (11) £28. 18*s*. 9*d*. (12) £94. 3*s*. 6*d*.

2. (1) R295. 6*a*. 6*p*. (2) £115. 3*s*. (3) R1598.
(4) £1194. 7*s* 6*d*. (5) R32. 12*a*. (6) £250. 2*s*. 6*d*.
(7) R148. 14*a* (8) £21. 16*s*. $7\frac{4}{5}$*d*. (9) R50. 10*a*.
(10) £107. 13*s*. (11) R178. 8*a*. 6*p*. (12) £5. 15*s*. 9*d*.
(13) R75. 0*a*. $11\frac{13}{25}$*p*. (14) £45. 12*s*.

3. (1) R91 11*a*. 9*p*. (2) R2901. 0*a*. 6*p*. (3) R158. 7*a*. 6*p*.
(4) £2. 1*s* 5*d*. (5) £4352. 9*s*. 6*d*. (6) £155. 16*s*. 8*d*.

4 (1) R2278. 14*a*. 8*p*. (2) R6086. 11*a*. 6*p*. (3) R5043. 9*a*. 4*p*.
(4) R60 12*a*. (5) £145. 0*s*. $11\frac{13}{16}$*d*. (6) £4754. 19*s*. 3*d*.
(7) £554. 11*s*. $1\frac{1}{3}$*d*. (8) £397. 2*s*. $2\frac{2}{5}$*d*.

Ex. 141. (pp. 289-290.)

1. R4550.	**2** R5000.	**3.** R22750.	**4.** £556. 13*s*. 4*d*.
5. £236-6-8.	**6.** R6050.	**7.** R1333-5-4.	**8** R1500.
9 £375.	**10.** £187. 10*s*.	**11.** R7500.	**12.** £10800.
13 $5\frac{1}{2}$ yrs.	**14.** 10 yrs.	**15.** 100 days.	**16.** $4\frac{2}{3}$ yrs.
17. $2\frac{1}{2}$ yrs.	**18.** 6 yrs.	**19.** 10 yrs.	**20.** 6 p. c.
21. $4\frac{1}{4}$ p. c.	**22.** 20 yrs.	**23.** 25 yrs.	**24.** 16 yrs.
25. $66\frac{2}{3}$ yrs.	**26.** 10 mo.	**27.** $8\frac{1}{3}$ yrs.	**28.** $4\frac{1}{2}$ p. c.
29. 5 p. c.	**30.** 3 p. c.	**31.** $3\frac{3}{4}$ p. c.	**32.** 4 p. c.
33. 5 p. c.	**34.** $6\frac{2}{3}$ p. c.	**35.** $7\frac{1}{2}$ p. c.	**36.** R1000.

Ex. 142. (pp. 292-293.)

1. (1) R6243. 3*a*. 2*p*. (2) R10925. 9*a*. 7*p*. (3) R2522.
(4) R494. 8*a*. 8*p*. (5) R1711. 8*a*. (6) R4758. 3*a*. 6*p*.
(7) £576. 13*s*. 7*d*. (8) £18. 19*s* 11*d*. (9) £836. 4*s*. 6*d*.
(10) £234. 2*s*. 4*d*. (11) £99. 8*s*. 9*d*. (12) £904. 13*s*. 5*d*.

2 (1) R656-0-4. (2) £302-15-7. (3) £417-7-1 (4) R232-10-6.

3. (1) R1365-4-3. (2) R1748-8-10. (3) £104-1-8 (4) £722-8-2.

4. (1) R5195-15-2. (2) R6232-9-11. (3) £6060-11-11. (4) £5067-4-0.

Ex. 143. (p. 294.)

1. (1) R750. (2) R300. (3) R9000. (4) £400. (5) £705.
(6) £15. (7) R885. 15*a*. (8) R2609. 6*a*. (9) R17233. 5*a*. 4*p*.
(10) £198. 6*s*. 8*d*. (11) £220. (12) £1355.

2. (1) 13*a*. 4*p*. (2) R2. 8*a*. (3) R2. 5*a*. 4*p*. (4) £7.
(5) £10. (6) £26. 4*s*. $5\frac{1}{2}$*d*. (7) R1. 1*a*. 6*p*. (8) R16. 8*a*. 11*p*.
(9) £10. 10*s*. (10) £300. (11) £4. 6*s*. 8*d*. (12) £1990. 12*s*. 6*d*.

Ex. 144. (p. 293.)

1 (1) $4\frac{4}{9}$ mo. (2) 5 yrs. (3) 12 yrs (4) 3 yrs. (5) $5\frac{1}{4}$ yrs. (6) $3\frac{3}{4}$ yrs.
2. (1) 4 p c. (2) 5 p. c. (3) 8 p. c. (4) 7 p. c. (5) 3 p. c. (6) 4 p. c.

Ex. 145. (pp. 296-297.)

1. 2 mo. 2. 4 mo. 3 $2\frac{1}{2}$ p. c. 4. $6\frac{1}{4}$ p. c.
5. R2287. 8*a*. 6. £4763-14-$4\frac{1}{2}$. 7. R933-5-4. 8. £25620.
9. R4000, 5 p. c. 10. R3900; 5 yrs. 11. £16-10-$3\frac{33}{100}$. 12. $16\frac{2}{3}$ p. c.
13. 25 p. c. 14. R43. 12*a*. 15. R41. 10*a*. 8*p*.

Ex. 146. (p. 299.)

1. R87640. 2. R1. 2*a*. 1*p*. 3. R20. 0*a*. 11*p*. 4. R12. 3*a*.
5. £32. 6*s*. 7*d*. 6. £2. 18*s*. 1*d*. 7. £2. 14*s*. 8. R871. 14*a*. 3*p*.
9. R1. 8*a*. 3*p*. 10. R18933-9-4. 11. £1. 7*s*. 7*d*. 12. 3*d*.

Ex. 147. (p. 300.)

1. R125. 2. R25882. 8*a*. 3. £12000. 4. 736. 5. R864, R56736.

Ex. 148. (p. 302.)

1. R786 6*a*. 6*p*. 2. R12600. 3. £93. 15*s*.
4. £25000. 5. R40000. 6. £806800.

Ex. 149. (p. 305.)

1. £291-13-4. 2. £176-11-3. 3. £5530. 4. R3-2-6, R7751-12.
5. £1 (Amsterdam) = $14\frac{2}{27}$ fr. 6. £75. 7. 95256 fr.
8. 1555 fl. 2 kr. 9. Loss, £781. 5*s*. (through Amsterdam). 10 R25000.
11. £7500. 12. Loss, £2. 7*s*. 6*d*. 13. £7. 16*s*. 3*d*.

Ex. 150. (pp. 306-307.)

1. R1250. 2. £50. 3. 16 yrs. 4. 25 yrs. 5. 4 p. c.

Ex. 151. (pp. 310-311.)

1. (1) R4700. (2) R4560. (3) R4760. (4) £8307. 15*s*.
(5) R7774. (6) R2962. 4*a*. (7) £9165.
2. R7048. 8*a*. 3. R47263. 2*a*. 4. R14981. 4*a*.

5 (1) R2250. (2) £2150. (3) £8000.
(4) R10500. (5) R24000. (6) £1900.
6. R5000. **7**. £10000. **8**. £90. 5*s*. **9**. £21600.
10. R10000. **11**. R20000. **12**. R7750.

Ex. 152. (pp. 312-313.)

1. (1) R1729. (2) £96. (3) R1254. 4*a*. (4) £151. 10*s*.
2. £54-1-3$\frac{3}{50}$. **3**. R5013-4-2$\frac{168}{377}$. **4**. R1200 ; the latter by R60.
5. R670. 8*a*. **6** R872 7*a* **7**. R195082 ; 3$\frac{31}{48}$. **8**. R551. 4*a*.
9. No alteration. **10** R522 increase. **11** Nothing.
12 R243. 12*a*. ; R6093. 12*a*. **13** R10600 ; R6000.
14. 3933 : 3740. **15**. R58080.

Ex. 153. (pp. 314-317.)

1 (1) 3$\frac{1}{3}$. (2) 3$\frac{11}{13}$. (3) 3$\frac{373}{400}$. (4) 4$\frac{1}{6}$.
2. 3$\frac{707}{1536}$. **3** 3$\frac{1411}{3013}$. **4**. 92$\frac{5}{8}$. **5**. 97$\frac{1}{8}$. **6**. 93$\frac{3}{5}$. **7**. 96.
8 115$\frac{13}{24}$. **9** 86. **10**. 125. **11**. 90. **12**. 92$\frac{5}{8}$. **13** 2$\frac{29}{32}$.
14 £30-13-9 more (from the latter). **15**. R1430. **16** £14400 : 83$\frac{1}{3}$.
17 £4305. **18** £2950. **19**. R14965 ; R35235 **20**. £22050.
21 6$\frac{3}{4}$. **22**. R5610 **23**. 2$\frac{9}{35}$. **24** 37 ; R4255
25. Decrease, £69. **26** R326375. **27** 2*a*. gain.
28 Increase, £52. 10*s*. **29** Increase R322. 8*a*. **30**. R472500.
31. Increase, R53$\frac{4}{7}$. **32** The 1st ; R791700. **33** £500000.
34 10 p. c. **35** 108$\frac{13}{16}$. **36**. Increase, R50.
37 Decrease ; £106 5*s*. **38**. £69. 12*s*. **39** £201$\frac{1}{14}$ increase.
40. R65066. 10*a*. 8*p*. **41** 87$\frac{3}{31}$. **42** Decrease, 25 p c.
43 £33600, £27000. **44** £3381 ; £1794. **45**. £26-12-9$\frac{1}{4}$. **46** £2700.

Ex. 154. (p. 319.)

1. 255 yd. **2**. 100 mi. **3**. 216·333...ft. **4** 70 ft. 9 in. **5** 6 ft. 7$\frac{1}{2}$ in.
6. (1) 5. (2) ·6384. (3) 100. (4) 1985. (5) 1$\frac{9}{10}$.
7. (1) 1·732... (2) 9·899... (3) ·072... (4) 11·916... (5) 1·932... (6) ·268...

(Ex. 155. (p. 321.)

1. (1) 12, 15, 72, 279. (2) 31 . 93 ; 567 ; 1101.
(3) 907 , 3747 ; 6031. (4) ·013 ; 3·39 ; ·0921.
(5) 9·4402 ; 13·2077 ; 2·3360. (6) ·721. 1·462, 1 930, 1·842.
2. 1 ft. **3**. 2 ft. 10·6 in. nearly. **4**. R10. 11*a*. 1$\frac{1}{3}$*p*.

Ex. 156. (p. 321.)

1. 16, 1·1, ·97. **2.** 5, ·9, ·3. **3.** 4 ; 1·2.

Miscellaneous Examples VII. (pp. 323-328.)

1. 6*s.* $8\frac{1}{4}$*d.* nearly. **2.** 9·8. **3** The latter, R3-5-$6\frac{1}{2}$.
4. £33-6-8. **5** £90000. **6.** $113\frac{1}{7}$ sq. mi.
7. £600. **8.** *A*, 8*s.* 8*p.* ; £1 10*s.* 8*d.* **9.** R576. 14*a.*
10. £1200. **11.** £2424. 3*s.* $10\frac{14}{31}$*d.* **12.** 40 yr. : 30 yr.
13. *A*, R150 ; *B*, R120 ; *C*, R90 ; *D*, R60. **14.** R29. 8*a.* 9*p.* , R100.
15. 220 , 180. **16** *A*, £55 ; *B*, £$37\frac{1}{3}$; *C*, £21. **17.** $7\frac{1}{2}$ cwt.
18. £94. 6*s.* $1\frac{1}{2}$*d.* **19.** 768 sq. in. **20** R13040-15-1.
21. 28·168219. **22.** R4. 1*a.* 8*p.* **23.** *B*, by $\frac{10}{41}$ p. c.
24. £817 1*s* 8*d.* ; £1129. 3*s* 4*d.* **25** £281 19*s.* nearly. **26** £240 2*s.*
27. 2 : 3 : 1. **28** R2300. **29** 10 yrs.
30. Circuitously by 35·985 milreas. **31** 51. **32** 6 wks.
33. 23 oxen **34.** 48 ft. **35.** 160 mi.
36. £322 ; £227. 4*s.* ; £2060. 16*s.* **37.** 16 yrs ; 26 yrs. **38.** $17\frac{9}{7}$.
39. R$10\frac{4}{5}$. **40.** $4\frac{1}{2}$. **41.** $3\frac{96063}{282221}$. **42.** £220.
43. R29829 2*a.* **44.** R1470. **45.** £6720. **46.** £10.
47 50000000 qr. **48.** R13-5-4. **49.** 10. **50.** £12. 10*s.*
51. R1000. **52.** 2550 ; 2500. **54** £$1428\frac{4}{7}$; £50 ; $62\frac{1}{2}$.
55. 32. **56.** 197. **57** $8\frac{1}{3}$. **58** 6 mi.
59. £1000. **60.** 30 days. **61.** R5000.
62. R14508, R12090, R12896, R9672. **63.** R30 more from the former.
64. $32\frac{9}{17}$. **65.** 40 in. nearly. **66** £7995.
67. R10000 ; R12500. **68.** £20800 ; £10000.

Miscellaneous Exercises (pp. 328-358.)

1. Men, 17244999 ; women, 18034000. **2.** 2800. **3.** 37.
4. £100. 10*s.* $10\frac{3}{4}$*d* · 20 sq. yd. 4 sq. ft. 36 sq. in. **5.** 25 sec.
6 19*s.* 5*d.* **7.** 539. **8.** 17669. **9.** 2997972000 **10** 4096.
11. 64 days. **12.** 6*s.* 4*d.* **13.** £4. 9*s.* $8\frac{1}{2}$*d.* **14** 2592000.
15. $1\frac{1}{2}$*d.* **16.** 1*s.* 4*d.* **17.** 97353052. **18.** 237.
19. 1 ton 7 cwt. 3 lb. 3 oz. 9 dr. **20.** 54 boys ; 54 girls.
21. R7992. **22.** ·9 wks. **23.** 22275 words · 2025 lines
24. R6. 3*a.* 6*p.* ; R4. 2*a.* 4*p.* ; R2. 1*a.* 2*p.*
25. $\overline{\text{MMMDCCXXXI}}$CCCLXIV. **26.** £57. 12*s.* 6*d.*

27. Man, R723. 8*a*. ; woman, R241. 2*a*. 8*p*. ; child, R120. 9*a*. 4*p*.
28. 1 yr. 10 mo. 2 da. **29**. £6. **30**. 37, 13, 11, 7, 3^3 ; 1485.
31. £3. 3*s*. 4*d*. **32** 18. **33**. 1664, 64, 691200. **34**. 13.
35. £3311. 5*s*. 3*d*. **36**. 3990. **37**. *A*, R12. 8*a*., *B*, R37. 8*a*. ; *C*, R45.
38. $4\frac{3}{4}$*d*. **39**. Gold, 11 sr. 20 tolas ; silver, 3 sr. ; copper, 3 sr. 60 tolas.
40 9 in.. **41**. R75 ; R25. **42**. 216 ; £50. 8*s*. **43**. $37\frac{1}{2}$ min.
44. 35. **45**. R1. 8*a*. 9*p*. **46**. R218. 8*a*. 6*p*. **47**. 9.
48. 87241. **49**. 11 yrs. ; 24 yrs. **50**. 2652 sr. **51**. 2519.
52. 1545. **53**. 2201 **54**. 5040. **55**. 84. **56**. R1010.
57. 162. **58** $3\frac{1}{2}$ days. **59**. 68 shots ; $\frac{1}{4}$ oz. **60**. 495 sr.
61. £12. **62**. *A*, 100*d*. ; *B*, 90*d*. **63**. $10\frac{5}{13}$ min. **65**. 72963.
66. 143. **67**. 2 po. **68** 161. **69**. £13 7*s*. **70**. 142.
71. R802. **72**. 16. **73**. *C*, took R29-14-9 from *A* and 3*p*. from *B*.
74. £1150, £1600, £2100. **75**. 11. **76**. 33 yrs. **77**. R3520 ; 96 da.
79. 7 mi. **80**. 99 da. 10 hr. 40 min. **81**. *A*, R296313 ; *B*, R280582.
82. 1080. **83**. 21 yr. 5 mo. 4 da. **84**. R2. 7*a*. **85**. R2958. 12*a*.
86. 160. **87**. 261. **88**. 352. **89**. 38. **90**. 520200. **91**. 45.
92. 76. **93**. £64. **94**. 144936 ; 144540. **95**. 185741 or 185661.
96. 250 ft. **97**. 35. **98**. $4\frac{613}{1507}$. **99**. 6*a*. **100**. 180.
101. 12 min. **102**. $34\frac{3}{8}$ mi. **103** Sunday, 5-20 A. M. **104**. 600 wks.
105. 1400 sec. ; 36. **106**. 168. **107**. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}>1$; 3600, 2400, 1800.
108. R100. **109** R4384. 6*a*. **110**. $7\frac{71}{102}$ mi. from *B*.
111. £3586-13-4 £4035 ; £4304 ; £4483-6-8. **112**. £19200 ft.
113. $\frac{27}{272}$. **114**. R6346. 10*a*. 8*p*. **115**. 326400. **116**. 9707 ft.
117. R253125. **118**. 791 da. $22\frac{3}{4}$ hr. **119**. $\frac{11}{18}$. **120**. R75.
121. 1106835. **122**. *A*, £554 8*s*. ; *B*, £475. 4*s*. ; *C*, £302. 8*s*.
123. 240, 180. **124**. 20 min. **125**. 20 lb.
126. Christians 320000 ; Mahomedans 200000. **127**. 432. **128**. 75.
129. $4\frac{1039}{1932}$. **130**. *A*, R364 ; *B*, R136. 8*a*. ; *C*, R156 ; *D*, R214. 8*a*.
131. 22 hr. **132**. 408. **133**. R2. 4*a*.
134. Stalls 600, boxes 1000, pit 800, gallery 200 ; altogether 2600.
135. 45 days. **136**. R1502. 15*a*. 6*p*. **138**. 5 for 4d.
139. $1\frac{3}{2}$ ft. **140**. R15. **141**. 7899 mi. 1 f. 25 po. 1 yd. 6 in.
142. 25m. 6f. 6p. 2ft. 4fn. ; 42499 hhd. 12ga. 3qt. 1pt. 2gl.; 146916499 sq. in.
143. *A*, 162, *B*, 108, *C*, 72. **144**. 1. **145**. £2-13-$4\frac{1}{2}$; £2-13-$2\frac{3}{4}$.
146. (*a*) 1. (*b*) $3\frac{1}{16}$. **147**. £74. 6*s*. 9*d*. **148**. 56.
149. 7 sov. ; 14 $\frac{1}{2}$-cr.; 35 shil. ; 70 three-pence. **150**. 4-30 P. M.

151. 3 hr. 30 min. **152.** 12 mi. **153.** 25000. **154.** R37-0-7. **155.** R720.
156 £28. **157.** 132. **158.** 2*d.* **159** (*a*) $\frac{7}{11}$ (*b*) $27\frac{18}{101}$.
160. Captain, £287. 17*s.* ; an officer, £63-19-4 ; a man, £15-19-10.
161. 2419200. **162.** $\frac{284}{385}$, $\frac{4357}{5557}$; 7*s.* $2\frac{2}{5}d$.
163. 80. **164.** 5 o'clock, 1 o'clock. **165.** 997·23 oz
166 40. **167.** 4-30 A.M., March 13. **168.** £1. 4*s.* $4\frac{1}{2}d$.
169 £7 ; $4\frac{1}{4}d.$; £1. 10*s.* **170.** 126. **171.** $\frac{4}{11}$. **172.** 8*s.* 9*d.*
173 30. **174.** $4\frac{7}{32}$ mi. **175.** 350 ; 560. **176.** $14\frac{2}{3}$.
177. £11. 11*s.* $7\frac{1}{2}d$. **178.** Sunday, 14 times. **179.** 60.
180. 11985 crowns. **181.** 8. **182.** 175 runs. **183.** 24, £17. 10*s.*
184. *A*, R100 ; *B*, R150 ; *C*, R250 ; *D*, R500. **185.** £4-6-$1\frac{1}{2}$, loss.
186. $\frac{9}{110}$ **187.** R$\frac{19}{61}$. **188.** R16000. **189.** 10 mi. an hr.
190. Each son R2666-10-8 . eldest daughter R2000, others R1900 each.
191. $\frac{1}{10}$. **192.** 3 hrs. **193.** 2*s.* $8\frac{1}{2}\frac{8}{5}d$; 6*a.* ; c003273333.
194 £120. 4*s.* $2\frac{1}{4}d$. **195** $2\frac{30}{71}$ da. **196.** 35 da. **197.** 1.
198 104 boys, 46 girls. **199** 16·3. **200.** *A*, *B* each $74\frac{2}{17}$ da , *C*, $157\frac{1}{2}$da.
201. £4806. 15*s.* **202.** $64\frac{2189}{3000}$; 64·729$\dot{6}$. **203.** 24.
204. 24 da. **205.** 975. **206.** 250. **207.** 5. **208.** 16 da.
209. ·02601963, 128492 , ·$2\dot{7}\dot{0}$. **210.** 190 tons 1 qr. 6lb ; 6lb 7 oz.
211. 100000. **212.** 19*s.* $2\frac{43}{50}d$. **213.** 8 : 9. **214.** 9 : 8.
215. ·11157. **216.** 200 gui. ; *A*, 125 gui. ; *B*, 75 gui. **217.** £261360.
218. 1298·819$\dot{4}$ **219.** $\frac{1}{8}$. **220** 21000. **221.** ·3801730.
222. 45 mi. **223.** 18*s.* 9*d.* **224.** R33600 ; R33745. 14*a.* 8·64*p.*
225. £29-18-$8\frac{3}{4}$. **226.** £272. 5*s.* **227.** *A*, 15 da. ; *B*, 10 da. ; *C*, 12 da.
228. 20. **229.** Both R200. **230.** R83-5-4. **231.** 17 in.
232. $93\frac{3}{4}$. **233** £75. 7*s.* $9\frac{3}{4}d$. **234.** £10. 8*s.* ; 9·984*d.* ; 1 56*d.*; $\frac{1}{4}$.
235. *A*, R52. 6*a.* ; *B*, R157. 2*a.* ; *C*, R314 4*a.*
236 $3\frac{3}{4}$ min. ; 10·8 mi. an hour. **237.** 80. **238.** $18\frac{3}{4}$.
239. £3. 17*s.* **240.** £3661. 13*s* 4*d.* **241.** R52-12-$4\frac{4}{5}$.
242. 1 ton. 5 cwt. 13 qr. ; 12*s.* 11·776*d.* **243.** $1437\frac{1}{2}$. **244.** 112·64 yd.
245. 8908·2 fr. **246.** In 2 days more. **247.** 13*s.* 4*d.*
248. £31010. **249.** 124. **250.** 93696 ft. 3·855 in. **251.** 20 da.
252. 87 yd. ; $43\frac{1}{2}$ yd. **253.** £863. 6*s.* 8*d.* **254.** R42.
255. 1·68 lb. **256.** $2\frac{1}{2}$ p. c. **257.** 572. **258.** 8.
259. R4. 14*a.* **260.** ·$\dot{0}$00574... **261.** 9471. **262.** R390.
263. 10*a.* **264.** £52000. **265.** 2 mi. an hour.
266. Breadth, 14 ft. ; height, 12 ft. **267.** 4 hr. 6 min.

268. 3 : 13. **269.** 7200. **270.** £5200. **271.** $3\frac{3}{4}$; £1600.
272. *B*, by R30. 7*a*. $7\frac{1}{7}p$. **273.** $1\frac{1}{2}$ ft. **274.** 20°.
275. R2. 7*a*. **276.** 8 : 1. **277.** 8 : 13 **278.** 13 : 20.
279. R2800 , 4 p. c. **280.** 5 p. c. **281.** 15 da. **282.** 10 da.
283. 10, 25, 50, 75. **284.** 9 gal. **285.** R8. 6*a*. $2\frac{2}{3}p$.
286. R87. 8*a*. **287.** R472. 8*a*. **288.** R$99\frac{5}{18}$; R400.
289. Loses $\frac{1440}{1441}$ min. daily. **290.** 7 : 3. **291.** $3\frac{6}{11}$ mi.
292. 183 sq. ft. ; R915. **293.** $7\frac{4}{5}$ mo. **294.** R4125.
295. 4 p c. **296.** R121. 4*a*., increase ; R32916. 10*a*. 8*p*.
297. £222. 1*s*. $1\frac{3}{4}d$. **298.** 311·126 yd. ; 155·563 yd.
299 3*s* $6\frac{1}{2}d$. **300.** $3\frac{5}{9}$ ro. **301.** £3. 4*s*. 11*d*. **302.** £5270-16-8.
303. R3758. 12*a* **304.** At 54. **305.** £2. 4*s*. 6*d*.
306. £1. 1*s* **307.** $\frac{7}{9}$. **308.** 741 ; $2\frac{1}{3}$. **309.** 48 days.
310. £1714-15-3. **311.** 8·4 yr. **312.** In both cases, £8. 2*s*. 6*d*.
313. Horse, R350 ; cow, R140. **314.** £6352-11-$11\frac{1}{2}$. **315.** $2\frac{11}{32}$.
316. £14-10-$3\frac{1}{3}$. **317.** 240. sq. ft. **318.** £4-5-4. **319.** 8 p. c.
320. £5920 in 3 p. c. ; £1080 in 6 p.c. **321.** 58°. **322.** 23 ; 68.
323 19 : 10. **324.** $3\frac{3}{5}$ mi. an hr. ; $5\frac{23}{35}$ mi. an hr. **325.** $3\cdot4\frac{5}{8}$ P.M.
326. £48. **327.** R1152. **328.** £7995. **329.** £1. 4*s* ; 18*s*.
330. $3\frac{1}{2}$ days **331.** £101. 17*s*. $11\frac{13}{25}d$. **332.** Gains 6·15 p. c.
333. $\frac{3}{8}$ mi. **334.** 16*s*. 8*d*. ; 5*s*. 5*d*. **335.** £882. **336.** £2000.
337. £48400. **338.** After $16\frac{3}{4}$ days . 3-$57\frac{2}{9}$; 4-$2\frac{2}{9}$. **339.** 35 days.
340. $12\frac{1}{32}$ fr. **341.** £125. 2*s*. 6*d*. **342.** $15\frac{5}{8}$. **343.** £150000.
344. £653. 2*s*. 6*d*. **345.** 12*s*. 8*d*. **346.** 16 ft.
347. 164 guineas ; 156 half-guineas. **348.** Centres, 62 ; outers, 38.
349. *A*, 36 ; *B*, 40 ; *C*, 44. **350.** 99. **351.** 3 : 1.
352. R55000 ; R50781. 4*a*. **353.** Horse, £30 ; cow, £15.
354. 20 ft. 6 in. ; R42. 11*a*. 4*p*. **355.** 180 men.
356. 24 times. **357.** 50 days. **358.** $33\frac{1}{3}$. **359.** £264.
360. R16500. **361.** 3 times. **362.** 30 men. **363.** $31\frac{1}{2}$ mi. an hr.
364. R28476. 9*a*. **365.** $11\frac{1}{5}$. **366.** R500 ; R400. **367.** £5000.
368. £450 ; £102, increase. **369.** $11\frac{109}{121}$ in. **370.** 8 mi. an hr.
371. R10. 12*a*. **372.** $3\frac{5}{6}$ days. **373.** 98 gal.
374. *A* should pay R13. 8*a*. 11*p*., and *B* should pay R47. 3*a*. $10\frac{2}{3}p$., to *C*.
375. £4207. 10*s*. ; £5049. **376.** At 11*d*. **377.** 156 ; 3*s*. 3*d*.
378. *A*, R754. 6*a*. ; *B*, R1963. 2*a*. ; *C*, R2717. 8*a*. **379.** 80 yd.
380. *A*, R2400 ; *B*, R900 ; *C*, R240 ; *D*, R60. **381.** 20 mi.

382. 25 p. c. **383.** R10 **384.** R58. 8*a.* more from the latter.
385. *A*, R77. 8*a.*; *B*, R111. 4*a.*; *C*, R188. 12*a.* **386.** R45, R15, R9.
387. 91 st. **388.** Flour, $3\frac{9}{28}d.$; raisins, $9\frac{27}{28}d.$; suet, 1*s.* $5\frac{5}{7}d.$
389. £145. 4*s.* **390.** R20. 13*a.* 4*p.* **391.** £2000. **392.** £46. 16*s.*
393. 108 days. **394.** $2\frac{1}{5}$ ft. **395.** 680. **396.** 16 ft.
397. Gain, 4 p. c. **398.** 1 : 2. **399.** 6800 : 7221.
400. £1350; increase, £9. 5*s.* **401.** 299·919...gal.; 48125 oz.
402. 2740 yd.; $13\frac{83}{99}$. **403.** £1. 0*s.* 10*d.* **404.** 180 oz.; 4*s.* 2*d.*
405. £8000000. **406.** *A*, $16\frac{1}{2}$; *B*, 17; 405 hrs. **407.** 10 p. c.
408. 10 mi. **409.** 4 mi. **410.** £4030. **411.** R96. **412.** 792 yd.
413. 6 min. 6·08 sec. **414.** 15 hr. **415.** 108. **416.** 3 ft. 9 in.
417. 18, 27, 24, 30. **418.** 19 : 17. **419.** *A*, $9\frac{2}{5}$ hr.; *B*, $6\frac{6}{7}$ hr.
420. $1\frac{1}{2}$ mi. per hr.; 1 hr. **421.** £1-17-6; £7. 10*s.* **422.** £150. 15*s.*
423. *A*, R914-13-4 and *B*, R2458-14-8, nearly. **424.** R5565; R5940.
425. £$97826\frac{2}{23}$; increase, £$459\frac{18}{23}$. **426.** £11000. **427.** R1536.
428 1 rupee = 2·044 shillings nearly. **429.** $34\frac{68}{113}$ min. after 10.
430. 30 miles per hour.

Ex. 157. (pp. 361-364.)

1. (i) 42790 m. (ii) 1·434 m. **2.** (i) ·42872 kilog. (ii) ·03745 kilog.
3. (i) 45·25000525 ares. (ii) 7·518564 ares.
4. 5313 kiloletres, 5 hectolitres, 6 dekalitres, 8 litres, 6 decilitres, 2 centilitres, 5 millilitres.
5. 333·33.... **6.** 111·11...ler. **7.** 170 fr. 76·2 centimes. **8.** 20090 fr.
9. 5600 fr. **10.** 92·3... **11.** 26·51... **12.** 372 fr.
13. 2·75 centimes. **14.** 2·5 years. **15.** 14·2...p. c.
16. 4000 fr. stock; 200 fr. **17.** 1609. **18.** (i) 3 miles 543 yds. 1 ft. 9 in. (ii) 27 lbs. 3 oz. (iii) £21. 0*s.* 6*d.* (iv) £6. 1*s.* 4*d.* (v) 345 gallons 2 pints.
19. 4047 sq. metres nearly. **20.** 1016 kilogram, 48 grams, 117 milligrams.
21. 3051. **22.** £40000. **23.** 1037. **24.** ·00985 ton.
25. ·9941 metres. **26.** 6*s.* 6*d.* **27.** 29·33. **28.** 5*s.* 9*d.*
29. 6·5. **30.** 453·66 grams; ·00220428 lb. **31.** 30·479.
32. 981·42. **33.** 72·16 litres. **34.** 1 fr. 67 centimes.
35. R3. 1*a.* 4*p.* **36.** R39716. 10*a.* 8*p.* **37.** 1·2 metres. **38.** 1000.
39. 800 boys, 1800 girls. **40.** 322·6 litres; 711 lbs. 14·6 oz.
41. 4023 fr. 28 centimes. **42.** 265 : 156 : 20.
43. $\frac{7}{34}$ water, $\frac{27}{34}$ other liquid. **44.** 61·1 of the 1st, 38·9 of the 2nd.

45. 1,890,000 litres. **46.** $37\frac{1}{2}$ p. c. **47.** 37·5 kilometres.
48. 1·585 p. c. **49.** 1·02 grams. **50.** 93 Napoleons 15 francs.

Ex. 158. (pp. 365-366.)

1. (i) 1234. (ii) 20878. (iii) 8725. (iv) 10018.
2. (i) £5. 2fl. 8c. 4m. (ii) £5. 2fl. 5c. (iii) £1. 8fl. 2c. 5m. (iv) £3. 4fl. 5c. 5m. (v) £11. 6fl. 8c. 5m.
3. £62. 4fl. 9c. 8m. **4.** £42. 5fl. 3c. 9m.
5. £390. 9fl. 8c. 8m ; £90456. 2fl. 5c. **6.** (1) £310. 7fl. ·4c. 4·5m. (2) 19421·5. **7.** 6fl. 6c. 6·6m ; 4fl. 6c. 5·625m. ; £11. 4fl. 5c. 6·25m.
8. £1. 4s. 7·248d. ; £10. 13s. ·552d ; £12. 17s. 10·2d.; £120. 11s. 4·86d.
9. 25. **10.** ·008. **11.** £4. 4fl. 2c. 9m. **12.** £60. 6fl. 3c. 2·625m.
13. £56. 7fl. 5c. **14.** 3·2 p. c. **15.** £150 ; £5000.

CALCUTTA ENTRANCE PAPERS.

1858-90.

1. $33\frac{1}{3}$. **2.** $1\frac{159}{2401}$ oz. **3.** 5 : 22. **4.** $857\frac{1}{7}$ ac. **5.** £1350.
6. R6. **7.** ·00064 ; ·009, 400000. **8.** R9963. **9.** 7564. ·7071.
10. R6. **11.** $4\frac{4}{135}a$. **12.** In $25\frac{15}{22}$ min. **13.** $14\frac{27}{35}$ days.
14. £529. 4s. $7\frac{1}{4}d.$ $\frac{7}{13}q$. **15.** 540. **16.** 6800 : 7221.
17. £$79\frac{253}{625}$; 79·4048 ; ·3415. **18.** $329\frac{1}{70}$ yd. ; R1023-9-$7\frac{1}{5}$.
19. 45 men. **20.** R84-1-10 ; R16. 8a. **21.** 12 da.
22. £2-16-0$\frac{1}{4}$ ·9137890625q. **23.** ·10444637. **24.** R210.
25. $99\frac{5}{16}$; £176-4-$2\frac{170}{227}$. **26.** 19 mi. 836 yd. 2 ft. **27.** $9\frac{4}{5}$.
28. 440 mi. **29.** 401 : 544. **30.** £14. 7s. $11\frac{5}{6}d$. **31.** 16 years.
32. R15. $11\frac{16}{19}a$. ; 8091 cub. ft. **33.** $998999\frac{3947}{4940}$. **34.** $10\frac{2}{7}$ da.
35. The 2nd. **36.** R3250. **37.** R1597. 10a. 3p. **38.** R262. 8a.
39. R20. 11a. $2\frac{64}{81}p$. **40.** 19 yr. **41.** 5a. $7\frac{4}{5}p$. ; R5498. 7a.
42. 63 da. **43.** $3\frac{65}{144}$ cu. ft. ; $15\frac{115}{144}$ cu. ft. **44.** R66666-10-8 ; R108.
45. $\frac{1}{25}$. **46.** R35. 1a. 4p. **47.** R16540. **48.** R58. 2a. $3\frac{36}{53}$.
49. 9. **50.** $12\frac{1}{6}$ yd. ; R1. 12a. **51.** 200 da. **52.** $4\frac{1}{2}$ p. c.
53. £78. 15s. **54.** 125. **55.** 39 days. **56.** R3312 ; R219.
57. 1·00015. **58.** 104. **59.** 12 carts 19 times and 4 carts 18 times.
60. 68 men. **61.** Decrease, £11. 4s. 3d. **62.** 18.
63. 100, 20, 3, $\frac{4}{10}$, $\frac{5}{100}$, $\frac{6}{1000}$.
64. Each boy £4. 11s. ; each woman £13. 13s. ; each man £27. 6s.
65. 65 gal. ; 13 hr. **66.** *C* wins by $\frac{880}{3741}$ yd. **67.** R25. **68.** £15400.
69. 18 days. **70.** R35000. **71.** R$96\frac{2}{3}$. **72.** $28\frac{4}{7}$ yr.; R562. 8a. ; 75 p.c.

73. 18 ; $8\frac{1}{2}$ p. c. **74.** The 1st is better . £1342. 10*s.* ; $3\frac{93}{170}$ p. c.
75. R48659. 6*a.* **76** R12-12-$9\frac{3}{5}$; loss R$133\frac{1}{3}$. **77.** 10. **78.** R510.
79. $15\frac{5}{8}$ days. **80** £100 **81.** 51 days. **82.** R104. 4*a.*

91. **1.** (*a*) $\frac{18}{35}$; (*b*) $\frac{3}{8}$. **2.** 2·202642. **3.** R408. 3*a.* $4\frac{11}{112}$*p.*
4. $9\frac{9}{13}$ hr. **5.** R·20800. **6.** $8\frac{1}{3}$ yd.

92. **1.** $\frac{70}{135}$. **2.** 26219. **3.** ·312 ; ·098 ; ·998.
4 R1232 14*a.* nearly. **5.** £2500.

93. **1.** (1) $5\frac{67}{110}$; (2) 3. **2.** ·0789 ; $\frac{226}{405}$, $\frac{36}{55}$, 1·$\dot{1}$. **3.** £345. 17*s.* $3\frac{3}{4}$*d.*
4. R238. 3*a.* $2\frac{0}{7}$*p.* **5.** R90000 ; R7300$\dot{0}$.

94. **1.** £37. 0*s.* 8*d.* **2.** £491. 8*s.* **3.** 16*s.* 0·37$\dot{5}$01$\dot{3}$*d.*
4. ·9998. **5.** R6 each. **6** R124-10-$1\frac{1}{1}\frac{247}{825}$.

95. **1.** 1·0000. **2.** R12345. **3.** 3 fr. 84 centimes.
4. $\frac{7}{12}$. **5.** R47, increase ; ·68$\dot{3}$2876712$\dot{2}$.

96. **1.** 23704543, 8143. **2.** (i) $\frac{5}{12}$. (ii) ·075088. **3.** 2·2677.
4. R531. 3*a.* 10·064*p.* **5.** Loss, 1 6 p. c. **6.** R21735.

97. **1.** (*a*) ·0725. (*b*) $\frac{2}{15}$. **2.** Yes, an aliquot part ($\frac{1}{320}$); R32. 9*a.* $1\frac{1}{3}$*p.*
3. R20 **4.** 33 yr. 4 mo. **5.** R6 per share. **6.** 1·7774.

98. **1.** 20160. **2.** (*a*) $\frac{11}{16}$; (*b*) ·008$\dot{6}$. **3.** 234 ; 8·0608.
4. R250. **5.** $1\frac{1}{2}$ per cent. gain. **6.** R23400.

99. **1.** 25. **2.** (*a*) $1\frac{1}{16}$. **3.** R606. 11*a.* $9\frac{3}{4}$*p.*
4. 226 ; ·226. **5.** 60 mds. ; $3\frac{13}{20}$ p. c. **6.** R18.

1900. **1.** 42 minutes after. **2.** (*a*) 6. (*b*) $\frac{1}{24}$. **3.** 8.
4. £70. 14*s.* $11\frac{28}{48}$*d.* **5.** 125. **6.** R100 more.

1901. **1.** (*a*) 1·41$\dot{6}$. (*b*) ·565. **2.** (*a*) Yes. (*b*) £68. 15*s.* 9*d.*
3. 4lb. 8 oz. **4.** 4 p. c. **5.** 86·42. **6.** R122169.

1902. **1.** (*a*) Non-recurring decimal. (*b*) ·04 ; ·0$\dot{3}\dot{6}$.
2. (*a*) R15326. 10*a.* 8*p* ; (*b*) R7340. **3.** 35 boys.
4. 2·25. ; 8·729. **5.** (*b*) £100. **6.** The latter ; R60.

1903. **1.** (*a*) 1 ; (*b*) ·000568$\dot{1}$. **2.** (*a*) yes ; (*b*) £170. 19*s.* $4\frac{1}{2}$*d.*
3. $1\frac{4}{5}$ minutes. **4.** (*b*) $3\frac{1}{6}$; 1·7728...
5. $\frac{1}{4}\frac{3}{8}$ of a gallon. **6.** (*b*) R50.

1904. **1.** (*b*) 997920. **2.** (*b*) ·5. **3.** $22\frac{1}{2}$ days.
4. 217 yds. 2 ft. 4·8 in. **5.** £700. **6.** R151710.

1905. **1.** 165. (If the price of each mango be an exact number of pies ; otherwise the number of mangoes may be any between 152 to 226.)
2. (ii) $\frac{1}{4}\frac{3}{8}$. **3.** 4·4666.... ; ·7905... **4.** R4. 1*a.*
5. R110. 7*a*, $2\frac{1}{2}$*p.* ; 3 per cent. **6.** £3000.

1906. **1.** (2) 99679. **2** (2) (1) 1. (2) 2. **3.** R17. 2*a*.
4. $\cdot1\dot{6}$; $\cdot7745\ldots$ **5.** 17*s.* 6*d.* **6.** $11\frac{1}{9}$ p. c.

1907. **1.** 37128. **2.** (1) $\cdot\dot{1}$. (2) $\frac{1}{40}$. **3.** R46. 11*a.* $10\frac{43}{80}p$.
4. 110400. **5** R760. 8*a*. **6.** 13*s.* $3\frac{80581}{85331}d$.

1908. **1.** (1) Non-recurring decimal. (2) Re1. 8*a*. **2.** £934. 18*s.* 2*d*.
3. (1) $11\frac{109}{121}$ min. slow. (2) *D* wins by $7\frac{1819}{9483}$ yds.
4. (1) $2\frac{3}{25}$. (2) $\cdot5345\ldots$ **5** R1000. **6.** R425. increment.

MADRAS ENTRANCE PAPERS.

1857-88.

1. 4 ft. 2 in. **2.** £39 7*s.* 6*d* **3.** 342250lb. , 47 hr. 32 min. 5 sec.
4. $\cdot23007$ ..of a mile. **5.** 57°17′45″ nearly. **6.** Gain, $R25\frac{605}{833}$.
8. The weekly receipts per mile in 1858 were R6. 1*a.* $4\frac{241}{432}p$. more.
10. 11960 sq yd. 4 sq. ft. 20·41 sq. in. **11.** 56831327.
12. 26292. **13.** R1087. 8*a.* $11\frac{13}{25}p$
14 *A*, R22840 *B*, R11420 ; *C*, R3806-10-8 ; *D*, R7613-5-4
15. $1091495\cdot\dot{4}2857\dot{1}$. **16.** $26\frac{6}{19}$ per cent. **17.** R202-2-$0\frac{24}{25}$.
21. £478-18-$8\frac{96}{125}$. **22** 9 da. **23** 45 mi per hr. **25.** Steamer , 16 hr.
26. 9 hr. $37\frac{8}{11}$ min. **27.** $\frac{113}{375}$ greater by $\frac{1}{7810}$. **28.** $3\cdot2743\dot{6}$.
29 $2\frac{5}{8}$ miles. **30.** R120 **31.** 11 ft. $6\frac{1}{5}$ in.
32. $12\frac{1}{2}$. **33.** £5888. **34** R16666-10-8. **35** 20. **36.** 8 hr.
37. 226875 lb. **38** R32 7*a*. **39.** 25 per cent.
40. $\frac{2}{5}$. **41.** 1st. child R5184, 2nd R2592, 3rd R1728.
42. R3. **43.** 32 days. **44.** 60.
45. £2428-15 ; £1238-13-3 ; £1190-1-9. **46.** 1561 ft.
47. 6 hr. 59 min. 15 sec. **48** R9-7-3. **49.** $8\frac{3}{4}$. **50** 3*a.* $6\frac{6}{25}p$.
51. 220 yd. by 165 yd ; area = $7\frac{1}{2}$ ac **52.** R2790. 10*a.* ; [illegible]
53. *A* receives 6*s.* 8·0325*d.* more than *B*. **54.** 12 lacs ; R5.
55. $36\frac{2}{3}$ miles ; 37 minutes past 8 A. M. **56.** $16\frac{2}{3}$ days.
57. £419. 19*s.* 3*d*. **58** 6*p* **59.** £233-17-10 ; $5\frac{1}{5}d$. **60.** 2 years.
61. 12*s.* **62.** 4 sq. in. **63.** 70 oz **64** 22 miles. **65.** 76 ac.
66. 35 measures. **67.** 18 mi. **68.** R770 ; 1 p.c. **69.** £1000 ; 2 yr. ; $2\frac{1}{2}$yr.
70. R13680. **71.** 8 yd. ; 3 hr. **72.** Faster 99 yd. ; slower 77 yd.
73. 17*s.* 6*d.* **74.** 30 sr. **75.** £250 ; 4 p.c. **76.** £690.
77. $277\frac{1}{3}$ cu. in. **78.** 2 : 1. **79.** $2\frac{1}{2}$ miles.
81. $10\frac{1}{3}$ da. ; $4\frac{76}{245}$ cu. ft. **82.** R46. 8*a*. **83.** $3^2, 5, 7^2, 11^2, 13^2$; 5.
84. 15 ft. **85.** £1600. **86.** 67 yd. ; 125. **87.** 9 p c.
88 $13\frac{1}{3}$ days. **89.** 13 per cent. **90.** £49. 16*s.* **91.** $4\frac{1}{6}$ measures.

92. 52 days. **93.** R9180. **94.** R$1\frac{67}{125}$. **95.** R276-5-3, 31440.
96. 27 days. **97.** 25 shares. **98.** $6\frac{2}{9}$ ft **99.** 3 miles.
100. 3*a*. **101.** Width $18\frac{1}{2}$ ft.; height $14\frac{1}{2}$ ft. **102.** $3\frac{1}{2}$ p. c.
103. Each child ; R960 ; each brother, R495. **104.** R666. 12*a*.
105. R37350. **106.** 14 yrs. **107.** R10020. 5*a*.
108. R28500. **109.** Slower $15\frac{3}{22}$ miles ; faster $29\frac{19}{22}$ miles.
110. 13 ft. 4 in. **111.** R1771. 12*a*. **112.** 675 lb.
113. 27 gal. **114.** 27 days. **115.** *A*, 4 hr. 20 min.; *B*, 7 hr. 35 min.
116 £190. **117.** 1000 yd. **118.** The side of the cube is 7 in.; 7776.
119. *A*, R480 ; *B*, R533-5-4 ; *C*, R466-10-8 ; $1\frac{1}{3}$ p. c. **120.** £8.
121. $4\frac{1}{6}$ p. c. **122.** R211. 9*a*. **123.** $\frac{95}{192}$; £64.
124. 11*s*. $10\frac{1}{2}$*d*. **125.** R1920. **126.** 3*s*. 9*d*. **127.** 4 yrs.
128. £5000. **129.** ·9196 ; £16. 10*s*. **130.** 3500000.
131. 1-13 P M, 2nd July. **132.** 80 men. **133.** £180.
134. R17. 8*a*. **135.** 520344000 cu. ft.; $1\frac{47}{88}$ in. **136** R7 2*a*.
137. Increase, R502. 8*a*. **138.** R500000. **139** 500400.

89. **1.** £14005-2-$4\frac{1}{4}$; R103991-12-7. **2.** $\frac{241}{281}$.
3. ·$08\dot{2}73\dot{0}2\dot{9}$; 6*s*. $9\frac{3}{4}$*d*. **4.** R1730-13-$6\frac{1}{4}$. **5.** R48 2
6. £1694-13-9. **7.** R280000. **8.** 10 days.
9. 7500274. **10.** £39-3-9. **11.** 2·08925.

90. **1.** 342 ac. 2 ro. 39. po. 2 sq. ft. 36 sq. in.; 160 yds. **2.** 1·5.
3. R975358-9-$2\frac{1}{6}$. **4.** 30 wks. **5.** 14273. **6.** 4 mo.
7. Increase, £397. **8.** 12 cwt. 1 qr 19 lb. 4 oz., £662. 10*s*.
9. 343 : 169. **10.** 19487·171.

91. **2.** R1-11-8. **3.** R1-10-$2\frac{41}{181}$. **4.** 9 ; 46·$\dot{9}47\dot{1}77$.
5. 12 min. **6.** R6-6-4, R158. **7.** £291-9-$5\frac{1}{4}$.
8. 20 min. (afternoon). **9** 10*d*. **10.** R9180.

92. **2.** $\frac{3}{4}$. **3.** 5*s*. 3*d*.; ·0037115025. **4.** R67567-9-$7\frac{1}{8}$.
5. £416-13-4. **6.** 3·700965. **7.** 3221625 tons.
8. R355-13-4. **9.** R55-8-4. **10.** $3\frac{3}{35}$% **11.** 25640000.

93. **2.** $\frac{1}{4}$. **3.** 5*s*. $2\frac{3}{4}$*d*. nearly. **4.** R1593. 4*a*. 5·6*p*.
5. R93333. 5*a*. 4*p*. **6.** £986. 17*s*. $6\frac{1}{2}$*d*. **7.** R23. 12*a*. 4*p*.
8. $4\frac{1}{2}$ mi. per hour. **9.** R1062. **10.** 14·1625%. **11.** 9 0073210.

BOMBAY ENTRANCE PAPERS.

2. ·$0097\dot{2}$. **3.** 15·404 ft.
4. Each man, R$83\frac{1}{3}$; each woman, R$55\frac{5}{9}$; each child R$27\frac{7}{9}$.
5. $622\frac{2}{9}$. ac. **7.** Increase, R500. **8.** R4169-9-$10\frac{2}{3}$; $29\frac{1}{4}$ yrs.

9. Tons 4 cwt. 3 qr. 4 lb. 13 oz. 10. 7·6 ft.
12. 1111$\frac{1}{9}$. 13. R140 ; 360 two-anna pieces ; 960*ps*.
14. 2164 6*a*. 2$\frac{1}{2}$*p*. 15. R179$\frac{089}{059}$. 16. R1. 3*a*. per 100.
17. 2265625 metres. 18. 7369. 19. 2*s*. 4*d*.
20. *A*, *B*, *C*, R615 , *D*, R410. 22 215·484. 23. R333. 5*a*. 4*p*.
24. £11. 11*s*. 5$\frac{49}{103}$*d*. 25. R18. 26. 5656567742.
27. Saltpetre, 840 lb. ; sulphur, 112 lb. ; charcoal, 168 lb.
28. *A*, R3451$\frac{17}{25}$, *B*, R2876$\frac{2}{5}$. *C*, R10462$\frac{23}{25}$. 30. 1584 lb.
31. R52. 32. Increase by R3. 7*a*. 4$\frac{104}{109}$*p*.
33. *A*, R22222$\frac{2}{9}$; *B*, R33333$\frac{1}{3}$; *C*, R44444$\frac{4}{9}$; *D*. R46153$\frac{11}{13}$;
E, R30769$\frac{3}{13}$; *F*, R23076$\frac{12}{13}$.
34. 12 shares ; R1460. 35. 53 hr. 36. R1636363$\frac{7}{11}$; increase R10000.
37. 14*a*. 3*p*. 38 6$\frac{7}{17}$ months. 40. 2, 2, 5, 3, 3, 7, 43.
41. 788 423. 43 90 men. 44 3 yr. 8 mo 24 da.
45. R41. 10*a*. 8*p* , 8 cwt. 1 qr. 20$\frac{5}{8}$ lb. 46 4 p. c.
47. 9 days. 48. Increase, R428. 49. R25600.
50. R[illegible] ; R600 : R2100. 51. £585$\frac{5}{47}$.
53 $\frac{1}{32}$; $\frac{27}{37}$. 54. £606. 55. 4$\frac{1}{21}$. 56 9$\frac{13}{23}$, R2956. 4*a*. 57. 83149.
58 5$\frac{130}{309}$, decrease. 59. 1055 subscribers.
60 R1500. 61. 7$\frac{1}{2}$ hrs. 62. £2376. 5*s*. 63. 30780.
64. [illegible] 65. 55$\frac{1}{2}$. 66. £213. 12*s*. 67 R14586.
68. [illegible] 4*d*. 69. R26 70. 7$\frac{1}{32}$ p. c. ; R46000 Govt. stock , 4$\frac{5}{16}$ p. c.
71. 36 mi. and 24 mi an hr. 73. $\frac{134}{201}$; $\frac{130}{293}$. $\frac{1345}{2017}$. 74. 10·017.
75. 6792$\frac{24}{63}$ 76 3$\frac{9}{25}$*s*. , 4$\frac{8}{25}$*s*. ; 5$\frac{19}{25}$*s*. ; 7$\frac{17}{25}$*s* 77. R18750 ; R56 .4*a*. increase.
79. Tea R2. 8*a*, sugar 2*a*. 8*p*. 80. *A*, R850, *B*, R846, *C*, R1182.
81. £650. 82. £818. 8*s*. 83. 9$\frac{93}{220}$ miles.
84 R7678. 2*a*. ; 10*a*. 2·85*p*. 85. £1500. 86. £215. 8*s*. 9$\frac{15}{17}$*d*.
87. £20. 88. £2890. 10*s*. 90. R10. 91. 3$\frac{1451}{11804}$ p. c.
92. 26 coolies. 93. 2 years. 94. £273. 8*s*. 9*d*.
95. £1508. 15*s*. 7$\frac{1}{16}$*d*. 96. 1 min. 97. R2646. 98. 4 p. c.
99. 24 posts. 100. 9$\frac{3}{4}$ wk. ; £341. 5*s*. 101. £4328. 2*s*. 6*d*.
102. 77$\frac{35}{36}$ yd. 103. 22 yd. 104. £86. 11*s*.
105. £2. 9*s*. 0$\frac{1}{4}$*d*. $\frac{19}{77}$*q*. 106. £15. 16*s*. 2*d*. 107. £7. increase
108. 4 p. c. 109. 113 boys. 110. 1$\frac{1}{4}$ cwt. ; £1. 1*s*. 6*d*.
111. £72. 6*s*. 8*d*. 112. In the latter ; £457. 5*s*. 1$\frac{5}{8}$*d*.

87. 1. 5, 7, 11, 13 ; 22·9999891208453038625795$\dot{8}$. 2. 192$\frac{1}{4}$ ft.
3. 1$\frac{3}{5}$ 4. (i) 17 ft. 6$\frac{1}{4}$ in. (ii) 3·3752. 5. £5103. 6. 3$\frac{179}{187}$.

88. 1. 6. 2 19s 3d. 3. 20 mo.
4. 20 : 9 ; 5s. 1½d. per oz. 5. 5a. 4p.

89. 1. 10d. from 1st. ; 4s. 3¼d. from 2nd. 2. 8s. 4d. 3. 10
4 94298·11. 5 4 p. c.

90. 1. 2. 2. $5\frac{?}{7}$ ft × $5\frac{3}{7}$ ft. × $5\frac{5}{7}$ ft. 3 5-15′.
4 R32000. 5. 3 : 13.

92. 1 (i) $\frac{1}{6}$. (ii). ·8$\dot{3}$ 2. 100 lbs. ; 2 cwt. ; 3 qt.
3. 10-30 P M yesterday ; 10h. 30′ 50″. 4. R640.

93. 1. ·00502083 ; 15a. 7·44p. ; $\frac{1}{7}$ 2. 10. 3. 3 hr. 30 m
4. £259. 3s 5¼d. 5 401 : 544.

94. 1. (i) 24 (ii) $\frac{59}{115}$. 2. £32-14-3$\frac{9}{?}$; 3 3$\frac{9}{13}$ mo
4 £3-2-0$\frac{24}{?}$ 5. $\frac{19}{21}$ 6. 2 P.M , 23 Aug ; 1-46′, 2-16′.

95. 1 146097 da. 2. 156. 3. 30.
4. 1$\frac{1}{13}$ da. 5. R2160. 6. 772 , 15a. 3p.

PUNJAB ENTRANCE PAPERS.

1875-90.

1. 15 2. R3281 1a. 6p. 3. R800. 1a. 9p
4 22$\frac{2}{7}$ sr. for a rupee. 5. 220 da. 6. 1s. 10·76d.
7 338 sq. ft 8 2·115 cu in. 9. 316·277 yd. 10 R14. 13a.8$\frac{4}{?}$$p$
11 34 hrs. 12 140 ; 170 ; 190. 13. R567. 8a decrease
14. 8a. 11$\frac{2987}{4180}$$p$ 15 £9. 15s 16. 2420 sq yd 17. The latter
18 She loses. 19 R9. 9a 8$\frac{1}{4}\frac{9}{3}$$p$ a md.
20 R3762$\frac{14}{27}$; 2667$\frac{219}{257}$; R2213$\frac{111}{263}$; R1956$\frac{133}{?}$.
21 R11029. 6a. 7$\frac{1}{17}$. 22. 16 days 23 17 p.c 24 R14 8a. 6$\frac{6}{7}$$p$
25. £1324. 13s. 9$\frac{1}{?}$$d$. 26 8320 men. 27. R86$\frac{37}{?}$. 28. 2$p$.

91. 1. (1) 1$\frac{1}{24}$. (2) 11. 2. ·00622 .. 3. 10 years
4. £200 ; 5 yrs. 5 45 gal.

92. 1 1$\frac{1}{701}$; $\frac{355}{113}$. 2. 5·90625. 3. 218972·16 gal
4. £10166$\frac{2}{3}$; £6000. 5. 17s. 3d.

93. 2. 13·713729902. 3. ·41421. 4. R17-5-9½. 5. 32 mi.

94. 1. ·$\dot{5}7142\dot{8}$, ·$\dot{4}2857\dot{1}$. 2. 44 ft. ; 33 ft. 3 £471-1-6$\frac{3}{4}$.
4. R987-10-5$\frac{1}{2}\frac{?}{?}$, R987-8. 5. 10·04987.

95. 1. 1$\frac{1}{14}$, 1·0$\dot{7}1428\dot{5}$. 2. 1·25lb. 3. £391, £529, £1311
4. 2·2360679. 5. 76 yds. 7½ in., 38 yds. 3¾ in.

96. 1. R63. 9a. 1$\frac{1}{6}$$p$. 2. $\frac{65}{315}$. 3. 2·2136 lbs.
4. 11400000 ; 17100000 ; 3800000. 5. R3930.

971. £63, £60 and £54. 3. 2·4142135...
4. 100 days. 5. 540.

ALLAHABAD ENTRANCE PAPERS.

1889-90.

12 min $40\frac{10}{11}$ sec. 2. 8 days.
$39\frac{3}{4}$ miles from the starting place. 4. 115 cu in. 5. $11\frac{9}{11}$ in.
9. 3. $64\frac{7}{16}$ gal. 4. 5. 5. 9999, $7\frac{10}{11}$.
9. 2. (a) 12, (b) ... 846714... 3. $56\frac{2}{3}$ 4. £3. 5. 1·0001.
9. 2. 2 fur 12 po 1 yd. 3. R3. 7a. $4\frac{2}{7}p$. 4. £14. 1s. $3\frac{3}{8}d$.
5. £350 11s 8d. 6. 1869. 7. 79·032; $8\frac{7}{15}$.
4. 1. (a) 999 × 807 = 806193. (b) − 1. 2. (a) ·0069. (b) 5·0596.
3. 444 mi. 4. R555.
95. 1. (b) 4 ft. 2. (a) ·0626... (b) 1·4634... 3. R5700.
4. R10 6a. 6p., R3. 7a. 6p. 5. 250 yds.
96. 1. (a) $1\frac{3}{16}$. (b) 250. 2. (a) ·020409375. (b) 200·001.
3. 178 hrs. 52 min 30 sec. 4. $7\frac{1}{7}$ miles. 5. R2793 2a.

HIGHER EXAMINATION PAPERS.

1. R12960, R11220. 2. £1239,... 3. R2...
4. 10a. 8p. 5. R6000000. 6. R27. 15a. ...p. 7. R40.
8. ... 9. ... p. c. 10. R53. 7a. $2\frac{2}{5}p$. 11. 29040.
12. R16500. 13. 9a. 14. 50. 15. 1132 gr.
16. 8 : 1. 17. 112. 18. R3; R$2\frac{1}{2}$; R2.
19. 10-30 P.M. yesterday; 10-$36\frac{3}{5}$ P.M., 45 days later. 20. B; $\frac{1}{312}$.
21. £1334. 11s ...d. 22. £200. 23. $12\frac{4}{5}$ da. 24. £1782.
26. 8 yds. 27. 18s. 28. 17s. 6d. 29. R77. 30. 18s.
31. 26800 ft. 32. 120. 33. $\frac{1}{3}$ min. 34. $7\frac{1}{2}$ min. 35. 4-25 P.M.
36. 1 : 4. 37. 8284. 38. £$2686\frac{2}{3}$. 39. $18\frac{2}{9}$ p. c.
40. 29 min. $37\frac{1}{2}$ sec nearly. 41. $33\frac{1}{3}$ p. c. 42. Just passes.
43. R2 44. £350. 45. £1650; £1540 46. £2394.
47. 72 men; 288 women. 48. Copper, 39 parts; gold 146 parts.
49. 200 c. ft. 50. 13. 51. £14700. 52. 12 p. c.
53. 12 days. 54. £$309\frac{1}{11}$, £$271\frac{17}{33}$; £$19\frac{13}{33}$.
55. 1 mi. from the summit towards A. 56. £271.
57. 8400. 58. 1510 oz. 59. £$19\frac{5}{7}$. 60. £480.
61. 13 oxen. 62. $78\frac{1}{11}$ days. 63. 33·24 in. 64. $33\frac{1}{3}s$.

CAMBRIDGE UNIVERSITY PAPERS.

1881-92.

1. £105. **2.** Brussels. **4.** 5 : 4. **5.** Loss by £11. [illegible]s.
6. 32000 ft. **7.** 2·73. **8.** £80. **10** Equal. **11** 4.
12. Loss by £3. 10s. **13.** 50 tons. **14.** 6 : 1. **15** $133\frac{13}{20}$ hrs.
16. (i) 13 ac. 1 ro. 1 po. $1\frac{?}{7}$ sq. yd. 4 sq. ft. 108 sq. in. (ii) 317[illegible]
17. $3\frac{1}{2}$ per cents. **18.** 10s. 6*d*., 9s ; 1s. 6*d*. **19** 12s., 10s. 6*d*. ; 5s. [illegible]*d*.
20. 75 p. c. **21.** £41-14-$9\frac{1}{4}$*d*. nearly. **22** $\frac{3}{4}$. **23.** 8 p. c.
24. 50 mi. per hr. **25.** £1. 2s. 2*d*. **26.** $\frac{3}{25}$. **27.** $119\frac{7}{8}$.
28. $\frac{1}{6}$. **29.** £2. 2s. **30.** 5 days. **31.** £60 : £80.
32. Incr. £18. 15s. **33.** £60 ; £65 **34** £1200.
35 £132 ; £66, £33 ; £99. **36.** £21037. 10s.
37. £208 ; £104 ; £52 ; £156. **38** £12177. **39.** $8\frac{13}{17}$ cwt.

93 (A). **1.** £3716. 8s. **2.** 105. **3.** $\frac{13}{28}$, $\frac{4}{7}$, $\frac{3}{7}$, $\frac{15}{24}$. **4** $\frac{1}{2}$; (i) $\frac{2}{3}$, (ii) 4[illegible]
5. £4 3s. $1\frac{1}{2}$*d*. **6.** ·02714, $\frac{669}{1480}$. **7.** 30 min.
8. 100 days. **9.** £433. 6s. 8*d*.
10 1494944 tons. 3 cwt. 3 qr. 20 lb. **11.** £66
(B). **1.** £4684. **2.** 105. **3.** $\frac{13}{036}$, $\frac{17}{80}$, $\frac{3}{8}$, $\frac{4}{9}$.
4. $\frac{1}{15}$; (i) $\frac{2}{3}$; (ii) $\frac{27}{74}$. **5.** £21. 6s. $1\frac{1}{2}$*d*.
6. ·00247689 ; 19·053 ; $\frac{009}{740}$. **7.** 18 min. **8.** 80 days.
9. £266. 13s. 4*d*. **10** 1349095 tons 19 cwt. 2 qr. [illegible] lb.
11. £280, £5600.

94 (A). **1.** Thirteen thousand, seven hundred and eighty-three.
2. 23712. **3.** £2. 9s. 3*d*. **4.** $\frac{3}{3?}$. **5.** 3 : 1.
6 $5·01\dot{3}$; 1500 **7.** £4897. 8s. 9*d*. **8.** £[illegible]8. 10s. 1*d*.
9. 9s. 2*d*. **10.** £23. 10s. $6\frac{5}{6}\frac{1}{4}$*d*. **11.** £[illegible]. 17s. 5*d*. increase.
(B). **1.** Thirty-three thousand, four hundred and seventy-four.
2. 11856. **3.** £17. 10s. 7*d*. **4.** $7\frac{1}{7}$ **5.** 4 : 1.
6. 3·765 ; ·002898437̇5 **7.** £6561. 16s. [illegible]*d*. **8** £434. 7s. 6*d*.
9. 14s. 7*d*. **10.** £4. 3s. $5\frac{1}{16}$*d*. **11.** [illegible]32. 15s. 1*d*. increase.

95 (A). **3.** 50 **4.** ·002197 ; $9·98\dot{4}$. **5.** £368. 19s. 11*d*.
6. 7067. **7.** £4. 16s. 6*d*. **8.** £1489. 6s. 8*d*. nearly.
9. 1-30 P.M. **10.** £229. 10s. **1**[illegible] £338. 16s. $9\frac{3}{4}$*d*.
(B). **3.** $\frac{2}{3}$. **4.** ·001728 ; $9·5\dot{3}5\dot{9}$. [illegible] £257. 16s. $9\frac{1}{2}$*d*.
6. 8404. **7.** £3. 1s. 8*d*. [illegible] £415. 5s. $0\frac{1}{4}$*d*. nearly.
9. 2-36 P.M. **10.** £364. [illegible]1. £279. 4s. $8\frac{3}{4}$*d*.

96 (A). 1. 157. 2. 640. 3. $2\frac{1}{2}$. 4. ·216 ; 14*s*. $10\frac{2}{3}$*d*.
5. £37. 7*s*. $1\frac{1}{4}$*d*. 6. £112. 19*s*. 7*d*. 7. $29\frac{3}{5}$ p. c.
8. $5\frac{5}{11}$ min. past 7. 9. $95\frac{1}{4}$. 10. 352 ft.
(B). 1. 153. 2. 66[illegible]. 3. $1\frac{2}{3}$. 4. ·164 ; 13*s*. $8\frac{2}{5}$*d*.
5. £74. 17*s*. $2\frac{7}{32}$*d*. 6. £175. 9*s*. 11*d*. 7. $23\frac{1}{18}$.
8. $49\frac{1}{11}$ min. past 3. 9. [illegible]$\frac{1}{2}$. 10. $293\frac{1}{3}$ ft.

MADRAS UNIVERSITY.

1894. 1. $\frac{1}{17}$. 2 31250 ; ·0904054. 3. 4882 tons 16 cwt. 28lbs.
4 6*a* $1\frac{1}{2}$*p* 5. R885. 14*a*. 3*p*. 6. £598-15*s*. 3*d*.
7. $38\frac{1}{8}$ p.c. 8. R3. 3*a*. 9. R38. 2*a*. 8*p*. 10. 3·36 p c.
1895. 2. $\frac{1}{81}$. 3. ·4481. 4. R3359. 2*a*. $10\frac{1}{2}$*p*.
5 R872881. 9*a*. 3*p*. nearly. 6. 2 yds. 1 ft. 7 in nearly.
7. £625. 8. R27348-12*a*. 9. R78645. 13*a*. 4*p*.
10. $5\frac{7}{16}$ p. c. 11. 12 ft. nearly.
1896. 2. $\frac{31}{70}$. 3. ·421875 ; R123. 11*a*. $3\frac{1}{9}$*p*.
4. R2,55,16,373. 9*a*. $11\frac{1}{5}$*p*. 5 114 men. 6. R1041. 10*a*.8*p*.
7. R2,20,000 stock. 8. R3. 13*a*. 6*p*. 9. 12·986.
10 R2 8*a*. 11. 70·605009.
1897. 2. $\frac{15}{16}$. 3. R3. 2*a*. 7*p*. 4. R33,862. 12*a*. $3\frac{3}{8}$*p*.
5. R430. 13*a*. 4*p* 6. R4556-4*a*. 7. R535 increase.
8 Each man R7 ; each woman R4. 6*a*., and each child R2. 10*a*.
9. 11·072 p.c. 10. R152. 9*a*. 7*p*. 11. $460\frac{1}{2}$ yds.
1898. 1. (*a*) R57,53,809. (*b*) R9,77,192. (*c*) R9,58,051 ; R9,69,783.
2. $1\frac{1}{42}$. 3. R1. 2*a*. 1*p*. ; ·0021875. 4. R545. 7*a*. $11\frac{7}{12}$*p*.
5. 6th July, mid-night. 6. R6875. 7. R1200.
8. R1. 1*a*. 11*p*. nearly. 9. 753·06 Lakhs of rupees.
10. 70848000 and 76161600. 11. 12·06.

PUNJAB UNIVERSITY.

1898. 1. 107 0129615898. 2. Monday. 3. R66. 10*a*. 8*p*. 4 1.
5. R66. 10*a*. 8*p* gain.
1899. 1. ·764. (*a*) $102·19163497809240\dot{3}\dot{8}$. 2. 6 yds.
4. 4 sovereign 6 half crown 22 shilling. 5. The former.
1900. 1. 2000·301. 2. R7352. 15*a*. nearly. 3. 10 yds by 22 yds.
1901. 1. 7 hrs. 2. £5. 0*s*. 4*d*. 3. 32490 ; 2057 ; 1·579...
4. 12 p.c. 5 79 : 49.

1902. **1.** $3 \times 5 \times 7 \times 11 \times 13 \times 37$. **2.** 5760.
3. R4800. **4** R2. 7*a* 9*p*. **5.** 15 p.c.

ALLAHABAD UNIVERSITY.

1897. **1.** 47 ; 127041. **2.** $\cdot 2\dot{8}\dot{3}$ **3.** £291. 17*s*. 6*d*. nearly.
4. £315 **5.** $22\frac{1}{4}$3 90625 Francs.

1898. **1.** (i) 2*a*. (ii) $\frac{1}{12}$ (*a*) 29. **2.** (*a*) $3\frac{2}{3}\frac{101}{162}$. (*b*) $\cdot 4\frac{1}{2}$.
3 $3\frac{1}{7}$, 001. **4.** £1. 9*s* $4\frac{1}{2}$*d*. nearly. **5.** £125.

1899. **1.** $4\frac{1}{4}$; ·63. **2.** 42 68. **3.** £425. 1*s*. $5\frac{3}{4}$*d*-nearly.
4. 60 days **5.** 2·2360 ; ·7071. **6.** $3\frac{149}{505}$*d*.
7 (*a*) 25 p c loss (*b*) 80.

1900. **1.** ·0067143... **2.** (*a*) $\cdot 2\dot{9}95670\dot{6}$. (*b*) $\frac{1}{40}$.
3. (*a*) R197 12*a*. $0\frac{3}{4}$*d*. (*b*) 3 165. **4.** R3200 ; R3889·62.
5. R289 5*a*. $0\frac{1}{2}$*p*. nearly.

1901. **1.** (*a*) $1\frac{3}{110}$ ft. (*b*) £8 1*s*. **2.** (*a*) $\frac{3}{35}$. (*b*) ·03391...
3. 160 yds. **4.** R625. **5.** R16800.

1902. **1.** ·000279 G.C.M. ; 25434·713 L.C.M. **2.** 44. **3** R328.
4. *A* 48 and *B* 84. **5.** $7\frac{7}{30}$ days. **6.** $8\frac{1}{8}$ years.